ANALOGY BETWEEN SCALAR AND VELOCITY FLUCTUATIONS IN A SLIGHTLY HEATED AXISYMMETRIC MIXING LAYER

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ABSTRACT

The paper is an experimental and analytical investigation of the analogy between the variance of the passive scalar field, θ^2 and the dynamical field, in particular the total kinetic energy q^2 (the sum of variances of the three velocity components). The analogy between the transport equation of θ^2 and q^2 is most likely to be valid under the constraint of statistical homogeneity (negligible effect of pressure) and for a Schmidt (or Prandlt) number equal to 1 (molecular or viscous diffusion predominates in the same range of scales).

Experimental data were inferred from simultaneous hot- and cold-wire measurements in a slightly heated axisymmetric shear layer, for which similar initial and boundary conditions were imposed for both θ and q (i.e., the mean temperature and velocity gradient are both present).

We show clear experimental evidence that:
i) at large scales, for which the production is dominant (either shear or mean temperature gradient), the analogy between the kinetic energy and the temperature variance is satisfactory.
ii) the smallest scales, especially for locations where the magnitude of the mean temperature and velocity gradient decreases, become shear-independent, and the analogy is not tenable.

We provide an analytical explanation for this behaviour, based on a simple model which is reasonably well-validated against experimental data. It is shown that:
i) local isotropy is not necessary for the similarity $\theta - q$ to be valid;
ii) the main factor which allows the similarity to hold is the production term in the one-point kinetic energy budget equation. When only production is present, a simple closure of this term based on a Prandtl-type model leads to simple, analytical solutions and the similarity can be explained.
iii) when other effects (e.g., decay) are present, departures from similarity can occur.

CONTEXT

It has been widely postulated that the passive scalar field θ should reveal some degree of analogy with the dynamical field since θ is simply convected by the velocity vector $\vec{u}$. Fulachier & Dumas (1976) were first to elucidate such an analogy in the slightly heated boundary layer. More precisely, they identified that the analogy has the best prospect of being satisfied when the temperature field is compared to the total kinetic energy q^2 (the sum of fluctuations of the three velocity components). This analogy was further validated from experiments in several laboratory and natural shear flows (Fulachier & Antonia (1984)).

The analytical justification for this relies on the analogy that exists in the transport equations of θ and q. As mentioned by Fulachier & Antonia (1984) and Antonia *et al.* (1997), the analogy in the transport equation of θ^2 and q^2 is however only valid under the constraint of statistical homogeneity (negligible effect of pressure) and for a Schmidt (or Prandlt) number equal to 1 (molecular or viscous diffusion predominates in the same range of scales).

Arguably, the best candidate for experimentally observing the analogy between θ and q is grid turbulence in which the scalar (temperature) is injected through the use of a heated mandoline. However, Danaila & Antonia (2009) and Danaila *et al.* (2012) demonstrated that in grid turbulence, scalar and dynamical fields do not behave similarly even though their transport equations are analogous (see also Chassaing *et al.* (2002)).

Hence, the key ingredient which results in an analogy between scalar and velocity fluctuations is apparently associated with the inhomogeneity or forcing at large scales, and specifically the mean shear which is present in most of the experiments of Fulachier & Antonia (1984) but not in grid turbulence or more generally shearless turbulence as encountered on the axis of wakes and jets. This statement is rather speculative and the conditions under which the analogy is satisfied, e.g. the specific role played by the mean shear, are worth investigating.

This is the main objective of this paper which exploits experimental data inferred from simultaneous hot- and cold-wire measurements in a slightly heated axisymmetric shear layer. The mixing layer appears to be well suited for highlighting the influence of the mean shear on the degree of analogy between statistics of θ and q since (i) it is a shear-driven turbulent flow with (ii) similar initial and boundary conditions for both θ and q, i.e. the mean temperature and velocity gradient are both present and are of same sign. (iii) The mean shear is inversely proportional to the streamwise distance. The analogy between scalar and velocity fluctuations can therefore be tested as a function of the varying mean shear magnitude.

EXPERIMENTAL RESULTS

Experimental set-up and measurements.

Experiments were performed in a mixing layer associated with a slightly heated round jet, Thiesset *et al.* (2014). The jet nozzle has a diameter of $D = 55$mm, and the jet exit velocity U_0 was set to 12.3m.s^{-1}. The corresponding Reynolds number $Re_D = U_0 D/\nu$ is $46,700$ (ν is the kinematic viscosity). The temperature excess $\theta_0 \approx 15^o C$ on the jet centerline. The ratio Gr/Re_D^2 (Gr is the Grashof number) was about 3.5×10^{-3} indicating that temperature can be considered as a passive scalar. Simultaneous velocity and temperature measurements were performed at six different downstream distances from the jet nozzle $1.5 \leq x/D \leq 4$ and for several transverse positions across the shear layer. The longitudinal u and transverse v velocity components in the x and y direction respectively were measured using a X-wire probe, consisting a two Wollaston (Pt-10%Rh) wires of diameter 2.5μm and typical length of 0.5mm. Velocities u and v were measured together with the temperature θ for which a Wollaston (Pt) wire of nominal diameter $d_w = 0.6\mu$m, was used. A square-wave injection technique was adopted for the determination of the frequency response of the cold wire.

Experimental assessment of the analogy in a slightly heated mixing layer.

The analogy between θ^2 and q^2 is assessed both in spectral and physical spaces. We first define the normalised spectra as $E_{\beta\beta} = F_{\beta\beta}/\overline{\beta^2}$. $F_{\beta\beta}$ is the power spectral density of $\beta \equiv u, v, \theta$ and the overbar denotes averaging. The spectral energy distribution of the total kinetic energy is $F_{qq} = F_{uu} + F_{vv} + F_{ww}$. For structure functions, the normalisation reads $D_{\beta\beta} = \overline{[\beta(x+r) - \beta(x)]^2}/\overline{\beta^2}$, where r is the spatial separation along the streamwise direction. The latter is calculated from temporal signals and by using Taylor's hypothesis. Since only u and v were measured, axisymmetry along the x axis is invoked, e.g. $D_{qq} = D_{uu} + 2D_{vv}$. The w spectra measured by Wygnanski & Fiedler (1970) in the mixing layer indicate that this assumption is closely satisfied over almost all wavenumbers (see their Fig. 26).

Figures 1(a) and 1(b) show spectra and structure functions of u, v, q and θ at $x = 2.5D, y = y_{0.5}$ ($y_{0.5}$ is the transverse position at which the mean longitudinal velocity $\overline{U} = U_c/2$, with U_c the mean longitudinal velocity at $y = 0$). The wavenumber and spatial separation are normalized by δ, the momentum thickness of the shear layer. In agreement with the studies of Fulachier & Dumas (1976); Fulachier & Antonia (1984), the analogy between the spectral distributions of θ^2 and q^2 is very well satisfied at nearly all scales. However, the energy distribution of θ at a given scale differs significantly from that of either u and v. This gives further strength to the appropriateness of the analogy between θ^2 and q^2. Note however that the spectral distributions of u, v, θ and q^2 collapse in a narrow range of wavenumbers ($k\delta \sim 0.5$), where a bump is easily discernible. This bump of energy is associated with the presence of coherent Kelvin-Helmholtz vortices which results from the presence of the mean shear. This underlines that the energy production mechanism of u, v and θ acts at the same wavenumber and is rather similar.

The spectra and structure functions at $y = y_{0.5}$ and $1.5D \leq x \leq 4D$ are displayed in Figs 2(a) and 2(c) together with the ratios $E_{qq}/E_{\theta\theta}$ and $D_{qq}/D_{\theta\theta}$ presented in Figs. 2(b) and 2(d). A careful analysis of Figs 2(a) and 2(c) indicates that the analogy between θ^2 and q^2 is well satisfied at all scales, in the range $1.5 \leq x/D \leq 3$, before a rather sudden change in behaviour beyond $x = 3.5D$ where the analogy is now only satisfied at large scales, but not at small scales. As indicated in Figs. 2(b) and 2(d), departures from a perfect agreement between E_{qq} and $E_{\theta\theta}$ or D_{qq} and $D_{\theta\theta}$ can be as large as 25% at small scales.

In summary, the present measurements illustrate the effect of the mean shear on the degree of analogy between q and θ. As the magnitude of the mean temperature and velocity gradient decreases, the small scales start becoming shear-independent, whilst the largest scales continue to be driven by production effects. Spectra and structure functions at different y, not presented here, corroborate this. We have thus clear experimental evidence that in flows and at scales for which the production is dominant (either shear or mean temperature gradient), the analogy between the kinetic energy and the temperature variance holds satisfactorily. Note that these results hold when r is measured along only one (virtual) spatial direction, that of the mean flow. These scales are artificially homogeneous, because they are derived from one-point temporal measurements, with the help of Taylor's hypothesis. Further limitations of this will be discussed later.

ANALYTICAL ARGUMENTS FOR ANALOGY IN A MORE GENERAL CONTEXT

The previous section was devoted to a comparison between statistics of q and θ inferred from hot-and cold-wires measurements. This is however somewhat limiting because the increment $\vec{r}$ can only be calculated from temporal measurements and Taylor's hypothesis, and therefore, $\vec{r} \parallel \vec{e}_x$, the direction of the mean flow. In the following, we provide analytical arguments for the '$\theta^2 - q^2$' analogy, for any flow, and for the general case when $\vec{r}$ is indeed a vector.

To do this, transport equations for $D_{qq}(\vec{r})$ and $D_{\theta\theta}(\vec{r})$ will be written and compared.

We assume the classical incompressibility hypothesis $\rho =$ constant, as well as $\nu =$ constant. Instantaneous Navier-Stokes equation (free from external forcing) is first written for the **total** velocity U_i, viz.

$$\frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j} = -\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\nu \tau_{ij}\right], \tag{1}$$

with $\tau_{ij} = \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}$. We now consider two points of the flow, $\vec{x}^+$ and $\vec{x}^-$, separated by the increment $\vec{r}$ such as $\vec{x}^+ = \vec{x}^- + \vec{r}$. For economy of writing, the following abbreviations are applied

$$\begin{cases} U_i^\pm = U_i(\vec{x}^\pm) \\ P^\pm = P(\vec{x}^\pm) \\ \tau_{ij}^\pm = \tau_{ij}(\vec{x}^\pm) \\ \partial_i^\pm = \partial x_i^\pm. \end{cases}$$

.

Following the procedure given in Hill (2001) or Danaila *et al.* (2004), Eq. (1) is then written at these two points, viz. in $\vec{x^+}$

$$\partial_t U_i^+ + \partial_j^+(U_j^+ U_i^+) = -\partial_i^+ P^+ + \partial_j^+(\nu^+ \tau_{ij}^+) \tag{2a}$$

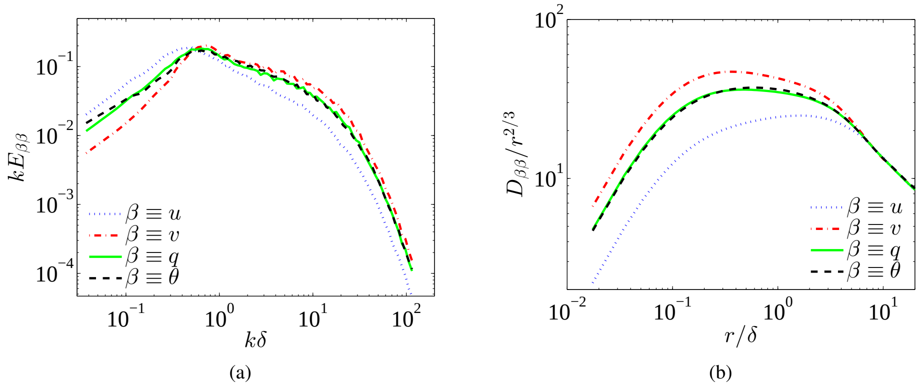


Figure 1. (a) Pre-multiplied spectra $kE_{\beta\beta}(k)$ and (b) compensated structure functions $D_{\beta\beta}/r^{2/3}$ of $\beta \equiv u, v, q, \theta$ at $x = 2.5D$ and $y = y_{0.5}$.

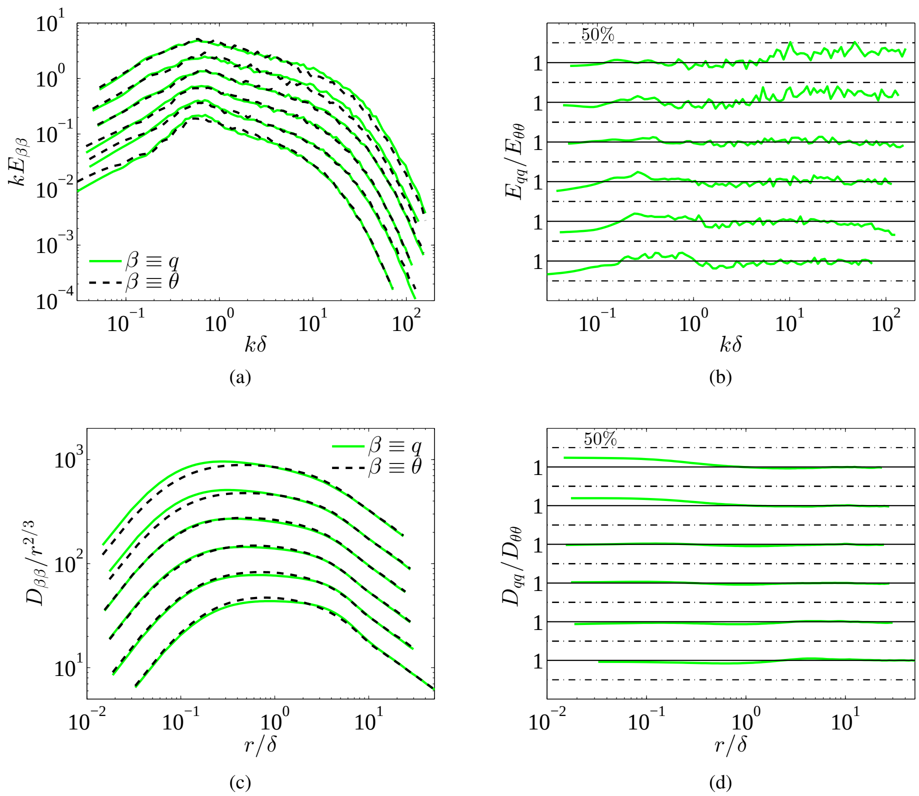


Figure 2. (a) Pre-multiplied spectra and (c) compensated structure functions of q and θ for $1.5D \leq x \leq 4D$, $y = y_{0.5}$. In (a) and (c), spectra and structure functions have been shifted upwards by a factor 2. Ratios (b) $E_{qq}/E_{\theta\theta}$ and (d) $D_{qq}/D_{\theta\theta}$ for $1.5D \leq x \leq 4D$, $y = y_{0.5}$. The dash-dotted lines correspond to a departure of 50%.

and in $\vec{x^-}$

$$\partial_t U_i^- + \partial_j^+ (U_j^- U_i^-) = -\partial_i^- P^- + \partial_j^- (\nu^- \tau_{ij}^-). \quad (2b)$$

Because $\vec{x^+}$ and $\vec{x^-}$ are independent points, then $\partial_i^+(\,)^- = \partial_i^-(\,)^+ = 0$. We further consider that the mean velocity field is sufficiently uniform for its spatial increments to be negligible compared with those of the random field, namely $\Delta\overline{U}_i \ll \Delta u_i$. Moreover, we consider the shear to be uniform at each point. Hence, subtraction of Eq. (2b) from Eq. (2a) provides

$$\frac{D}{Dt}\Delta u_i + (\partial_j^+ u_j^+ + \partial_j^- u_j^-)\Delta u_i + \Delta\left(u_j \partial_j \overline{U}_i\right) = -(\partial_i^+ + \partial_i^-)\Delta P + (\partial_j^+ + \partial_j^-)\Delta(\nu\tau_{ij}), \quad (3)$$

where $\Delta() = ()^+ - ()^-$ and $\frac{D}{Dt} \equiv \partial_t + \overline{U}_j \partial_j$ is the material derivative. Note that the mean velocity as well as its gradients with respect to a fixed, laboratory frame, are considered to be the same for the two points. This equation is then multiplied by $2\Delta u_i$ and (time) averaged. After straightforward calculations, Danaila *et al.* (2004), the final result is

$$\frac{D}{Dt}\overline{(\Delta u_i)^2}+\frac{\partial}{\partial X_j}\overline{\frac{u_j^+ + u_j^-}{2}(\Delta u_i)^2}$$
$$+\frac{\partial}{\partial r_j}\overline{\Delta u_j(\Delta u_i)^2}+2\overline{\Delta u_i \Delta u_j}\frac{\partial \overline{U_i}}{\partial x_j}$$
$$=$$
$$-2\partial_{X_i}\overline{\Delta P \Delta u_i}+2\nu\frac{\partial^2}{\partial r_j^2}\overline{(\Delta u_i)^2}$$
$$-2\overline{\varepsilon}^+ - 2\overline{\varepsilon}^-, \qquad (4)$$

where $\overline{\varepsilon}$ is the mean energy dissipation rate and superscripts $+$ and $-$ indicate values at the two points. Note that each term in Eq. (4) depends on the vector $\vec{r}$.

The transport equation for $\overline{(\Delta\theta)^2}$ is

$$\frac{D}{Dt}\overline{(\Delta\theta)^2}+\frac{\partial}{\partial X_j}\overline{\frac{u_j^+ + u_j^-}{2}(\Delta\theta)^2}$$
$$+\frac{\partial}{\partial r_j}\overline{\Delta u_j(\Delta\theta)^2}+2\overline{\Delta u_j \Delta\theta}\frac{\partial \overline{T}}{\partial x_j}$$
$$=$$
$$+2a\frac{\partial^2}{\partial r_j^2}\overline{(\Delta\theta)^2}$$
$$-2\overline{\varepsilon_\theta}^+ - 2\overline{\varepsilon_\theta}^-, \qquad (5)$$

where $\overline{\varepsilon_\theta}$ is the mean dissipation rate of the scalar variance, a is the thermal diffusivity and $\overline{T}$ is the mean temperature. For the sake of simplicity, the mean temperature gradient (as the mean velocity gradient) is considered uniform, but a similar development may be carried out for mean temperature gradients (or, shear) slightly inhomogeneous over large scales. Again, all terms depend on the vector $\vec{r}$.

The first two conditions that are required for the analogy to be valid are that the Prandtl number $Pr \equiv \frac{\nu}{a} \approx 1$, and the pressure diffusion term in Eq. (4) is negligible. The latter may be neglected in locally homogeneous flows, Hill (2001). Note that local homogeneity along any space direction is a less restrictive requirement than local isotropy. Flows developing sufficiently far from the boundaries will easily satisfy this condition, in contrast to a flow close to and along a direction normal to the wall. However, as shown by e.g. Antonia & Kim (1991) and Antonia *et al.* (2009), v contributes little to $\overline{q^2}$ near the wall and therefore the contribution $\partial_y\overline{\Delta P \Delta v}$ remains negligible. In other words, along the directions with strong inhomogeneity, the contribution to the pressure-diffusion term is expected to remain small. Therefore Eqs. (4) and (5) are mathematically analogous for locally homogeneous but anisotropic flows.

Further, three flow categories may be distinguished.

I) First, the purely decaying flows, in which the production terms are absent, such as decaying grid turbulence in which the passive scalar is injected through a mandoline. For these flows, albeit homogeneous and isotropic, the analogy between q and θ is not tenable and an explanation was provided using the energy transport equations in both spectral space (Danaila & Antonia (2009)) and real space (Danaila *et al.* (2012)). The key-point is a simple closure for the energy transferred at each scale by turbulent fluctuations, based on a characteristic time which accounts for the strain of the largest scales.

II) Second, we consider flows dominated by large-scale production effects. The decay and turbulent diffusion terms may be considered as negligible. Therefore, the equations for α ($\equiv u_i$ or θ) formally written for scales which are either within the RSR (Restricted Scaling Range) or exceed those scales in that range:

$$\frac{\partial}{\partial r_j}\overline{\Delta u_j(\Delta\alpha)^2}+2\overline{\Delta u_j\Delta\alpha}\frac{\partial \overline{M}}{\partial x_j} = -2\overline{\varepsilon_\alpha}^+ - 2\overline{\varepsilon_\alpha}^-, \qquad (6)$$

where M is the mean value of the variable α (either $\overline{U}_i$ or $\overline{T}$). Since local isotropy does not hold, each term depends on $\vec{r}$. Dividing Eq. (6) by $\overline{\varepsilon_\alpha}^+ + \overline{\varepsilon_\alpha}^-$ and using an energy transfer model (Danaila *et al.* (2012)) results in

$$\frac{\frac{\partial}{\partial r_j}\overline{\Delta u_j(\Delta\alpha)^2}}{\overline{\varepsilon_\alpha}^+ + \overline{\varepsilon_\alpha}^-} \propto \frac{Q_\alpha(\vec{r})}{\tau(\vec{r})}, \qquad (7)$$

where $Q_\alpha(\vec{r})$ is the energy of α effectively transferred at scale $\vec{r}$. For isotropic flows, this quantity was defined such as, Danaila *et al.* (2012),

$$Q_{\alpha,iso}(r) \propto r\frac{d\overline{(\Delta\alpha)^2}}{dr}. \qquad (8)$$

For anisotropic flows, a possible definition of $Q(\vec{r})$ is as follows

$$Q_{\alpha,aniso}(\vec{r}) \propto \vec{r}\nabla_{\vec{r}}\overline{(\Delta\alpha)^2}(\vec{r}). \qquad (9)$$

An experimental validation for Eq. (7) was provided in decaying grid turbulence by Danaila *et al.* (2012). Further investigations are necessary to test this model in shear turbulence.

The function $\tau(\vec{r})$ represents the characteristic time at scale $\vec{r}$ and can be defined from the local characteristic strain rate felt by each scale, which accounts for the total strain imposed by the larger scales. One expression was provided and discussed in Thiesset *et al.* (2013). Because $\tau(\vec{r})$ only depends on the velocity field, together with the hypothesis that the Prandtl number is very nearly equal to 1, it is obvious that τ is the same for both the dynamic and scalar fields, independently of whether or not local isotropy holds.

Therefore, with these hypotheses, it is straightforward to show that when the production terms are dominant, say along a particular direction y, functions $\overline{(\Delta q)^2}(\vec{r})$ and $\overline{(\Delta\theta)^2}(\vec{r})$ behave similarly if and only if functions $\overline{\Delta u\Delta v}\frac{d\overline{U}}{dy}/(\overline{\varepsilon}^+ + \overline{\varepsilon}^-)$ and $\overline{\Delta\theta\Delta v}\frac{d\overline{T}}{dy}/(\overline{\varepsilon_\theta}^+ + \overline{\varepsilon_\theta}^-)$ behave similarly. Because, from the one-point energy budget equations, $\frac{d\overline{U}}{dy}/(\overline{\varepsilon})$ and $\frac{d\overline{T}}{dy}/(\overline{\varepsilon_\theta})$ are constants, it follows that one sufficient condition for the similarity $\theta^2 - q^2$ to hold, is that functions $\overline{\Delta u\Delta v}$ and $\overline{\Delta\theta\Delta v}$ behave similarly for any increment $\vec{r}$. Evidence for the reasonable analogy between $\overline{\Delta v\Delta u}$ and $\overline{\Delta v\Delta\theta}$ is provided by Fig. 3, at different x. The analogy impairs as x decreases, most likely because the shear strength decreases, whereas the decay effect increases. These functions were calculated for separations along $\vec{r} \parallel \vec{e}_x$. Departures from a better collapse may be attributed to the presence of the decay in this flow. Had these structure functions been evaluated for real, spatial separations, possible discrepancies among them might be attributed to the spatial variation of the mean gradients

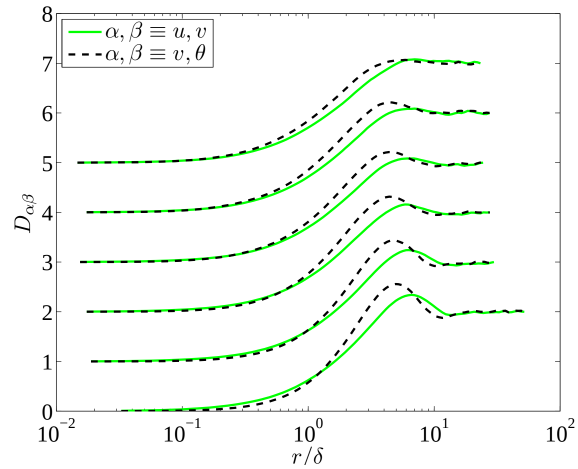


Figure 3. Distribution of $\overline{\Delta v \Delta u}$ and $\overline{\Delta v \Delta \theta}$, normalised with respect to $r^{2/3}$, as functions of r/δ, for $1.5D \leq x \leq 4D$, $y = y_{0.5}$ shifted upwards for increasing values of x.

(dU/dy and dT/dy). Because these functions are calculated from hold/cold wires measurements which are one-point measurements, these gradients are artificially constant for all scales.

Therefore, we have demonstrated, albeit after making some reasonable hypotheses, that one sufficient condition for the second-order structure functions of q and θ to be similar in flows dominated by production effects, is that functions $\overline{\Delta u \Delta v}$ and $\overline{\Delta \theta \Delta v}$ behave in a similar fashion over scales $\vec{r}$.

We now examine more closely the transport equations for $\overline{\Delta u \Delta v}$ and $\overline{\Delta \theta \Delta v}$. In flows dominated by production effects only, transport equations for these functions can be written. The methodology is classical and we will only describe it briefly. Starting from the Navier-Stokes equations, the transport equation for Δu is written and multiplied by Δv. Symmetrically, the Navier-Stokes equation for v is used to derive the transport equation for Δv which is then multiplied by Δu. Adding both equations and time averaging, the transport equation for $\overline{\Delta u \Delta v}$ is obtained. The most dominant term is $\overline{(\Delta v)^2}\frac{dU}{dy}$. A similar development for the transport equation for $\overline{\Delta \theta \Delta v}$ may be carried out and, the most dominant term is $\overline{(\Delta v)^2}\frac{dT}{dy}$. If dU/dy and dT/dy are almost constant over the scales considered, which is artificially the case when Taylor's hypothesis is used, then the forcing terms in the transport equations for $\overline{\Delta \theta \Delta v}$ and $\overline{\Delta u \Delta v}$ are proportional. Therefore, $\overline{\Delta \theta \Delta v}$ and $\overline{\Delta u \Delta v}$ behave in a similar fashion over the largest scales, at which the production term is dominant.

This result may be further reinforced if a Prandtl-type mixing-length model is written, viz.

$$\overline{\Delta u \Delta v} \propto \overline{Q_v}^{1/2} r \frac{dU}{dy}, \tag{10}$$

and

$$\overline{\Delta \theta \Delta v} \propto \overline{Q_v}^{1/2} r \frac{dT}{dy}, \tag{11}$$

where $Q_v = r\frac{d\overline{(\Delta v)^2}}{dr}$. This model brings further arguments to the analogy between $\overline{\Delta \theta \Delta v}$ and $\overline{\Delta u \Delta v}$, and hence to that between $\overline{(\Delta \theta)^2}$ and $\overline{(\Delta q)^2}$.

The model is reasonably supported by the experimental data, as illustrated by Fig. 4(a) which compares measured $\overline{\Delta v \Delta u}$ with Eq. (10). There is good agreement for all x. Similarly, Fig. 4(b) compares the measured $\overline{\Delta v \Delta \theta}$ with the distribution of $\overline{\Delta v \Delta \theta}$ obtained from Eq. (11). The agreement is reasonable for large scales, but impairs for the smallest scales.

III). Third, we consider the widest variety of flows, in which the production and the decay, as well as other effects, coexist. Therefore, the $\theta^2 - q^2$ analogy is not expected to hold perfectly. The degree with which the analogy is satisfied is reflected by the contribution of the production term to the one-point energy budget, which represents the large-scale limit for the two-point energy budget. Formally, the one-point energy budget equation for $\alpha(\equiv q^2, \theta^2)$ can be formally written as

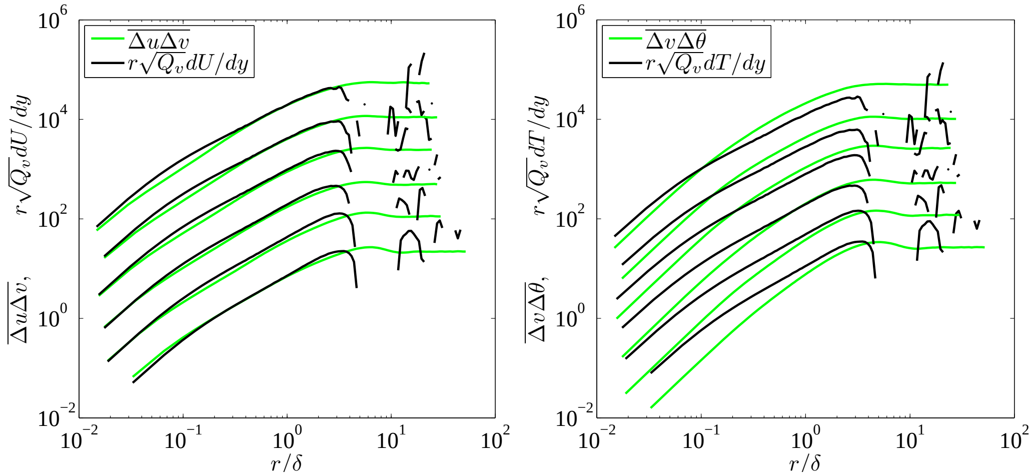
$$\overline{\varepsilon}_\alpha = Prod_\alpha + Decay_\alpha + Diff_\alpha + Molec_\alpha, \tag{12}$$

where *Prod* stands for the production, *Decay* for the decay term, *Diff* for the diffusion (turbulent and pressure-diffusion), and *Molec* represents the molecular effects.

The ratio $\mathscr{R}_\alpha = \frac{Prod_\alpha}{\overline{\varepsilon}_\alpha}$ is a relevant parameter to describe the degree at which the analogy is expected to hold. For $\mathscr{R}_{q^2} \approx \mathscr{R}_{\theta^2} \approx 1$, the production term is dominant and the analogy should hold along each direction, provided the Schmidt number is nearly 1. For small values of $\mathscr{R}_{q^2}$ and $\mathscr{R}_{\theta^2}$, the production term is negligible and the analogy cannot hold.

1 CONCLUSIONS

We presented experimental and analytical investigations of the analogy between the the passive scalar field θ and the dynamical field, in particular the total kinetic energy q^2. The analogy between the transport equations of θ^2 and q^2 is most likely to be valid under the constraint of statistical homogeneity (negligible effect of pressure) and for a Schmidt (or Prandlt) number equal to 1 (molecular or viscous diffusion dominates in the same range of scales).



(a) (b)

Figure 4. Distributions of a) $\overline{\Delta v \Delta u}$ and the proposed model, and b) $\overline{\Delta v \Delta \theta}$ and the proposed model, as a function of r/δ, for $1.5D \leq x \leq 4D$, $y = y_{0.5}$ and shifted upwards for increasing values of x.

Simultaneous hot- and cold-wire measurements were made in a slightly heated axisymmetric shear layer. They illustrated the effect the mean shear exerts on the analogy between θ^2 and q^2. As the magnitude of the mean temperature and velocity gradient decreases, the small scales become shear-independent, whilst the largest scales continue to be driven by production effects. There is clear experimental evidence to show that in flows and at scales for which the production dominates (either shear or mean temperature gradient), the analogy between the kinetic energy and the temperature variance holds reasonably well.

We have also provided analytical explanation for this behaviour, based on a simple model which is reasonably validated against experimental data. Specifically,
i) local isotropy is not necessary for the similarity $\theta^2 - q^2$ to be valid.
ii) a key factor for the similarity to hold is the presence of the production term in the one-point kinetic energy budget equation. When only production is present, a simple closure of this term based on a Prandtl-type model leads to simple, analytical solutions to be obtained.
iii) when other effects (e.g., decay) are present, departures from similarity have been observed.

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