

OPEN LOOP CONTROL OF A TURBULENT BACKWARD FACING STEP BY DBD PLASMA ACTUATORS

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ABSTRACT

A non-thermal surface plasma discharge (dielectric barrier discharge) is installed at the step corner of a backward-facing step ($U_0=15$ m/s, $Re_h=30000$, $Re_\theta=1650$). Wall pressure sensors are used to estimate the reattaching location downstream of the step and also to measure the global wall pressure fluctuation coefficients. A parametric study is performed where the voltage amplitude, the burst frequency and the duty-cycle of the input high voltage are investigated separately regarding their performance. PIV is used to analyse the impact on the dynamic of the flow. The minimum reattaching position is achieved by forcing the flow at the shear layer mode through 2D periodic perturbations in the initial region at $St_h=0.25$ ($St_\theta=0.013$), where a large spreading rate is obtained by increasing the periodicity of the vortex street and by enhancing the vortex pairing process. A recirculation reduction by 22% is observed. Pressure fluctuations are maximized for an actuation at half the shear layer mode ($St_h=0.125$ or $St_\theta=0.006$). In this case, the vortex pairing goes along with sequences of single vortex growing by fluid entrainment during their convection.

I. INTRODUCTION

Despite their limitations, flow control devices are of great interest to the aeronautical industry. The number of studies investigating mechanical, fluidic or plasma actuators is increasing over the years, this confirming the needs for researches focused on turbulent flow manipulation by active control. One of the interests of plasma actuators is the principle of electro-mechanical conversion that implies fast energy transfers. By adjusting the electrical parameters of the electrical signal applied to the air-exposed electrode, the amplitude, frequency and duty-cycle of the periodic flow fluctuations induced by the plasma can be easily tuned.

The present investigation concerns the manipulation of a turbulent flow separation downstream of a backward-

facing step (BFS) by a surface plasma discharge. This simple geometry leads to a complex separated and reattaching flow, and depending on the underlying physics, different Strouhal numbers (St) can be defined. A primary periodic organization arrives from the separated shear layer that develops from the separation point. This separated shear layer evolves as a typical mixing layer in its earlier stage of formation with a typical 'shear layer mode' of instability at about $St_\theta=0.012$ (where θ is the momentum thickness of the boundary layer at the trailing edge) (Hasan, 1992). This implies the development of vortical flow structures, organized in a periodic manner, that entrain irrotational fluid from the non-turbulent regions bounding the shear layer. Initially formed at high frequency, the vortices can pair reducing the frequency signature of the vortical flow structure with the downstream position. The impact of the coherent flow structures to the bottom wall is then a periodic phenomenon (usually a value about $St_{X_r}=0.6$ is reported as in Cherry et al., (1984); Driver et al., (1987); Hudy et al., (2003) among many others, with X_r the position of the mean reattaching flow). The reattachment location is highly unsteady partially due to the periodic impact of the vortical flow structures. However, as it reported in Lee and Sung (2001) or Kya and Sasaki (1985), a flapping motion of the shear layer contributes to the widening of the separated shear layer. A flapping motion is a low frequency phenomenon with frequency signature at about $St_h\sim 0.02$. The BFS configuration is then a rich environment in terms of periodic motions and therefore it offers a lot of flow control scenarios

The present study proposes an experimental parametric investigation regarding the capability of plasma discharge for minimizing the mean flow reattachment location and maximizing the wall pressure fluctuations. The mean flow reattachment is identified by using a series of unsteady pressure sensors installed on the bottom wall of the BFS model. These sensors are also used for computing the spatial integral of the wall pressure fluctuation coefficients, defining a global estimator of the fluctuations in the flow. These two quantities define the two objectives functions which are optimized by a parametric study. The

voltage amplitude, the burst frequency and the duty-cycle of the electrical signal applied to a linear dielectric barrier discharge are the design variables of the optimization problem. When the optimized forcing conditions have been identified, time-resolved flow measurements are conducted to reveal the influence of the periodic optimized forcing on the vortex flow organization and dynamic.

EXPERIMENTAL SETUP

The model includes a step having a height, h , of 30 mm and a spanwise length of 300 mm. This model is installed in a closed-loop wind-tunnel having a moderate turbulent intensity (1%). The dimension of the test section is 300x300x1000 mm³. All measurements are performed for a free-stream velocity of 15.6 m/s. This results in a Reynolds number (based on h) of 30000. A glued zig-zag tripper (thickness of 300 μ m) is installed 10h upstream of the step corner in order to guarantee a fully turbulent boundary layer at the separation point. At the step corner, the momentum thickness is 1.65 mm ($Re_\theta=1650$) and the shape factor is about 1.6.

The step model is made of a 3-mm thick machined PMMA piece. So, the model is used as dielectric barrier directly. The electrode arrangement is introduced in Figure 1. The air-exposed and the grounded electrode have a width of 15 and 10 mm respectively and the gap between them is fixed at 2 mm. By construction, the electrode protuberance into the flow is limited to the electrode thickness (i.e. 60 μ m).

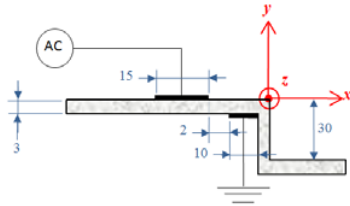


Figure 1 : Sketch of the DBD plasma actuator with details on electrode arrangement and co-ordinate system

High voltage power has a maximum output voltage of +/- 30 kV and a maximum output current for AC peak of +/- 40 mA (Trek 30/40) A signal generator card (NI, PXI-5402) is used to generate the waveform command send to the power amplifier. In the present investigations, the electrical parameters of interest are the voltage amplitude, the burst frequency and the duty-cycle. A low frequency gate function is used to trigger the ac sine voltage ($f_{ac}=2$ kHz). This gate function allows the plasma discharge to be alternatively turned off and on at frequencies in the range of the periodic fluctuations of the natural flow.

The bottom wall downstream the step has been equipped with 55 pressure taps distributed along the streamwise direction from $x/h=1$ to $x/h=9$. The distance between two successive taps is 4.5 mm resulting in a spatial resolution of 0.15h. A set of 32 unsteady pressure sensors (bandwidth of 2 kHz for max pressure of 250 Pa) are used to compute the streamwise distribution of the rms pressure fluctuations normalized by the inflow dynamic pressure.

Then, the wall pressure fluctuations are presented in coefficient form:

$$C_p = \frac{\sqrt{\frac{1}{n} [(P_1 - P_0) + (P_2 - P_0) + \dots + (P_n - P_0)]}}{0.5\rho U_0^2}$$

where P_0 and U_0 refers to the reference value of the static pressure and streamwise freestream velocity, respectively. The location of the mean reattachment point is one of the objective functions of interest in this study. Here, this location, X_R , is estimated by the position of the C_p peak along the stream wise direction on the wall (Figure 2). In order to extend the optimization to one of the unsteady components of the flow, a second objective function is considered. The spatial integral of the wall pressure fluctuation coefficient C_p is performed all along the bottom wall (grey region in Figure 2). This integral is an estimator of the global flow variations in the shear layer developing from the step corner. It is considered that the amplitude and frequency of the pressure fluctuations at the bottom wall are mainly caused by the intensity and integral length scale of the vortical flow structures embedded in shear layer and the flapping character of the reattaching flow as it is discussed in Cherry et al. (1984) and (1983). Then, higher value of C_p , and thus higher value of the C_p spatial integral, is supposed to be related to a new organization or amplification of the unsteady character of the separated flow.

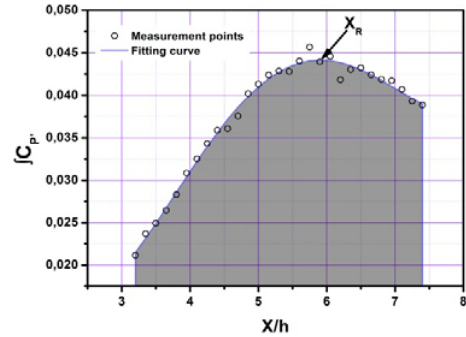


Figure 2 : Typical streamwise distribution of the wall pressure fluctuation coefficients C_p along the bottom wall downstream the step model for $U_0=15$ m/s.

This study includes time-resolved particle image velocimetry performed for the optimal control parameters. The flow downstream of the BFS is measured by a fast PIV system composed by a high-speed camera (Photron, APX-RS), a single head Nd:YLF high-speed laser (Quantronix, Darwin-Duo), a triggering unit (EG, R&D Vision) and a PC running Davis V8.2 software (Lavisision). The laser (laser sheet of 1 mm) is placed above the BFS at mid-span. The camera is operated at 2000 full frames per second at 1024x1024 pixels². The flow is seeded with atomized dielectric oil resulting in 0.5 μ m trackers. The two velocity components ($-1 < x/h < 8$, $-1 < y/h < 1.3$) are computed using a cross-correlation algorithm with adaptive multipass, interrogation windows of 64 x 64 to 16 x 16 pixels and an overlap set to 50%. This results in a spatial resolution of 2.15 x 2.15 mm² for the vector fields. Acquisitions start 10 seconds after the plasma has been turned on and they last 7.5 seconds.

II. EXPERIMENTAL RESULTS

The results of the parametric investigations using the pressure sensors as flow monitor are presented first. From this preliminary study, optimized forcing flow conditions can be defined. Then, flow measurements are conducted for the best identified forcing conditions.

II.1 Optimization by parametric study

The objective is to define the most promising actuation regarding the two objective functions (minimization of X_R and maximization of the wall fluctuating pressure) by investigating the influence of the electrical parameters of the DBD located at the step corner. The first design variable is the *burst frequency* (Figure 3). In absence of plasma discharge, the flow naturally reattaches at a mean position $X_R=5.9h$ which fully agrees with the background literature cited before and our recent LDV and PIV measurements presented in Sujar-Garrido et al. (2015).

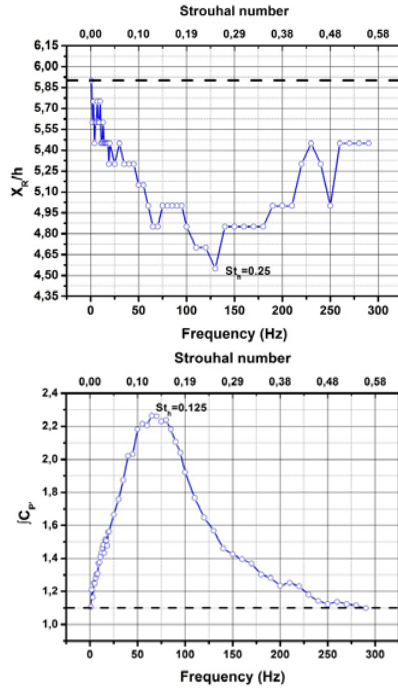


Figure 3 : Results of the parametric by burst modulation regarding the objective function related to flow reattachment location (top) and objective function about the integral of the pressure fluctuations (bottom). The voltage amplitude is 20 kV and the duty-cycle is 50%.

With the plasma discharge turned on, the recirculation bubble is systematically reduced. However, a minimal peak is found for a burst frequency set to 130 Hz ($St_h=0.25$, $St_0=0.013$). This frequency relates to the shear layer mode of instability, a mode well-known for reducing the reattachment location of separated turbulent shear layers (Chun and Sung, 1996). Here, from an initial location of 5.9h, the reattachment moves upstream to

4.55h for a forcing at the shear layer mode (reduction of the reattachment length of 22%). In a reduced manner, the flow is also sensitive to periodic excitations at the first harmonic of the shear layer mode (see the sudden peak at $St_h=0.48$ in Figure 3a). In this case of periodic forcing at the first harmonic of the most effective frequency, the recirculation length is reduced by 15%. Excitation at the fundamental scaled on the step height ($St_h=0.25$ here) is in agreement with the 0.2-0.4 range identified by Bhattacharjee et al. (1986) where the flow is forced by a loud speaker installed on the top wall of the test section. The present results are also similar to those presented in Chun and Sung (1996) that identified the optimal local forcing by loud speaker at $St_h=0.27$ ($Re_0=1470$) or with the recent results of Pourdyoussefi et al. (2014) by using linear plasma actuator on the step corner. In fact, this excitation at the shear layer mode also corresponds here to the step mode of instability of the separated shear layer as it is detailed in Hasan (1992), Chun and Sung (1996). At the initial stage of its development, the shear layer is composed of vortices separated by a short wavelength. The shear layer develops and grows by vortex pairing up to a certain spatial location. At this location, the vortex amalgamation ends leading to a final shedding frequency (as the preferred mode in jet flow) that have been called step mode in Hasan (1992). Here, despite initial turbulent conditions, the flow positively reacts to a forcing at the step mode.

A second peak reduction in X_R can be observed in Figure 3. For a periodic perturbation at 65 Hz, the recirculation bubble ends at 4.85h but the most remarkable event is the significant peak in the pressure fluctuations as it is shown by the bottom plot in Figure 3. At the best frequency (~65 Hz), the spatial pressure fluctuation is increased by 105%. This frequency corresponds to a Strouhal number $St_h=0.125$ ($St_0=0.006$, the first sub-harmonic of the shear layer mode), but also to a Strouhal number based on the reattachment location equal to $St_{XR}=0.6$. This physical frequency corresponds to the frequency of the vortical flow structures when they impact the bottom wall Chery et al. (1984); Driver et al. (1987); Hudy et al (2003). The present results demonstrate that the DBD is effective when it is operated in burst mode and the frequency of the the two most remarkable periodic forcing conditions has been identified for our experiment.

The second variable of the parametric study is the *duty-cycle value* of the burst signal. Here, it is varied from 10 to 90% while the mean reattachment and pressure fluctuations are recorded (Figure 4). The voltage amplitude is maintained at 20 kV peak while the burst frequencies are equal to 130 Hz or 65 Hz, the two optimal frequencies identified from Figure 3. At low duty-cycle, the amplitude of the superimposed periodic flow caused by the discharge is weak, therefore these duty-cycles have only minor influence on the reattachment. A same conclusion occurs when too high duty-cycle is used because in this case the actuator approaches a quasi-steady forcing. Finally, duty-cycle has to be set between 50 and 70% to minimize the size of the recirculation bubble at $St_h=0.25$. For the case of excitation at $St_h=0.125$, a single optimum is found at 50% for maximizing the pressure fluctuations.

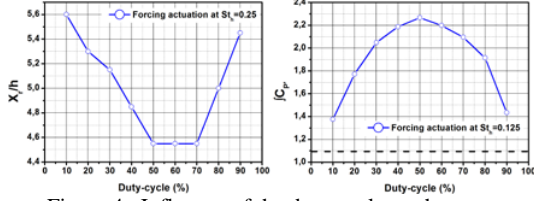


Figure 4 : Influence of the duty-cycle on the mean reattachment position (top) and the spatial integral of the pressure fluctuations (bottom) for 65 Hz ($St_h=0.125$), left and 130 Hz ($St_h=0.25$), right

The last design variable of this study is the *high voltage amplitude* used to produce the surface plasma discharge (Figure 5). At low voltage (<10-12 kV), the gas is ionized but the induced momentum transfer is not strong enough to modify the wake flow. Between 12 and 22 kV, there is a quasi linear reduction of the recirculation length X_R . The spatial integral of the pressure fluctuations increases linearly with the applied voltage beyond a certain threshold, but a saturation effect can be observed at the higher voltage amplitudes that have been tested.

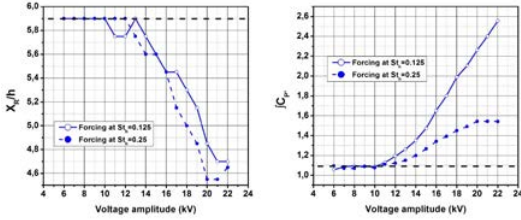


Figure 5 : Influence voltage amplitude on the mean reattachment and pressure fluctuations

II-2 Dynamics of the turbulent flow field

Two periodic perturbations are selected for maximizing the pressure fluctuations or minimizing the reattachment location, namely $St_\theta = 0.013$ ($St_h = 0.25$) and $St_\theta = 0.006$ ($St_h = 0.125$). For these conditions, the flow dynamic is detailed by a time-resolved PIV study.

To visualize the temporal evolution of the separated shear layer, the fluctuating velocity v' is extracted from the PIV fields close to the wall, at $y/h=0.05$ (Figure 6). In such a representation, the vortical flow structures presents a signature composed of alternations of positive and negative v' velocity component. In all the plots, the convection of the vortical flow structures is clearly visible. The convection velocity, U_c , is $0.55U_0$ for the natural flow and it is slightly reduced to $0.5U_0$ when the plasma discharge is applied. The plasma discharge reinforces the amplitude of the velocity fluctuations. In particular, a strong regularization of the vortical flow structures of the shear layer is observed for a periodic forcing at the shear layer mode $St_\theta = 0.013$ (Figure 6c). The time evolution of v' clearly exhibits periodic oscillations caused by the vortical flow structures passing through the measurement line in all the measurements. By forcing the inlet flow conditions with small amplitude perturbations at $St_\theta = 0.013$, the flow quickly organizes in a periodic manner (from $x/h=1$) and it exhibits a strong periodization between $x/h=2$ and $x/h=4$. Beyond the $x/h=4$ position, the

frequency signature is halved probably due by the vortex pairing process. Nevertheless, the vortical flow structures persists even downstream of the reattachment point. For this actuation, the shear layer dynamics is dominated by the perturbations imposed at the step corner. The wavelength between two successive flow structures (for $x/h>2$ and $x/h<4$) indicates a shedding frequency of about 125 Hz ($St_\theta \approx 0.013$). This frequency shows that the linear plasma discharge imposes the shedding frequency in the separated shear layer. An actuation at $St_\theta \approx 0.006$ (Figure 6b) also affects the flow dynamics, but its influence is not as clear as the regularization imposed by forcing the shear layer mode. The amplitude of the fluctuating v' velocity is clearly reinforced, even more than forcing at $St_\theta = 0.013$, but the shedding regularization is less pronounced. The flow structures increase in size when they are convected, but their mechanism of growing is not clear from Figure 6b.

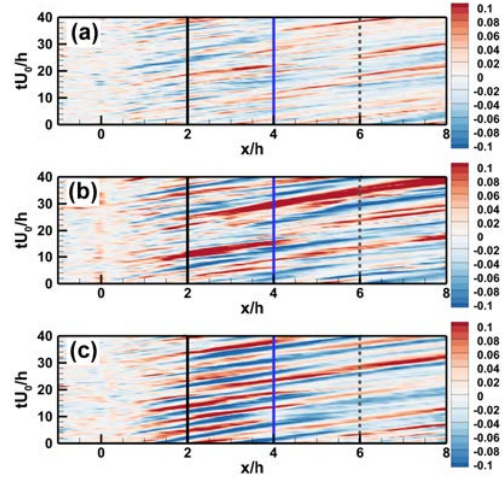


Figure 6 : Time evolution of the fluctuating velocity v' (at $y/h=0$) along the bottom wall and zoomed views at $x/h=2$, 4 and 6 for the natural flow (a), periodic forcing at $St_\theta = 0.006$ (b) and $St_\theta = 0.013$ (c).

The evolution of the frequency signature of the vortical flow structures is shown in Figure 7 by computing the power density spectra of the fluctuating component v' at three locations along the shear layer mean centreline. Unfortunately, the flapping motion of the separated shear layer cannot be clearly investigated because the low frequency fluctuations are not well resolved due to a limited recording time with PIV. However, relevant information can be extracted regarding the evolution of the flow structures when they are convected. At the beginning of the shear layer development ($x/h=2$), the turbulent energy of the baseline flow is small and no discrete frequency dominates. The periodic behaviour of the shear layer is not installed at this location. Beyond this point, the turbulent energy increases, but the frequency content of the turbulent flow is complex without noticeable most amplified mode. For the baseline flow, the vortex pairing and the shear growing by fluid entrainment coexist, but the size of the vortical flow structures is quite small resulting in a low turbulent energy.

An illustration of the vortex pairing process in case of an unforced flow is illustrated in Figure 8. By plotting the vector field in a zoomed view together with Γ_2 criterion as it is defined by Graftieaux et al. (2001), this figure shows how the vortical flow structures pair and it also demonstrates that the large scale vortices resulting from the pairing are trapped in the recirculating region.

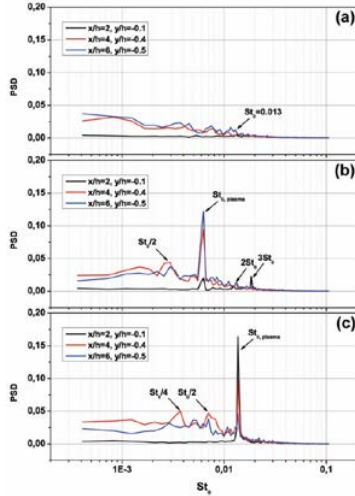


Figure 7 : Power density spectra of the fluctuating velocity component v' for the natural flow (a), forcing at $St_0=0.006$ (b) and $St_0=0.013$ (c).

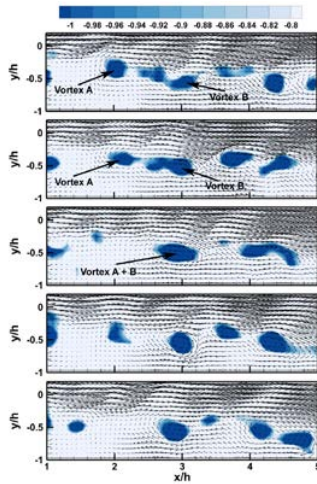


Figure 8: Illustration of the vortex pairing in the separated shear layer for the unforced flow. Each frame is separated by 2.5 ms.

With the plasma actuator operated for exciting the flow at $St_0=0.006$ (Figure 9), a periodic vortex shedding at the forcing frequency and its two first harmonics is observed in the first stage of the shear layer formation ($x/h=2$ in Figure 7). Downstream this position, a strong and sharp peak is observed at the forcing frequency, and a trace of a vortex pairing process is also visible at $St_0=0.003$. The statistical character of an analysis based on power density spectra computed over a long flow sequence indicates that the growing of the shear layer mainly relates to a pairing mechanism. However, with this forcing mode the vortical flow structures can increase in size when they are

convected as it is observed in Figure 6 by the continuous enlargement of positive and negative velocity regions at some instants. This particular growing mechanism is illustrated in the time-resolved flow sequence in Figure 8. The vortical flow structures are already formed at $x/h=1$, they are stretched during their convection but they remain coherent all along the shear layer development. In their final stage, the flow structures can have a size corresponding to the step height. Thus, the upper part of the vortical flow structure is in direct interaction with the main flow velocity that helps the flow structure to gain rotation speed and finally promotes a low resident time in the mean recirculating region. Downstream of $x/h=3$, vortex dislocation occur, the large scale vortex being separated in discrete vortices with smaller scales.

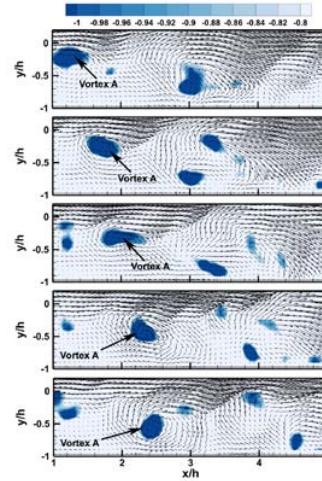


Figure 9: Illustration of the vortex pairing in the separated shear layer for periodic forcing at $St_0=0.006$.

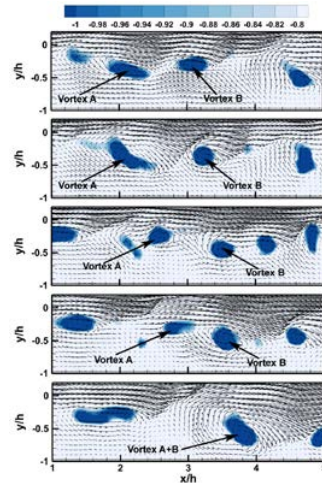


Figure 10: Illustration of the vortex pairing in the separated shear layer for periodic forcing at $St_0=0.013$.

For the actuation at the shear layer mode ($St_0=0.013$, Figure 10), the organized development of the shear layer get new insight from the power density spectra analysis. In the earlier stage of the vortex formation ($x/h=2$), the influence of the actuator is clearly evidenced

by the high amplitude peak observed at the forcing frequency. The shear layer oscillations are fully periodic without a trace of high frequency content or low frequency vortex pairing. At $x/h=4$, the peak at the forcing frequency is damped while the vortex pairing process becomes significant as demonstrated by the peaks at $St_0/2$ and $St_0/4$. The vortex pairing mechanism observed for this actuation is illustrated in Figure 8. The pairing of vortices results in a single large-scale vortical flow structure that fills the space from the bottom wall up to the upper region of the separated shear layer. As it was observed for actuation at $St_0=0.006$, the vortical structures are large and their rotation is reinforced by the entrainment in the upper region of the flow structure. Due to the size of the flow structure and the entrainment of the convective flow, the resident time in the recirculating region is reduced. Both the reduced resident time and the larger spreading of the shear layer caused by the amalgamation of adjacent vortices (Oster and Wynansky, 1982) contribute to a reduced recirculation length. Furthermore, as it is indicated in Wynant and Browand (1974), the pairing process also enhances the regions of flow trapped between two successive vortical flow structures increasing further the transfer from the non-turbulent region to the recirculation zone. The large reduction in reattachment position with the plasma discharge comes from the increase in vortex pairing caused by the local forcing at the shear layer mode. The changes in the flow dynamics and in the signature of the vortical flow transported by the convective flow are similar for a linear plasma discharge as it is used here or for sinusoidal velocity perturbations introduced by a spanwise slit placed at the step corner (Chun and Sung, 1996). This confirms further the bi-dimensional character of the actuation by a single linear DBD actuator.

III CONCLUDING REMARKS

The present experimental study interests in optimizing the influence of a linear plasma actuator on the vortex development of a separated shear layer formed in the wake of a backward-facing step. Wall pressure and time-resolved measurements are conducted to detect the mean flow characteristics and analyse the influence of the discharge on the flow dynamics.

The parametric study has shown that the recirculation bubble can be minimized by imposing 2D periodic perturbations in the initial region at the shear layer mode ($St_0=0.013$) or at the step mode $St_h=0.25$ both corresponding here to a same physical frequency of 130 Hz.. This is the most effective actuation, leading to a recirculation reduction by 22%. A second excitation mode was observed when the plasma operates at $St_h=0.125$. In this case the wall pressure fluctuation coefficients are maximized all along the bottom wall.

The most effective actuation at $St_h=0.25$ corresponds to a forcing of the shear layer mode ($St_0=0.013$) that has widely been reported as the best option for minimizing the recirculation region. This is confirmed by the present investigation where the earlier formation of the shear layer, the strong regularization of the vortex street, and the enhancement of the vortex pairing process are responsible for the shortened flow recirculation. The influence on the turbulent flow has also

been investigated for the actuation parameters optimizing the wall pressure fluctuation coefficients. This forcing ($St_0=0.006$) corresponds to the first sub-harmonic of the shear layer mode. It reduces the amalgamation of vortices and the shear may spread also by the entrainment of the external flow.

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