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A FAST AND DIRECT METHOD FOR CHARACTERIZING HYDRAULIC ROUGHNESS

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ABSTRACT

We describe a fast direct numerical simulation (DNS) method that promises to directly characterize the hydraulic resistance of any given rough surface, from the hydraulically smooth to the fully rough regime. The method circumvents the unfavorable computational cost associated with simulating high-Reynolds-number flows by employing minimal-span channels (Jiménez & Moin, 1991). Proof-of-concept simulations, employing the parametric-forcing roughness model (Busse & Sandham, 2012), demonstrate that flows simulated in minimal-span channels are sufficient for capturing the downward velocity shift predicted by flows in full-span channels. Owing to the minimal cost, we are able to conduct parametric DNSs with increasing roughness Reynolds numbers while maintaining a fixed roughness height that is 40 times smaller than the half-channel height. When coupled to an unstructured-grid code, the present method promises a practical, fast and accurate tool for characterizing hydraulic resistance directly from profilometry data of rough surfaces.

INTRODUCTION

Scientists have, for years, been cataloging the relationship between surface roughness and hydraulic resistance, the former pertaining to geometry while the latter to fluid dynamics (see reviews by Jiménez, 2004; Flack & Schultz, 2010). The cataloging never ends because each rough surface is unique. In order to make predictions in full-scale conditions, it is necessary to establish the equivalent sand-grain roughness k_s of a given surface, which relates the drag increment of a given surface to an equivalent surface composed of uniform sand grains. Once k_s has been determined, it is possible to predict the drag penalty at application Reynolds numbers using either the Moody chart (Moody, 1944) for pipe and channel flows, or developments of this for boundary layers (Prandtl & Schlichting, 1955; Granville, 1958). Generally speaking, the approach to date has been to first identify a particular rough surface of scientific or engineering interest, and then to characterize its hydraulic resistance through well-controlled laboratory experiments that simulate various flow conditions. By exposing the rough surface to an increasing range of flow speeds, until such point beyond which the resistance coefficient C_f becomes constant, referred to as the 'fully rough' asymptote, it is possible to ascertain k_s. A convenient dimensionless group here is the equivalent roughness Reynolds number $k_s \equiv k_s U_\tau/\nu$, where the friction velocity, $U_\tau \equiv \sqrt{\tau_0/\rho} \equiv U_\infty\sqrt{C_f/2}$; τ_0 is the wall drag force per plan area; ρ is the density; and U_∞ is the freestream or bulk velocity; and ν is the kinematic viscosity. For accurate predictions, it is not sufficient to merely establish k_s, rather the increment of C_f caused by surface roughness must be mapped with respect to k_s^+ at all conditions from the dynamically smooth up to the fully rough regimes, which generally covers the approximate range $5 \lesssim k_s^+ \lesssim 100$. Over the last century or so, this painstaking and time-consuming procedure has been repeated for many surfaces of interest and in this way, a database of roughness is amassed in the published literature over time.

One problem here is that there is no widely applicable function that relates k_s, a dynamic parameter, to readily observable or measurable geometric properties of the surface, such as the root-mean-square roughness height k_{rms} or average roughness height k_a. Certain classes of surfaces, say sand-grain roughness, may exhibit an approximate proportionality between k_s and some physical surface length scale, but as a general rule this proportionality will not hold across different surface topologies. There are numerous attempts in the literature to formulate more complicated functions, often involving some measure of mean surface height such as k_{rms} or k_a and other properties relating to the shape or arrangement of roughness elements such as skewness, effective slope, solidity and so on (see review by Flack & Schultz, 2010). Though many of these parametrizations have some success in describing the particular class of surfaces for which they were formulated (e.g. painted or sanded surfaces), none are widely applicable across the almost limitless range of surface topologies that are encountered in engineering and meteorological applications.

The present method to directly evaluate k_s represents a paradigm shift away from such parametrizations that are based on a handful of geometrical factors. Recognizing that all roughness geometries are unique, and that a one-size-fits-all formulaic solution is proving elusive, we have sought an approach to minimize the expense involved in experimentally determining k_s. The approach relies on the direct computation of hydraulic resistance by direct numerical simulation (DNS), which we presently show can be made substantially cheaper and faster than previously thought. We circumvent the otherwise-prohibitive cost of straightforward DNS by employing minimal-span channels (Jiménez & Moin, 1991), the rationale of which is discussed in the following.

Normally, a straightforward and direct computation of roughness drag using DNS employing full-span channels is extremely expensive as it entails simultaneously capturing both the bulk flow, which scales with the half-channel height, h, and the near-wall flow around the roughness elements, which scales with the characteristic roughness height, k. Given a rough surface of fixed blockage ratio $k_s/h \lesssim 1/40$ (Jiménez, 2004), a complete characterization of hydraulic resistance requires parametric simulations that sweep through the roughness Reynolds numbers, $k_s^+ \equiv kU_\tau/\nu \approx 5$ to 100, corresponding to the hydraulically smooth and the fully rough regimes, respectively. For the blockage ratio, $h/k_s = 40$, this means performing parametric simulations at the friction Reynolds numbers, $Re_\tau \equiv hU_\tau/\nu = k_s^+(h/k_s) \approx 200$ to 4000, which are currently unfeasible for routine engineering computations. Recall that the cost of DNS, counting the number of spatial and temporal degrees of freedom, scales unfavorably as Re_τ^3 (Pope, 2000, § 9.1.2). For grids conforming to the surface of the roughness elements, this cost is further exacerbated by the need for increased mesh density, and reduced time steps.

However, the extreme cost associated with conventional DNS employing full-span channels seems unnecessary. The quantity of interest from an engineering point of view is the retardation in the mean flow over roughness relative to the smooth-wall flow. This relative flow retardation or downward velocity shift, ΔU, occurs mostly in the vicinity of the roughness layer, but holds constant above a few roughness heights, well into the log layer (if it exists) and the wake region, cf. Townsend's outer-layer similarity hypothesis (Townsend, 1976). This suggests that a simulation of only the near-wall region and its interaction with the roughness geometry is required in order to extract $\Delta U^+ \equiv \Delta U/U_\tau$, which is known as the (Hama) roughness function. This distillation of the problem is consistent with the observation that ΔU^+ does not depend on the bulk flow but only on k^+ and other details of the roughness geometry.

A framework for simulating only the near-wall dynamics is the minimal channel as first described by Jiménez & Moin (1991) and is currently receiving renewed attention in various contexts of understanding wall-bounded turbulence (Flores & Jiménez, 2010; Hwang, 2013; Lozano-Durán & Jiménez, 2014). Presently, we exploit this framework for measuring ΔU^+ by fully resolving the near-wall Navier–Stokes dynamics and its interaction with the (modeled) roughness geometry. The prohibitive cost of conventional DNS is alleviated by use of these minimal-span channels, which are designed to preclude the bulk flow that scales with h. Without the bulk flow, the cost of DNS with roughness now only scales as k_s^{+3}, which is quite feasible for the engineering task at $k_s^+ \approx 5$ to 100. In general, the computational cost is $(h/k_s)^3$ times less than that of a conventional DNS in a full-span channel.

In the remainder, we show results from simulations using the parametric-forcing roughness model of Busse & Sandham (2012) that demonstrate the veracity of the present method.

SIMULATIONS

We solve the following Navier–Stokes equations of motion between two no-slip, impermeable walls at $z = 0$ and $2h$:

$$\frac{\partial u_i}{\partial t} + \frac{\partial u_j u_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j^2} + f(t)\delta_{i1} - \alpha_i F(x_3,k) u_1 |u_1|, \quad (1)$$

$$\frac{\partial u_j}{\partial x_j} = 0, \quad (2)$$

where u_i is the velocity; t is time; x_j is the spatial coordinate; ρ is the density; p is the pressure; $f(t)$ is the spatially uniform, time-varying, pressure gradient that drives the flow at constant mass flux; and the last term on the right-hand side of (1) represents the body force due to roughness. Here (x_1,x_2,x_3) or (x,y,z) are taken as the (streamwise, spanwise, wall-normal) coordinates. The effect of any roughness geometry, which includes both pressure and viscous drag, can always be formally written as a forcing term on the right-hand side of the Navier–Stokes equation but, for the present purposes, the form adopted here is meant to represent a generic roughness in the spirit of the parametric-forcing model of Busse & Sandham (2012). The roughness forcing is active only in the streamwise direction and opposes the flow, $\alpha_i = \alpha\delta_{i1}$, and $F(x_3,k)$ is the shape function that depends on the characteristic roughness height k, chosen here to be the step function

$$F(x_3,k) = \begin{cases} 1 & x_3 < k \quad \text{or} \quad 2h - x_3 < k, \\ 0 & \text{otherwise}. \end{cases} \quad (3)$$

In general, the shape function depends on the geometry of the roughness being modeled. For example, MacDonald *et al.* (2014) show that an exponential shape function closely models a three-dimensional single-mode sinusoidal roughness. The roughness factor, α, is thought to scale with the roughness density, that is, the frontal area per unit volume (Nikora *et al.*, 2007; Busse & Sandham, 2012), measured in inverse-length (area per unit volume) units. Presently, $\alpha = 1/(40k)$ and $k = h/40$ for all the simulations. This simple roughness model has the advantage of appearing completely homogeneous to the flow over it. This idealized roughness, which retains only information about the roughness height k, will be used to show that k imposes further restrictions on the minimal span of the computational domain. Periodic boundary conditions are imposed in the streamwise and spanwise directions with respective domain sizes, L_x and L_y. The parameters for the 20 separate simulation cases for this study are documented in table 1. The streamwise and spanwise grids are uniform, the wall-normal grid is stretched with the cosine mapping and the chosen resolutions are comparable to other DNSs (Moser *et al.*, 1999; Bernardini *et al.*, 2014). The streamwise domain sizes are large enough to accommodate the near-wall streaks, which are 1000 wall units long.

RESULTS

Mean velocity profiles at fixed friction Reynolds number

The mean velocity profiles, plotted in inner wall units, $U^+ \equiv U/U_\tau$ and $z^+ \equiv zU_\tau/\nu$, from all the simulations are shown in figure 1. We first focus on figure 1(c), which

Table 1. Simulation cases listed with nominal Re_τ. Each of these 10 cases is run using smooth and rough walls, that is, a total of 20 separate simulations. In the smooth cases, $\alpha = 0$, while in the rough cases $\alpha = 1/(40k)$ and $h/k = 40$. Δz_c is the wall-normal resolution at the channel centre, where it is coarsest.

	Re_τ	L_x^+	L_y^+	N_x	N_y	N_z	Δx^+	Δy^+	Δz_c^+
Minimal	180	3707	116	384	24	192	9.7	4.8	2.9
Minimal	395	3707	116	384	24	192	9.7	4.8	6.5
Full	590	3707	1854	384	384	256	9.7	4.8	7.2
Minimal	590	3707	116	384	24	256	9.7	4.8	7.2
Full	950	5969	2985	640	640	384	9.3	4.7	7.8
Minimal	950	3581	112	384	24	384	9.3	4.7	7.8
Minimal	2000	3707	116	384	24	768	9.7	4.8	8.2
Minimal	2000	3707	232	384	48	768	9.7	4.8	8.2
Minimal	2000	3707	463	384	96	768	9.7	4.8	8.2
Minimal	4000	3707	463	384	96	1024	9.7	4.8	12.3

shows four mean velocity profiles, all at $Re_\tau \approx 590$, the two in gray from minimal-span channels with $L_y^+ \approx 116$ and the two in black from full-span channels with $L_y/h \approx 3\pi$. The two solid lines, one full-span and one minimal-span, correspond to the smooth-wall channels and the two dashed lines, one full-span and one minimal-span, correspond to the (modeled) rough-wall channels. Other parameters, including the grid resolutions and streamwise domain lengths, are held fixed. It is evident that the full-span and minimal-span simulations agree in the near-wall region, $z^+ \lesssim 46$. This has been previously shown for the smooth-wall case by Flores & Jiménez (2010); Hwang (2013). With the top of the roughness-forcing region located by the dashed vertical line at $k^+ = Re_\tau/(h/k) \approx 590/40 \approx 15$, it can be seen that the full- and minimal-span profiles also agree for the rough-wall case in the vicinity of the roughness, suggesting that the minimal flow accurately captures the essential physics of the near-wall roughness-affected region. Consistent with Flores & Jiménez (2010); Hwang (2013), the minimal-channel profiles exhibit an exaggerated wake. Further, the minimal-channel profiles diverge from the full-channel profiles at precisely the same location, $z_c^+ \approx 46$, regardless of whether the wall is rough or smooth. The agreement in the profiles below z_c suggest that the minimal flow in this region is unconstrained by the minimal span, behaving as if it were in a full-span channel. Above z_c, the minimal flow is constrained and, in the absence of eddies up to size z that typically characterize this location in full-span channels, the mixing of momentum is significantly reduced, leading to a profile with a much sharper increase in mean velocity compared to the usual log behavior in full-span channels. Interestingly the profiles in the region above z_c are both constrained in exactly the same way. This is further supported by plotting the downward velocity shifts relative to the smooth-wall case, $U_s^+ - U_r^+$, shown in the inset of figure 1(c). Above the roughness-forcing region, $z^+ \gtrsim k^+ \approx 15$, the shift is parallel and impervious to the minimal-span constraint, suggesting a kind of generalized outer-layer similarity in the sense of Townsend (1976).

Typically, for profiles at unmatched Re_τ, the Hama roughness function ΔU^+ is obtained by the shift in the log region where it is well defined. However, this is unnecessary in the present simulations at matched Re_τ since the shift $U_s^+ - U_r^+$ is constant everywhere above the roughness and is therefore well defined, so that we can unambiguously set $\Delta U^+ = (U_s^+ - U_r^+)|_{z>k}$. The behavior of $U_s^+ - U_r^+$, where it increases in where $z < k$ and remains constant where $z > k$, is consistent with the expectation that it is the dynamics of the near-wall roughness-affected flow alone that sets ΔU^+. And further, the result that $U_s^+ - U_r^+$ is virtually identical in full- and minimal-span simulations supports the idea that the minimal flow faithfully captures the relevant flow dynamics that sets ΔU^+. This demonstrates the efficacy of the present method.

Effect of span

Many of the aforementioned behaviors in figure 1(c) at $Re_\tau \approx 590$, $k^+ \approx 15$ extend to other Reynolds numbers, e.g. $Re_\tau \approx 950$, $k^+ \approx 24$ (figure 1d). In particular, the roughness-affected region is faithfully predicted by the minimal-span simulations, and ΔU^+ can be unambiguously determined by evaluating $U_s^+ - U_r^+$ wherever $z > k$ for both full- and minimal-span simulations. The minimal- and full-span profiles still diverge at $z_c^+ \approx 46$ for $Re_\tau \approx 950$, as with $Re_\tau \approx 590$ (compare figures 1c, d), because the minimal span is held fixed at $L_y^+ \approx 116$, consistent with previous studies (Flores & Jiménez, 2010; Hwang, 2013) that show $z_c \propto L_y \approx 0.3$–$0.4 L_y$. With L_y held fixed, and with increasing Re_τ, there would be a point where $k > z_c$. This suggests a criterion in order for the minimal-channel method to work as intended, namely, that k needs to be smaller than z_c. This criterion makes physical sense. When the roughness is no longer immersed in the natural unconfined flow below z_c, it follows that ΔU^+ can no longer be accurate because ΔU^+ is a measure of the interaction between the roughness and the natural unconfined flow. The simulations at $Re_\tau \approx 2000$, 4000, respectively in figures 1(e, f), with various L_y, quan-

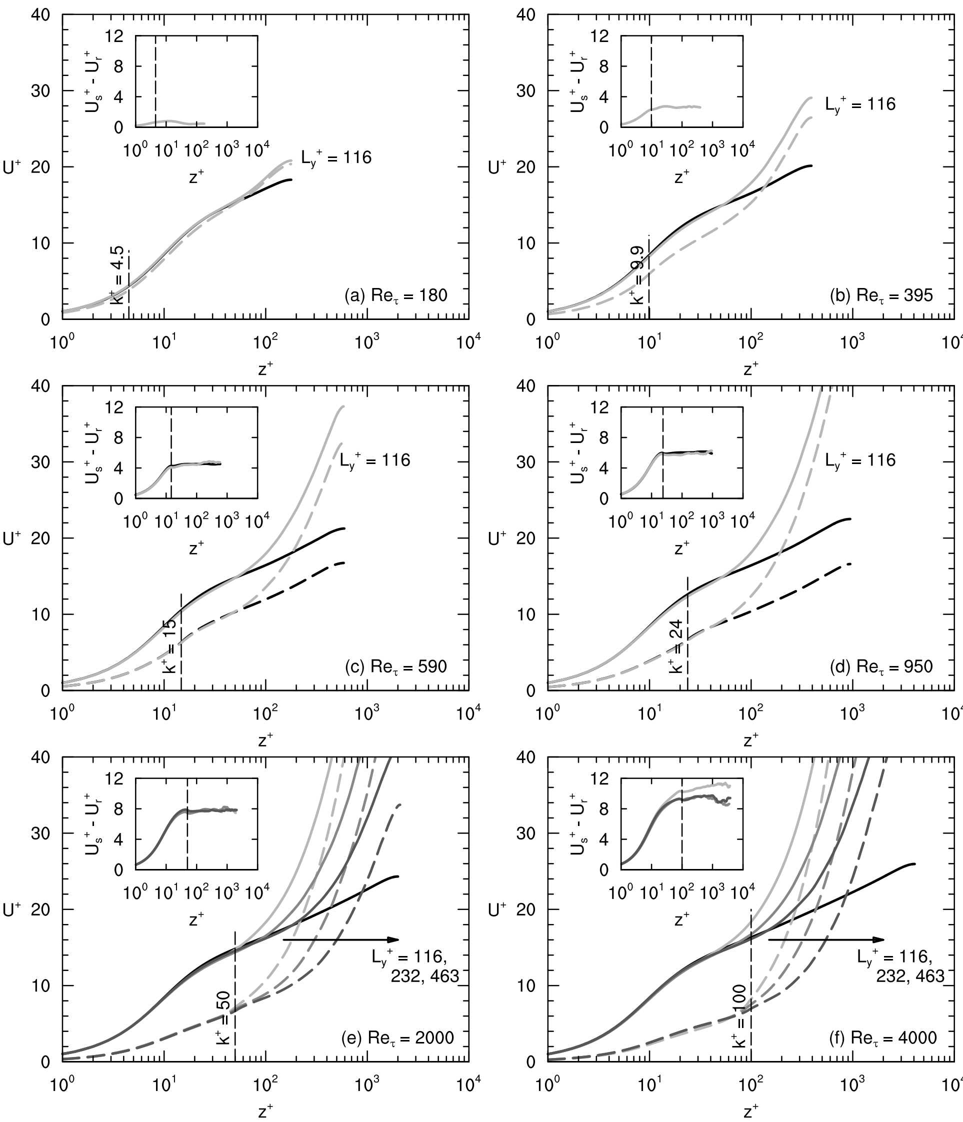

Figure 1. Mean velocity profiles of simulated turbulent channel flow at (*a*) $Re_\tau \approx 180$, (*b*) $Re_\tau \approx 395$, (*c*) $Re_\tau \approx 590$, (*d*) $Re_\tau \approx 950$, (*e*) $Re_\tau \approx 2000$ and (*f*) $Re_\tau \approx 4000$ through smooth-wall (solid) and $h/k = 40$ rough-wall (dashed), minimal-span (gray) and full-span (black) channels. The vertical dashed line marks the top of the roughness-forcing region, $z < k$. The inset shows that the velocity shift, $U_s^+ - U_r^+$, stays the same for both minimal- and full-span channels above the roughness-forcing region. The smooth-wall full-channel $Re_\tau \approx 180$, 395 profiles in (*a*, *b*) are from Moser *et al.* (1999); the smooth-wall full-channel $Re_\tau \approx 2000$ profile in (*e*) is from Hoyas & Jiménez (2006) and the smooth-wall full-channel $Re_\tau \approx 4000$ profile in (*f*) is from Bernardini *et al.* (2014).

tify this criterion. As L_y^+ increases with values 116, 232 and 463, the location where the minimal- and full-span simulations diverge, z_c^+, increases with values 46, 93, 185, that is, $z_c \approx 0.4L_y$. It is reassuring that the minimal-span smooth-wall profiles capture more and more of the full-channel profiles of Hoyas & Jiménez (2006) and Bernardini *et al.* (2014) as L_y increases (solid lines in figures 1*e*, *f*). For $Re_\tau \approx 2000$ (figure 1*e*), the thinnest minimal-span is $L_y^+ \approx 116$, corresponding to $z_c^+ \approx 0.4\,(116) \approx 46$, which is only slightly below the top of the roughness-forcing region at $k^+ \approx 50$. The criterion, $k < z_c \approx 0.4L_y$, is only slightly violated, leading to undetectable discrepancies in $U_s^+ - U_r^+$, as shown in the inset. However, for $Re_\tau \approx 4000$ (figure 1*f*), the top of the roughness forcing region $k^+ \approx 100$ is unequivocally larger than $z_c^+ \approx 46$ of the thinnest minimal channel, and clear discrepancies result in $U_s^+ - U_r^+$, as shown in the inset. Once the spans are widened to $L_y^+ \approx 232$, 463, such that $z_c^+ \approx 93$, $185 \gtrsim k^+ \approx 100$, the criterion $k < z_c$ is more or less satisfied, and $U_s^+ - U_r^+$ collapses, yielding a well-defined ΔU^+. Although the criterion, $k < z_c$, is developed here with the present modeled homogeneous roughness, where k measures the distance between the top of the roughness-forcing region and the hydraulic origin, we expect that, in practice, a conservative criterion would be $k_t < z_c$, where k_t is the maximum peak-to-valley roughness height.

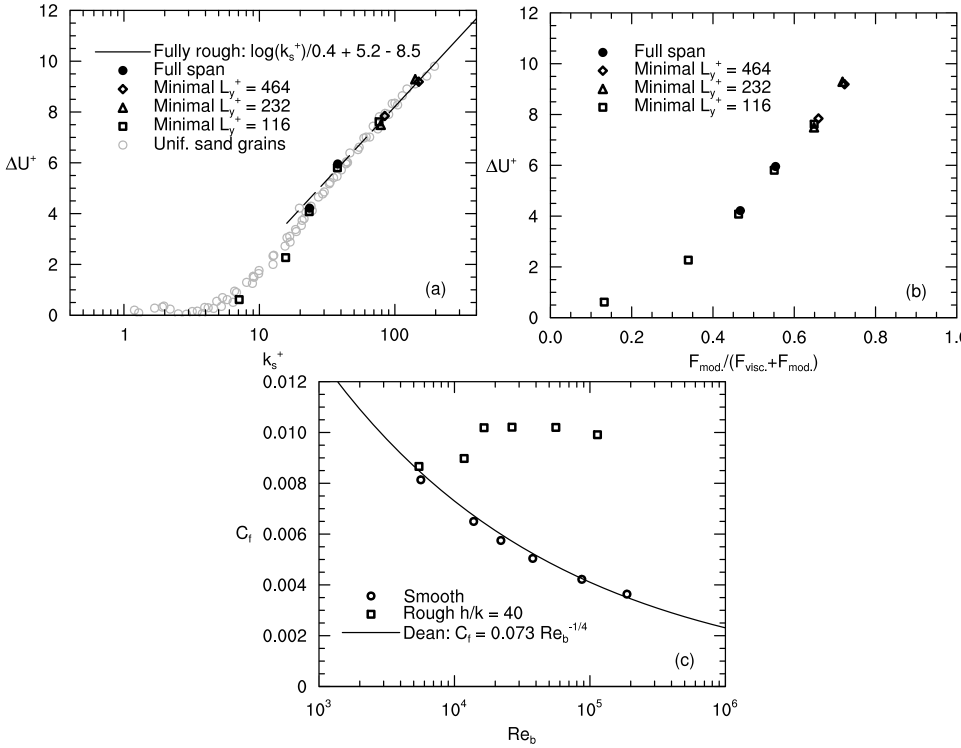

Figure 2. (*a*) Characterization of hydraulic roughness using the Hama roughness function, ΔU^+, for turbulent flow through rough channels with fixed $h/k = 40$. Increasing values of k^+ are obtained by increasing $Re_\tau = h^+$. The equivalent sand-grain roughness is $k_s = 1.6k$. The data from the uniform sand grain experiment of Nikuradse is extracted from Jiménez (2004). (*b*) Hama roughness function against modeled drag fraction. (*c*) Skin-friction coefficient, $C_f \equiv 2U_\tau^2/U_b^2$, versus bulk Reynolds number, $Re_b \equiv 2U_b h/\nu$, showing that C_f for the rough cases approach a constant for large Re_b, as expected in the fully rough regime. The simple fit for the resistance law is taken from Dean (1978).

Hama roughness function and skin-friction coefficient

A sweep in roughness Reynolds numbers, from $k^+ \approx 5$ to 100, corresponding to $Re_\tau \approx 180$ to 4000 (table 1) is needed in order to fully characterize the roughness transition, from the hydraulically smooth to the fully rough regime. The Hama roughness function, ΔU^+, for such a sweep is shown in figure 2(*a*). Only ΔU^+ from simulations satisfying $k < z_c$ are shown. Here, even the $Re_\tau \approx 4000$ case is feasible owing to the significant saving in computational cost associated with the minimal-channel technique. As a comparison, the smooth-wall full-channel profile in figure 1(*f*) requires a simulation with $8192 \times 4096 \times 1024$ grid points (Bernardini *et al.*, 2014) while the minimal-channel case at the same Re_τ with $L_y \approx 463$ only requires $384 \times 96 \times 1024$ grid points, amounting to a factor of 910 saving in number of grid points. The horizontal axis is $k_s = 1.6k$, where the constant 1.6 relates this particular (modeled) roughness height k with parameter $\alpha = 1/(40k)$ to its dynamic behavior measured against the equivalent sand-grain roughness k_s. The constant 1.6 can be determined owing to the availability of data in the fully rough regime that can be matched to the fully rough asymptote of uniform sand grains, $\log(k_s^+)/\kappa + A - 8.5$, where $\kappa \approx 0.4$, the von Kármán constant and $A \approx 5.2$, the smooth-wall log-law intercept. It is interesting to note that the top-hat version of the parametric-forcing model (Busse & Sandham, 2012) that is employed in the present study closely models the uniform-sand-grain roughness behavior in the transitionally rough regime, $4 \lesssim k_s \lesssim 70$. It is often thought that the pressure drag of the roughness elements dominates the viscous drag in the fully rough regime, $k_s^+ \gtrsim 70$–100. This idea is quantified using the present roughness model. Figure 2(*b*) shows the modeled drag fraction $F_{mod.}/(F_{mod.} + F_{visc.})$ versus ΔU^+, where $F_{mod.} \equiv \int_0^k L_y^{-1} \int_0^{L_y} L_x^{-1} \int_0^{L_x} \alpha u_1 |u_1| \,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$ and $F_{visc.} \equiv \nu \mathrm{d}U/\mathrm{d}z|_{z=0}$. The model $F_{mod.}$ can be interpreted as a pressure drag because it scales as the square of velocity. As expected, the drag partition undergoes a transition from a viscous-drag-dominated regime to a modeled-drag-dominated regime as ΔU^+ increases. However, even at $\Delta U^+ \approx 9$ corresponding to $k_s^+ \approx 160$, we observe that $F_{mod.}/(F_{mod.} + F_{visc.}) \approx 0.72$, indicating that a residual influence from viscous drag remains. However, figure 2(*c*) shows that the skin-friction coefficient, C_f, appears to already reach a constant value, independent of Re_b, a standard method used to show that a flow is in the fully rough regime. The bulk velocity, U_b, for a full-span rough channel flow is used in C_f and Re_b; U_b is readily estimated from $U_{r,full} = U_{s,full} - (U_{s,min} - U_{r,min})$, where all profiles are at matched Re_τ, which are available in this case.

CONCLUSIONS

We have presented a novel, fast and direct method for characterizing the hydraulic resistance of any given surface roughness. The way in which a particular roughness transitions from the hydraulically smooth to the fully rough regime is, to the first approximation, described by how the roughness geometry interacts with the near-wall flow. The method presented herein shows that this interaction, so far as the mean drag is concerned, is captured in the absence of the bulk flow using the idea of minimal-span channels.

The dynamic drag characterization of a rough surface is encapsulated in the ΔU^+, which we show can be accurately determined using minimal-span channels. The fact that ΔU^+ is plotted against k^+ (or k_s^+) in the literature and not against $Re_\tau = h^+$ acknowledges that ΔU^+, to a large extent, depends only on the roughness-affected near-wall flow. The minimal-span channel is a method for simulating this near-wall flow of thickness $O(k)$ without resolving the outer scale $h \gg k$, thereby breaking the curse of the (outer) Reynolds number. The savings in computational cost for the present method are possible because capturing only the near-wall flow requires far less grid points than capturing the full flow. The present method can be used to obtain the drag characteristics of many surfaces very quickly.

We propose that all the following criteria must be simultaneously satisfied when choosing the minimal span, L_y:

(i) $L_y > 100\nu/U_\tau$, the minimal span must be wide enough to accommodate the near-wall streaks;

(ii) $L_y > k/0.4$, the minimal span must be wide enough to immerse the roughness in unconfined wall turbulence; and

(iii) $L_y > \lambda_y$, the minimal span must be wide enough to capture the widest features of the roughness, λ_y.

Criterion (i) is fairly well established (e.g. Jiménez & Moin, 1991; Hwang, 2013), necessary to capture the interaction, and perhaps the destruction (Jiménez, 2004), of the near-wall streaks by the roughness elements as k^+ increases. Criterion (ii) is rigorously demonstrated in the present study, and stems from the distance-from-the-wall (z) scaling in the log region where eddies have spans or sizes, here set by the domain span L_y, that scale with z. Although not explored in this study, criterion (iii) is presumably necessary if ΔU^+ is to capture the whole effect, from the widest scales, λ_y, to the thinnest scales, of the roughness geometry.

In many ways, the present idea is not new. Large-eddy simulation (LES) directly simulates or resolves the large-scale flow that are deemed to be dependent on large-scale geometry while modeling the small-scale flow that are deemed to be universal. Here, in the case of the minimal-channel method, the idea of LES is used in reverse, and can be interpreted as small-eddy simulation (SES), a term first coined, and used in the same context, by Jiménez (2003). The minimal-channel method directly simulates or resolves the small-scale flow that are deemed to be dependent on the small-scale roughness geometry while modeling the large-scale flow that are deemed to be universal. In LES, the small scales are understood through the phenomenological theory of Kolmogorov, while in SES (minimal-channel method), the large scales are understood through the phenomenological theory of Townsend. In other words, we are directly simulating the non-universal parts of wall roughness while leaving the universal parts to be described by Townsend's outer-layer similarity hypothesis.

In Chung *et al.* (2015), the efficacy of the present method is rigorously demonstrated by comparison with DNS of explicitly gridded roughness simulated using a finite-volume code.

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