SELF-PRESERVATION IN ZERO PRESSURE GRADIENT TURBULENT BOUNDARY LAYERS

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ABSTRACT

Starting with the Navier-Stokes Equation (NSE), we derived the conditions for self-preservation (SP) in a zero-pressure gradient (ZPG) turbulent boundary layer. The analysis showed that it is strictly not possible to obtain SP in a ZPG turbulent boundary layer, unless the viscous term is eliminated from the NSE. This can be achieved in a smooth wall boundary layer only when the Reynolds number (Re) approaches infinity. In the case of rough walls, it is noted that the viscous effects can be compensated by surface roughness and therefore, SP is achievable, irrespective of Re. In this case, SP analysis showed that velocity scale (u^*) must be constant and the length scale (l) should vary linearly with streamwise distance (x). These SP conditions are tested using experimental data taken over a similar streamwise fetch on a smooth wall and several types of rough walls. It is observed that complete SP in a ZPG turbulent boundary layer is possible when the roughness height (k) increases linearly with x, where both the SP constraints ($u^* = U_\tau$ = constant and $l = \delta \propto x$) are met. In the present rough wall study, U_τ is observed to remain practically constant in x and $\delta \sim x$ and appears to be the next best candidate for achieving SP.

INTRODUCTION

In general, researchers strive to obtain some kind of scaling parameters based on experimental and/or numerical data that enable them to model or predict turbulence quantities in flows that are experienced in real engineering applications but are beyond the scope of testing in the existing laboratory facilities. This is not surprising considering the turbulence closure problem. There are two commonly used approaches to obtain scaling parameters and there seems to be some confusion in the turbulence research community on the difference between the two approaches. The first of the two, scaling analysis (hereafter, SA), refers to finding a length scale and a velocity scale that leads to collapse of normalised mean turbulence quantities. This is usually done in an ad hoc fashion via dimensional analysis, empirical methods, asymptotic arguments and order of magnitude arguments. The second approach, self-preservation (hereafter, SP), seeks similarity solutions based on one length scale and one velocity scale as the flow develops in the streamwise direction. For example, the mean velocity and turbulence intensity profiles should not change with x when normalised by these scales [Townsend, 1976].

Although, both these approaches aim to find scaling parameters, they are fundamentally quite different. SA relies on certain assumptions about the governing equations or experimental data over a finite range of Re, and hence the scaling parameters are not uniquely determined. This is clearly evident in the literature, where different velocity and length scales have been proposed in the past. For instance, George & Castillo [1997] suggested that free stream velocity (U_1) is the correct velocity scale by arguing that it leads to a similarity solution of the mean momentum equation in the outer region in the asymptotic limit of infinite Re. Jones *et al.* [2008] argued, again based on asymptotic arguments, that U_τ is an equally valid velocity scale. In the same line of thought, Zagarola & Smits [1998] insisted that the ratio of outer and inner velocity scales must approach a constant value at very large Re and proposed $U_1\delta^*/\delta$ as the outer velocity scale (δ and δ^* are the boundary layer thickness and displacement thickness respectively). Similarly, a wide range of length scales have been suggested aside from the conventional inner length scale (ν/U_τ; U_τ is the mean friction velocity and ν is the kinematic viscosity of the fluid) and the outer length scale (δ). For example, Rotta [1962] used dimensional arguments to replace δ by $U_1\delta^*/U_\tau$ and Weyburne [2008] reasoned that δ^* is the correct length scale based on the requirement that the area under all the scaled velocity profiles in a self-preserving flow must be equal.

On the other hand, in SP approach, the constraints on the velocity and length scales are derived after considering if the governing equations can admit an SP solution. Hence, the scaling parameters obtained through SP are unique. For instance, Townsend [1976] obtained unique scaling parameters for free shear flows, such as, jets and wakes through careful analysis of the governing equations. Another major difference between the two approaches is that SP analysis is only applicable to a spatially evolving turbulent flow whereas SA is used on data taken in the same (streamwise developing) or different wind tunnel facilities. This implies that the initial conditions and the upstream boundary conditions can affect the way in which a flow approaches SP [Townsend, 1976].

Without proper understanding about how different flows develop in the streamwise direction, it would be incorrect to compare two kinds of flows that exhibit differences in scaling, for example, smooth and rough wall flows. In smooth wall flows, fluid viscosity plays a major role in determining the dynamics of a turbulent boundary layer by providing the boundary condition to the flow, and prevents the boundary layer to achieve complete SP. On the other hand, in a rough wall flow, surface roughness can eliminate the viscous effects to different degrees depending on the geometry and dimensionality of the roughness and therefore, SP is achievable. Such a care has not been exercised in most studies where comparisons are made between smooth and rough wall flows. Consequently, it has led to the controversy over "outer-layer" similarity hypothesis - whether or not the outer region of a turbulent boundary layer is affected by surface roughness. There is currently no definite consensus on the validity of this hypothesis, [e.g. Jiménez, 2004; Antonia & Djenidi, 2010]. One can cite Grass [1971], Raupach [1981], Ligrani & Moffat [1986], Bandyopadhyay & Watson [1988], Schultz & Flack [2003], Flack *et al.* [2005], Schultz & Flack [2005], Krogstad *et al.* [2005], Bakken *et al.* [2005] and Wu & Christensen [2007] whose experiments over different rough surfaces (sand paper, mesh screen, cylinders, spheres, 2D grooves) supported the outer-layer similarity. On the other hand, Krogstad *et al.* [1992], Shafi & Antonia [1997], Tachie *et al.* [2000], Keirsbulck *et al.* [2002], Tachie *et al.* [2003], Bhaganagar *et al.* [2004] and Djenidi *et al.* [2008] found differences in the outer layer between smooth and rough walls. This state of affairs only serves to underline the importance of identifying the appropriate scaling parameters.

We attempt here to obtain scaling parameters for a ZPG turbulent boundary layer through a formal analysis based on the NSE. Our motivation here is to understand if there are any fundamental differences in how different boundary layer flows evolve towards a SP state.

CONDITIONS FOR SELF-PRESERVATION

At first, we start carrying out the SP analysis for a steady state ZPG two-dimensional turbulent boundary layer for which the momentum equation in the streamwise direction read:

$$U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y} + \frac{\partial \overline{uv}}{\partial y} + \frac{\partial(\overline{u^2} - \overline{v^2})}{\partial x} = \nu\frac{\partial^2 U}{\partial y^2} \qquad (1)$$

where U, V are the streamwise (x-direction) and wall-normal (y-direction) components of the mean velocity respectively. $\overline{u^2}$, $\overline{v^2}$ and $\overline{uv}$ are the streamwise, wall-normal and shear stresses respectively. Here, the overline represents averaging in both time and across the spanwise direction. Following Townsend's formulation for SP [Townsend, 1976], we assume:

$$U_1 - U = u^* f(\eta) \qquad (2a)$$
$$\overline{u^2} = v^{*2} g_u(\eta) \qquad (2b)$$
$$\overline{v^2} = v^{*2} g_v(\eta) \qquad (2c)$$
$$\overline{uv} = v^{*2} g_{uv}(\eta), \qquad (2d)$$

where U_1 is the freestream velocity, u^* and v^* are two scaling velocities dependent on x but not necessarily equal and $\eta = y/l$ with l a scaling length varying with x. The functions f, g_u, g_v and g_{uv} are functions dependent on η only. Substituting expressions (2a - 2d) into (1) and using the continuity equation ($\nabla . U = 0$) to solve for V, leads to, after some trivial manipulations,

$$-U_1\frac{du^*}{dx}f + U_1\frac{u^*}{l}\frac{dl}{dx}\eta f' + u^*\frac{du^*}{dx}f^2$$
$$-\frac{u^*}{l}\frac{d(u^* l)}{dx}f'\left\{\int_0^\eta f ds\right\} + \frac{v^{*2}}{l}g'_{uv} + \frac{dv^{*2}}{dx}(g_u - g_v)$$
$$-\frac{v^{*2}}{l}\frac{dl}{dx}\eta(g'_u - g'_v) = -\nu\frac{u^*}{l^2}f'' \qquad (3)$$

where the prime and double prime superscripts respectively denote the first and the second order derivatives with respect to η. Note that we do not identify u^* with the friction velocity $U_\tau = \sqrt{\tau_w/\rho}$ (τ_w and ρ are the wall shear stress and the fluid density, respectively) or U_1 nor l with δ, the boundary layer thickness.

Multiplying all the terms of (3) by l/v^{*2} makes the coefficient of g'_{uv} equal to 1, and accordingly, SP is satisfied across the entire boundary layer if,

$$\frac{U_1 l}{v^{*2}}\frac{du^*}{dx} = C_1 \qquad (4a)$$
$$U_1\frac{u^*}{v^{*2}}\frac{dl}{dx} = C_2 \qquad (4b)$$
$$\frac{u^* l}{v^{*2}}\frac{du^*}{dx} = C_3 \qquad (4c)$$
$$\frac{u^*}{v^{*2}}\frac{d(u^* l)}{dx} = C_4 \qquad (4d)$$
$$1 = C_5 \qquad (4e)$$
$$\frac{l}{v^{*2}}\frac{dv^{*2}}{dx} = C_6 \qquad (4f)$$
$$\frac{dl}{dx} = C_7 \qquad (4g)$$
$$\frac{\nu u^*}{v^{*2} l} = C_8, \qquad (4h)$$

where the constants $C_i (i = 1...7)$ are all nonzero and independent of x. Solving (4g) yields $l \propto x$. On the other hand, taking the ratio of (4b) and (4h), we get,

$$\frac{U_1 l}{\nu}\frac{dl}{dx} = \frac{C_2}{C_8}, \qquad (5)$$

which leads to a contradictory result that $l \propto x^{1/2}$. In order to avoid this inconsistency, one has to omit either of the two conditions, (4g) and (4h). In other words, one has to drop either $\frac{\partial(\overline{u^2}-\overline{v^2})}{\partial x}$ or $\nu\frac{\partial^2 U}{\partial y^2}$ in eq. (1). The former is possible when one assumes that the magnitude of $\frac{\partial(\overline{u^2}-\overline{v^2})}{\partial x}$ is negligible in comparison to the other terms of eq. (1). The latter term (viscous term) is neglected using the asymptotic argument, $Re \to \infty$. In his study, George & Castillo [1997] used both these simplifications to conclude that U_1 is the velocity scale for the outer region of a turbulent boundary layer. Since, he omitted both the conditions (4g) and (4h), George & Castillo [1997] could not obtain a constraint for the length scale. Similarly, Townsend [1976] assumed that $\frac{\partial(\overline{u^2}-\overline{v^2})}{\partial x}$ is relatively small and omitted it in his analysis. At this point, it is important to note that such assumptions about the individual terms in (1) can only give solutions to a confined region of the boundary layer and are not valid across the entire boundary layer. Accordingly, eq. (3), now becomes

$$-U_1\frac{du^*}{dx}f + U_1\frac{u^*}{l}\frac{dl}{dx}\eta f' + u^*\frac{du^*}{dx}f^2$$
$$-\frac{u^*}{l}\frac{d(u^*l)}{dx}f'\left\{\int_0^\eta f ds\right\} + \frac{v^{*2}}{l}g'_{uv} + \frac{dv^{*2}}{dx}(g_u - g_v)$$
$$-\frac{v^{*2}}{l}\frac{dl}{dx}\eta(g'_u - g'_v) = 0, \qquad (6)$$

which holds in the region of the boundary layer where the effects of viscosity are negligible, *i.e.*, outside the near-wall region. This is totally consistent with Townsend's statement that *if the motion in the viscous layer or around the roughness elements allows mean velocities and stresses of self-preserving forms, self-preservation flow may be possible over the fully turbulent part of the flow* [Townsend, 1976]. This suggests that SP is possible in a smooth wall TBL only when Re is close to infinity. In this case, the result is the same as one obtains in SA by using asymptotic arguments. It is to be noted that this result and any other asymptotic results are not very useful, at least, for modelling purposes. Besides, such asymptotic results cannot be verified using experimental data.

An alternate way to eliminate the viscous effects is to add surface roughness, where the roughness removes the viscous effects when averaged in the streamwise direction over certain distance, for example, one wavelength (λ) of the roughness elements. This implies that one can obtain SP in rough wall boundary layers, irrespective of Re. However, one has to find such a surface roughness that can exactly compensate for the viscous losses. For this, we can rely on the SP analysis to obtain conditions that aid us in constructing such a roughness geometry. At first, we recollect from the above discussion that $l \propto x$ (since viscous effects are nullified due to surface roughness). Then, taking the ratio of (4c) and (4a), we get

$$u^* = \frac{C_3}{C_1}U_1. \qquad (7)$$

The constraint (7) suggests that U_1 can be a scaling velocity as it is constant in the present case (ZPG), yielding u^* = constant. George & Castillo [1997] noted that U_1 is the correct scaling velocity for the smooth wall, however, only valid in the outer region that is beyond the region of influence of viscous effects. On the other hand, in rough wall flows, Kameda *et al.* [2008] showed that U_τ reaches a constant value after some initial development length. This implies that U_τ can be treated as u^* in the case of rough wall flows. Note that this U_τ is primarily contributed by the form drag and has no contribution from the viscous drag. In summary, one has to satisfy the conditions, u^* = constant in x and $l \propto x$ in a rough wall, for it to achieve exact SP.

TESTING IN SMOOTH AND ROUGH WALLS

The above SP conditions are tested using experimental data taken over a similar streamwise fetch on a smooth wall [Kulandaivelu, 2012] and several types of rough walls ($\lambda = 4k$, Krogstad & Antonia [1999]; $\lambda = 8k$, current study; $k \propto x$, Kameda *et al.* [2008]). Here, λ is the spacing between two adjacent roughness elements and k is the roughness height. The experimental conditions of smooth and rough wall boundary layers are summarised in table 1.

In figures 1(a & b), we compare the distributions of mean velocity and velocity defect between smooth and rough wall flows using δ and U_τ as scaling parameters. Note that, δ_{99} is used for δ in both smooth and rough wall boundary layers. For the smooth wall data, U_τ has been determined by matching the logarithmic mean velocity profile to the constants $\kappa = 0.384$ and A = 4.17 [Chauhan *et al.*, 2009]. For the present rough wall, U_τ has been calculated using the static pressure measurements around one of the roughness elements [see Kamruzzaman *et al.*, 2014, for full details] and the error in the origin (d_0) is determined by calculating the centroid of the moments of the pressure forces acting on the roughness element (see Jackson [1981]; Leonardi *et al.* [2003], for a full description of the method). Note that the results from this method have been verified against the U_τ values evaluated using the Von-Karman momentum integral equation ($C_f/2 \approx d\theta/dx$) for a ZPG turbulent boundary layer [Kamruzzaman *et al.*, 2014]. In the measurements of Kameda *et al.* [2008], U_τ has been obtained independently using a drag balance.

Looking at figure 1(a), we note that the mean profiles scaled significantly better with U_τ and δ over the rough walls in comparison to the smooth wall case. This supports our previous observation that it is not possible to obtain exact SP in a smooth wall boundary layer. The better collapse over the rough wall data implies that the SP conditions are better satisfied in rough walls. In $\lambda = 4k$ and $8k$ cases, the constraints are only approximately satisfied while they are exactly met in the rough wall with $k \propto x$. This is better illustrated in figures 2(a & b), where the trends of U_τ and δ in x are respectively plotted. It is clear that U_τ is continuously decreasing in x in $\lambda = 4k$ and $8k$ cases and therefore, SP is not completely achieved. In the case of $k \propto x$, U_τ is fluctuating about some mean value and appears to have reached a constant value just after $x = 2.2$ m. Further, there is a clear linear trend of δ in x over this roughness. Interestingly, we found that U_τ is changing at a very slow rate in x in the $\lambda = 8k$ case and $\delta \sim x$ and hence, the rough wall ($\lambda = 8k$) appears to be the next best candidate for achieving SP. This is also justified based on the results from our previous study [Kamruzzaman *et al.*, 2014], where we noted that the pressure drag over this roughness alone contributed to the total drag, suggesting that viscous effects are almost nullified in this roughness geometry.

Looking further, the velocity defect plots shown in figure 1(b) seem to indicate that U_τ and δ are good scaling parameters across most of the boundary layer over both smooth and rough surfaces. This can be understood by considering the quantity $U_1/U_\tau = \sqrt{2/C_f} = S$. Here, S can be treated as the ratio of inner and outer velocity scales, namely, U_τ and U_1 respectively. We further note that S is constant (see, eq. (7) with $u^* = U_\tau$) for a self-preserving rough wall flow and its deviation from a constant value indicates the failure of the velocity scaling over the smooth walls. Now, when we compute $(U_1 - U)/U_\tau$ along the streamwise direction, we are actually removing those differences and hence there is a better collapse of the smooth wall velocity defect plots in figure 1(b). Nonetheless, we still note some deviations in the near-wall region over the smooth wall, where the viscous effects are dominant. No such deviations are seen over the rough wall. These observations seem to suggest that the surface roughness can significantly alter the region of influence of the viscous effects

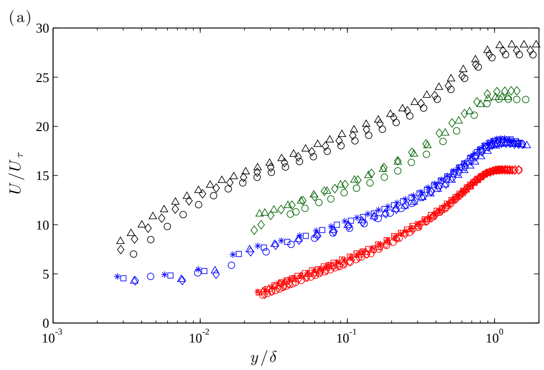

Figure 1: Comparison of (a) mean velocity and (b) velocity defect plots: smooth wall (black symbols, 1.6 m ≤ x ≤ 3.75 m, 7300 ≤ Re_θ ≤ 10850, Kulandaivelu [2012]); rough wall (green symbols, $\lambda = 4k$, 1.6 m ≤ x ≤ 2.6 m, 4000 ≤ Re_θ ≤ 5600, Krogstad & Antonia [1999]); rough wall (blue symbols, $\lambda = 8k$, 1.4 m ≤ x ≤ 2.8 m, 8100 ≤ Re_θ ≤ 14100, present study); and rough wall (red symbols, $k \propto x$, 1.56 m ≤ x ≤ 3.15 m, 4500 ≤ Re_θ ≤ 8600, Kameda *et al.* [2008]). U_τ and δ are the normalising velocity and length scales respectively. The symbols represent different streamwise locations. Note that mean velocity plots in $\lambda = 8k$ and $\lambda = 4k$ are shifted by 3 and 6 units respectively for clarity.

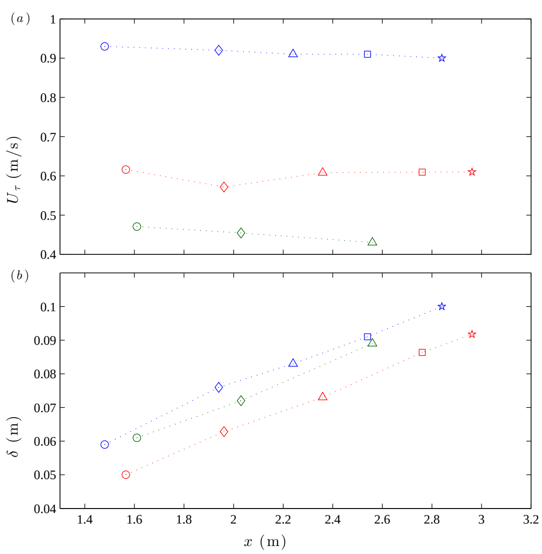

Figure 2: Streamwise variation of (a) U_τ and (b) δ in different rough walls compared in this study ($\lambda = 4k$, Krogstad & Antonia [1999]; $\lambda = 8k$, current study; $k \propto x$, Kameda *et al.* [2008]). See table 1 for symbols.

and thereby enable a flow to achieve SP.

CONCLUSIONS

An SP analysis of a ZPG turbulent boundary layer has been carried out starting with the Navier-Stokes equation. It is observed that exact SP is not possible in the case of smooth wall turbulent boundary layer, except when $Re \to \infty$. Further, our analysis showed that SP is possible over rough walls if the surface roughness can eliminate viscous effects completely. It is noted that the SP conditions are exactly satisfied when the roughness height is increasing linearly in x, which is also seen in the total collapse of mean velocity and velocity defect plots from experimental data over such a rough wall. The results from this study are significant as they give some insight into how a rough wall affects the viscous effects in the near wall region and provides a scope for further numerical/experimental studies to provide some physical explanation of this mechanism.

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x (m)	U_1 (m/s)	δ_{99} (m)	δ^* (m)	θ (m)	U_τ (m/s)	Re_τ	Re_θ	Symbol
Smooth wall								
1.6	20.11	0.047	0.0074	0.0055	0.74	2200	7030	○
2.65	20.12	0.060	0.0093	0.0069	0.72	2810	8900	◊
3.75	20.19	0.074	0.0112	0.0084	0.71	3380	10850	△
Rough wall ($\lambda = 8k$)								
1.48	13.89	0.059	0.0168	0.0091	0.93	3460	8120	○
1.94	13.82	0.076	0.0214	0.0119	0.92	4410	10500	◊
2.24	13.76	0.083	0.0222	0.0126	0.91	4480	11100	△
2.54	14.14	0.091	0.0244	0.0140	0.91	5300	12600	□
2.84	14.26	0.10	0.0268	0.0154	0.90	5830	14100	∗
Rough wall ($\lambda = 4k$)								
1.61	7.08	0.061	0.016	0.0089	0.47	1840	4005	○
2.03	7.04	0.072	0.019	0.0105	0.45	2100	4800	◊
2.56	7.02	0.089	0.022	0.0126	0.43	2450	5600	△
Rough wall ($k \propto x$)								
1.56	9.58	0.050	0.0136	0.0073	0.62	2006	4540	○
1.96	8.90	0.063	0.0168	0.0090	0.57	2520	5630	◊
2.36	9.46	0.073	0.0194	0.0106	0.61	2906	6590	△
2.76	9.48	0.086	0.0220	0.0122	0.60	3400	7520	□
3.15	9.00	0.097	0.0252	0.0139	0.61	4030	8570	∗

Table 1: Experimental conditions for the smooth wall data [Kulandaivelu, 2012] and several types of rough wall data ($\lambda = 4k$, Krogstad & Antonia [1999]; $\lambda = 8k$, current study; $k \propto x$, Kameda *et al.* [2008]).

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