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The physics of wall turbulence

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Abstract

The behaviour of wall-bounded turbulent flows is briefly reviewed, with emphasis on areas which remain open and which are distinct from the problem of turbulence in general. It is argued that the near-wall region is reasonably well understood, at least for smooth walls, but that its interactions with the outer flow are not, including the question of its asymptotic behaviour at large Reynolds numbers. The similarity properties of the logarithmic region are addressed next, in view of the recent controversy about its validity. It is concluded, from an analysis of experimental data for the fluctuation intensities, that the classical matching argument for the logarithmic law is probably correct. Finally, wall flows are identified as the seat of a second, spatial, energy cascade, different from the classical Kolmogorov one. It is conjectured that the large-scale intermittency of boundary layers might reflect a pattern-forming instability of this cascade, possibly related to certain anomalies observed in boundary layers over rough walls.

Key words: Turbulence; Wall-bounded flows; Similarity; Energy cascade.
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1 Introduction

Over the past decades a lot of work has been motivated by isotropic turbulent flows. While there is no doubt that they are important, and that they raise interesting scientific questions, the main purpose of this paper is to call attention to the related phenomenon of wall-bounded turbulence, and to the unsolved physical problems that it presents, different in nature from those of isotropic flows. It was actually in wall flows that turbulence was first recognized, and many of the earliest results refer to them [5,3]. They have continued to be the subject of intense technological attention, being closer than isotropic flows to problems of practical interest, and a lot is known about their behaviour, but their purely physical aspects have sometimes received less attention than those of the 'cleaner' isotropic case.

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The first important characteristic of shear flows in general is the presence of a mean velocity gradient, which makes the flow anisotropic and provides a continuous source of kinetic energy. Shear flows do not need large scale forcing to maintain a statistically steady state. Wall flows are also intrinsically inhomogeneous. Consider an infinitely long circular pipe. The flow is homogeneous in the axial and azimuthal directions, but the distance to the wall imposes a variable integral scale for the largest eddies, ranging from a fraction of the radius near the axis of the pipe to zero at the wall in the infinite Reynolds number limit. The energy is transferred locally from those large eddies to the Kolmogorov viscous cutoff, in much the same way as in isotropic flows, but the scale ratio across the cascade decreases as the wall is approached. Eventually, at any finite Reynolds number, the Kolmogorov and integral scales become of the same order, and the wall is isolated from the fully turbulent region by a thin viscous layer.

In equilibrium attached flows the local production of turbulent energy exceeds the local dissipation only in the viscous layer. In the core the opposite is true, and turbulence is maintained by energy that diffuses from the wall. Both regions are separated by a layer in which the production and dissipation of turbulent energy are approximately in equilibrium, and across which the flux of energy from the wall to the core is approximately constant. In the classical view, turbulence in this layer is self-similar, and the distance to the wall acts as a similarity scale, leading to a logarithmic velocity profile.

The viscous layer corresponds roughly to the dissipative range of scales of isotropic turbulence, the core plays the role of the energy-containing eddies, and the logarithmic region that of the inertial range. The classical cascade across different eddy sizes is superimposed on this spatial energy transfer. Most of the rest of this paper is devoted to exploring the similarities and differences between those two energy cascades, and their interactions.

An excellent elementary analysis of wall flows can be found in [28] while discussions of the experimental evidence are given in [23,7,29].

2 The near-wall and logarithmic regions

Very near the wall, the classical approximation is that the only relevant velocity and length scales are based on the viscosity ν, and on the friction velocity

$$u_\tau = (\tau_w/\rho)^{1/2}, \tag{1}$$

where τ_w is the friction stress at the wall, and ρ is the density of the fluid. This is essentially the Kolmogorov viscous scaling, and implies that all the near-wall observations should collapse in these 'wall' units. While this is true for some quantities, such as the longitudinal velocity fluctuations [17], there are significant residual Reynolds number effects in others, such as the wall-normal velocities. Figure 1 compiles data from boundary layers and plane channels spanning almost two orders of magnitude in Reynolds number. It is clear from it that there is considerable experimental uncertainty in the wall-normal velocity components, and that the question of whether they reach an asymptotic level at very high Reynolds numbers can still be considered open.

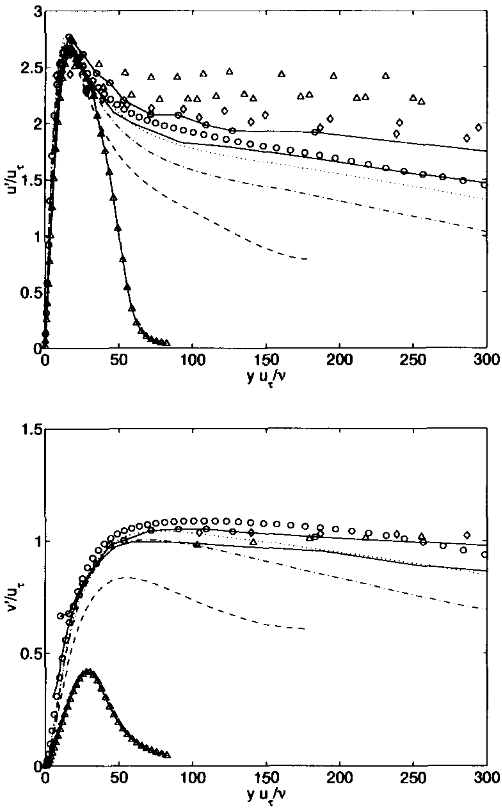

Fig. 1. R.m.s. longitudinal (top) and wall-normal (bottom) velocity fluctuations for various Reynolds numbers. Wall scaling. Lines are channels, characterized by $Re_\tau = u_\tau \delta/\nu$, where δ is the channel half-width. Symbols are boundary layers, characterized by $Re_\theta = U_\infty \theta/\nu$, where θ is the momentum thickness. To a very rough approximation $Re_\tau \approx Re_\theta/2$. ---- , $Re_\tau = 180$ [11]; —·— , $Re_\tau = 395$ [15]; ······· , $Re_\tau = 590$ [16]; —— , $Re_\tau = 708$; —○— , $Re_\tau = 1017$ [30]; —△— , $Re_\tau \approx 60$ [10, see text]; ○ , $Re_\theta = 1410$ [26]; ◇, $Re_\theta = 4981$; △ , $Re_\theta = 13052$ [25].

At issue is the influence in the near-wall region of the outer flow, where larger structures contain weaker velocity gradients but larger integrated energies. The main complication is that, while the wall-normal velocities are restricted by the presence of the wall, the same is not true for the tangential components, and large-scale tangential motions are possible even at the edge of the viscous layer. The interaction between the tangential large-scale 'inactive' motions and the wall-normal 'active' ones is not well understood.

Direct numerical simulations have revolutionized the study of turbulence at small to moderate Reynolds numbers. Not only do numerical experiments allow observations that cannot be easily done in the laboratory, but they permit 'thought' experiments in which the equations of motion, or their boundary conditions, are modified in unphysical ways to clarify particular aspects of a given phenomenon. It could argued that, as soon as a given turbulent flow becomes accessible to numerical simulations, it is likely to be fully understood within a few years. The limitation is, of course, that only simple flows at relatively low Reynolds numbers can be directly simulated with present computers [18], and that this situation is likely to last for some time.

Such numerical experiments have provided a lot of detailed information on the behaviour of the near-wall region, for which the local Reynolds numbers are low if the interaction with the outer outer flow is neglected. As we have noted above, this is a region of net turbulence production, and there is convincing evidence that it contains a self-sustaining autonomous 'engine', which can be studied in isolation by artificially suppressing the outer flow [10]. Such an 'autonomous wall' has been included in the compilation in figure 1 as the lowest Reynolds number case. In agreement with the trend of the other cases, it reproduces well the magnitude of the longitudinal fluctuations, but not that of the wall-normal ones, suggesting again that the former are not affected by the presence of an outer flow, while the latter are. It can also be shown that, if this regeneration cycle is interrupted in a turbulent channel, turbulence decays globally [9].

The mechanics of the minimal unit of this wall engine are reasonably well understood [8,24], to the point that fairly realistic low-dimensional models have been proposed [6]. It involves the interaction of low-speed streaks, which can be described as an array of long alternating streamwise jets near the wall, with shorter and more numerous quasi-streamwise vortices located near their edges. The vortices create the streaks by deforming the mean velocity profile, while the streaks eventually become unstable and regenerate the vortices. An overview of the structures present in the near-wall region can be found in [22], and [20] contains a recent collection of articles on their regeneration mechanisms.

The scaling arguments mentioned in the last paragraphs imply that, if the interaction with the outer flow can be neglected near the wall, any suitably invariant dimensionless group of variables should only be a function of $y^+ = yu_\tau/\nu$. As we move away from the wall viscosity should become less important, and the classical interpretation is that any such function should asymptote to a constant value. A particular example is the dimensionless mean velocity gradient, which should behave near the wall as

$$(y/u_\tau)\partial U/\partial y = G(y^+). \tag{2}$$

If we admit that there is a region in which (2) is still valid, but where $y^+ \gg 1$ and $G \to \kappa^{-1}$, we obtain the classical logarithmic velocity profile

$$U/u_\tau = \kappa^{-1} \log y^+ + A, \tag{3}$$

in which κ is a universal constant, and A depends on the particular boundary condition used at the wall. This equation has recently been shown experimentally to hold over three orders of magnitude in wall distance in very high Reynolds number pipes [31], but its theoretical foundation has been challenged. The new argument is that similarity does not imply that G should tend to a constant at large y^+, and that other behaviours are possible. In particular [2] suggests that $G \sim y^\alpha$ as $y \to \infty$, and derives an alternative velocity profile by fitting α to a particular set of experiments. Power laws are common as limiting behaviours, as in critical phenomena or in the anomalous scaling of structure functions in isotropic turbulence [27], but the deviations from the logarithmic law are here

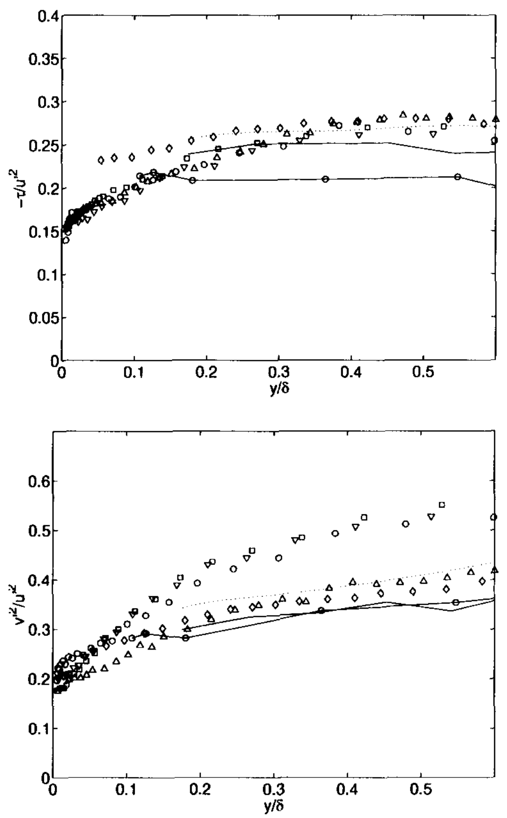

Fig. 2. Shear Reynolds stress $\langle u'v' \rangle$ (top) and wall-normal velocity fluctuations (bottom), normalized with the longitudinal velocity fluctuations. Outer scaling and only points with $y^+ > 100$. Symbols are as in figure 1, except for: ○ , $Re_\theta = 2 \times 10^4$; ▽ , $Re_\theta = 4 \times 10^4$; □ , $Re_\theta = 6 \times 10^4$ [4].

small enough that they are difficult to distinguish experimentally, and it is unlikely that the basic similarity argument can be settled from mean velocity data.

The arguments leading to (3) can be used on velocity fluctuations, and imply that ratios of different intensities, such as $\langle u'v' \rangle / \langle u'^2 \rangle$ and $\langle v'^2 \rangle / \langle u'^2 \rangle$ should be universal functions of y^+ near the wall. In the outer flow they would be functions only of y/δ, where δ is the boundary layer thickness. The overlap region would correspond to the limit $y/\delta \to 0$ of the outer flow, which should agree with the $y^+ \to \infty$ limit of the wall region, and the value of the different ratios should be universal constants. This is tested in figure 2 for boundary layers and channels over a wide range of Reynolds numbers. Only points where $y^+ > 100$ are included in the figure, which gives some support to a non-zero universal matching constant in both cases, as opposed to power-laws. This in turn supports the theoretical arguments behind (3), although it should be noted that the logarithmic behaviour of the velocity profile is observed experimentally for $y/\delta < 0.2$ in boundary layers, and for $y/\delta < 0.6$ in channels, and it is clear from the figure that the ratios are not constant, or even universal, in those ranges. It can actually be shown that, if $G(y/\delta)$ is plotted for the data in [31], it is independent of the Reynolds number but not of y/δ, as with the data in figure 2.

The conclusion is that the theoretical foundations of the logarithmic velocity law and, more interestingly, of detailed similarity, probably hold asymptotically at the wall itself, but that the impressive experimental confirmation of the former in [31] is only approximate. The problem is complicated again by the question of inner-outer layer interaction, since there is no reason for the 'inactive' motions mentioned above to be governed by y as a similarity variable, and their influence has to be considered when discussing the relevance of similarity.

3 The inverse energy cascade

A final issue which underlies most of the previous ones is the cascade mechanism by which energy is transmitted away from the wall, through the logarithmic layer, and into the core flow. There are fewer available data for energy budgets than for one-point velocity statistics, especially because the energy dissipation and pressure fluctuations are difficult to measure. The only reliable balances are those obtained from numerical simulations, and only recently have simulations become available which include short logarithmic layers.

If the velocities are separated into their ensemble averages and fluctuating parts, $u_i = U_i + u'_i$, the equation for the fluctuation energy $K = \langle u'^2_i/2\rangle$ can be written as [7,29,28]

$$(\partial_t + U_j\partial_j - \nu\nabla^2)K + \partial_j\phi_j = -\langle u'_i u'_j\rangle\partial_j U_i - \varepsilon, \tag{4}$$

where $\varepsilon = \langle(\partial_j u'_i)^2\rangle$ is the viscous dissipation. The first term in the right-hand side can be interpreted as the local production of turbulent energy, and

$$\phi_j = \langle u'_j(p' + u'^2_i/2)\rangle, \tag{5}$$

is a spatial energy flux. In parallel flows the only surviving flux is the cross-stream component, and figure 3 presents energy balances for three numerically simulated channels at increasingly high Reynolds numbers. It is clear that, at least in this Reynolds number range, the only net production of turbulent energy happens deep in the viscous layer, and that everywhere else dissipation dominates. This is best seen in the flux, which is positive (outwards) everywhere except very near the wall. In an intermediate region the flux is almost constant, especially at the highest Reynolds number, implying that production and dissipation are locally in equilibrium. This observed flux is a small fraction of the total local energy production, and instantaneous statistics show that it is the residue of a large-scale sloshing of energy towards and away from the wall, which is an order of magnitude larger than the mean. It is known that production and dissipation are the dominant term in the logarithmic layer [28], but similarity only requires that one should be proportional to the other. The reason why they are so exactly balanced in that region, even in the presence of a substantial spatial flux, can only be understood by requiring similarity for the energy flux. Momentum conservation implies that the shear stress $\tau = -\langle u'_1 u'_2\rangle$ is constant near the wall. If the flow is self-similar, the ratio $\phi_2/\tau^{3/2}$ should be constant, implying that energy flux itself should be constant and that production and dissipation

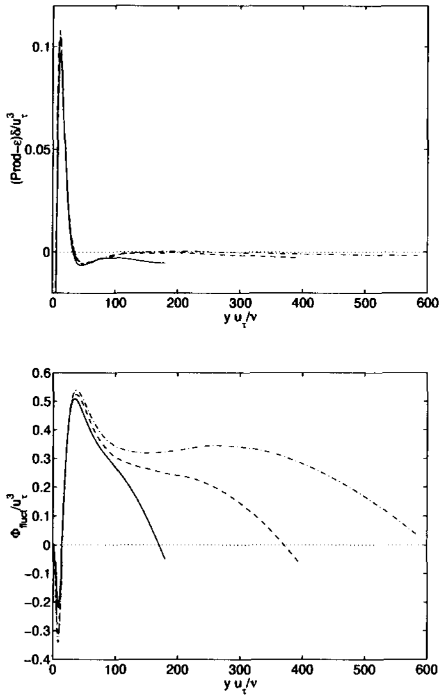

Fig. 3. (Top) excess local energy production, $-\langle u'v'\rangle \partial U/\partial y - \varepsilon$, and (bottom) turbulent energy flux, ϕ_2. Wall scaling. Numerical channels —— : $Re_\tau = 180$ [11]; ---- : $Re_\tau = 395$ [15]; —·— : $Re_\tau = 590$ [16].

should balance in detail. Note that this argument is subject to the same remarks made regarding figure 2.

In the central part of the channel dissipation exceeds production, which vanishes at the centre, and most of the turbulent energy in the central 30% of the channel is provided by the flux from the wall.

Because the energy-containing scales are small near the wall, while those at the core are large, this is an example of an inverse cascade in which, since the constant shear stress implies a constant momentum flux across the logarithmic layer, the transfer is as much of momentum as of energy.

The conceptual picture is that of a cascade organized by wall distance and by eddy size, where energy is transferred to smaller scales at any given location, and to larger ones away from the wall. Since it is known that the smallest scales of the flow are isotropic and carry no mean shear stress, it is only the larger ones that can be responsible for the momentum cascade. The latter therefore occurs in a finite range of longest wavenumbers at any non-zero distance from the wall, but that range becomes infinite with the Reynolds number as the wall is approached. Because of the different fluxes which are being transferred, the spectral slope of this cascade is different from the Kolmogorov one. In the small wavenumber limit, $ky \ll 1$, in which the structures are much larger than the distance from the wall, any dependence on y should drop out, and the spectrum can only depend on the momentum flux u_τ^2 and on the wavenumber k. The only dimensionally consistent combination is

$$E(k) \sim u_\tau^2 k^{-1}. \tag{6}$$

The range in which this formula is valid can be estimated from considerations of time scales as $k\delta > O(1)$ and $ky < O(1)$, where δ is the boundary layer thickness. Scales much larger than those are impossible because they don't fit in the boundary layer, while smaller ones do not interact with the wall and break down into smaller eddies by the usual Kolmogorov cascade. A fuller discussion, together with supporting experimental evidence, can be found in [19,13]. The k^{-1} spectrum is especially prominent in the longitudinal velocity fluctuations, and in the cospectrum of the shear stress $E_{12}(k_1)$ [21]. The k^{-1} range grows longer as the wall is approached, in agreement with the previous discussion, and the transition to the Kolmogorov regime is near $ky = 1$ in boundary layers. The k^{-1} range is less clear in the few available spectra for the spanwise velocity fluctuations, and it is almost fully absent from the wall-normal ones. This is presumably due to that, in the latter case, the integral scale is constrained by the presence of the wall and is never much larger than y, while the spanwise fluctuations carry no mean turbulent stress and do not therefore participate in the momentum cascade. Reference [19] includes an argument for the k^{-1} spectrum which is different from the one given here, but which is also equivalent to a momentum cascade.

Note that the previous discussion, and in particular the fact that the k^{-1} cascade is restricted to wavenumbers such that $ky < 0(1)$, imply that the momentum cascade is carried by eddies which are larger than the distance to the wall, and correspond to the 'inactive' motions previously mentioned, in spite of the implications of their name.

The two cascades are sketched in figure 4, with energy flowing locally from larger to smaller scales, and spatially towards larger scales farther away from the wall. Even if the second cascade is confined to the largest scales, the total number of degrees of freedom involved is potentially infinite. If we assume, for example, that the scales carrying the stresses are of the order of the wall distance y, most of the anisotropic degrees of freedom are localized close to the wall, where $y = O(\nu/u_\tau)$, and their number per unit volume is

$$N_{LS} \sim Re_\tau^2. \tag{7}$$

This increases with the Reynolds number only slightly more slowly than the *total* number of turbulent degrees of freedom, $N \sim Re^{9/4}$.

Note that the two transfer diagrams in figure 4 are equivalent, and that there is no implication that small eddies created near the wall survive to diffuse to the core region and transfer their energy there. Note also that, if the spatial cascade is restricted to eddies larger than $1/y$, the implied geometry is the one in the bottom part of figure 4, rather than one of freely 'floating' eddies across the boundary layer. This 'attached eddy' picture was first proposed by Townsend [29].

There are interesting questions regarding the presence or absence of intermittency in the Townsend cascade. The experimental evidence is that the large-scale velocity fluctuations are intermittent, in the sense that laminar regions alternate with turbulent ones [7, p. 586],

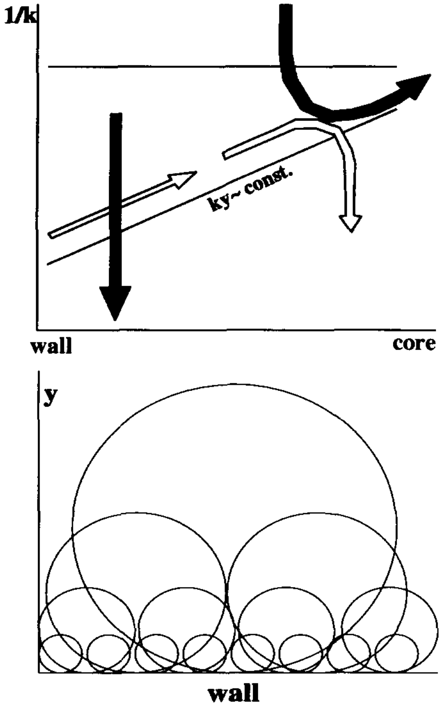

Fig. 4. Sketch of the two energy cascades in wall flows

but there is little evidence for Reynolds number dependence of this phenomenon, which is undoubtedly closer to large-scale pattern formation than to the small-scale intermittency of the velocity gradients in the direct energy cascade. If that were true, the inverse cascade mechanism would be due to the collective interaction of many small-scale structures near the wall, which could be modulated by the effect of the large-scale core structures passing over them. This process is probably not universal, since large scales seldom are, and since it would also depend on the details of the turbulence generation processes near the wall.

The autonomous wall cycle mentioned in the previous section applies to smooth walls, but many walls of practical interest are hydrodynamically rough. The near-wall mechanisms in this case are very different from the finely tuned regeneration cycle of smooth walls, at least at wall distances comparable to the roughness height, and fluctuation energy is injected into the flow directly from the roughness elements.

The classical view is that the effect of roughness is restricted to the neighbourhood of the wall and affects only the additive constant in (3), but there is some contrary experimental evidence suggesting that the detailed dynamics of the fluctuations in the logarithmic and core regions are different for smooth and rough walls [12]. The claim is, for example, that the correlation lengths of the outer flow structures are much shorter in rough boundary layers than in smooth ones. It is difficult to account for such a result in the classical model of a one-directional cascade, but it looks more natural in the pattern formation context mentioned above. If near-wall structures were modulated by the outer flow, it is easy to visualize how a large number of near-wall oscillators, representing the smooth wall regeneration cycle, could organize themselves into patterns with length scales comparable to the boundary layer thickness, which is the observed scale for outer-wall intermittency. Rough layers, whose near-wall mechanism is probably more akin to a random forcing, would either not self-organize or organize differently. This argument also implies that the detailed organization of pipes and channels, whose outer flows are influenced by the presence of opposing walls, should be different from those of boundary layers. While those arguments are suggestive, the experimental results of [12] require independent confirmation.

4 Conclusions

We have seen various open problems in the theory of wall-bounded turbulent flows. Foremost is that of inner-outer flow interaction, which implies the question of asymptotic Reynolds number independence.

We have also discussed the presence of a spatial energy cascade from the wall towards the outer flow. This is distinct from the classical Kolmogorov cascade and coexists with it, but it is a reverse cascade from smaller to larger eddies. It has been shown to be essentially equivalent to Townsend's 'attached eddy' hypothesis [29]. We have argued that such inverse cascades can only be intermittent as a result of pattern-forming instabilities, mediated by long-range collective interactions through their largest members, and we have conjectured that such an instability could explain certain anomalies observed in boundary layers over rough walls. Note that this brings us back to the question of inner-outer flow interaction.

We have finally reviewed the classical similarity assumptions for the logarithmic layer, in view of the recent suggestions in [2] that they might be invalid. We have found that the key hypothesis of the original derivation of the logarithmic law, that there is a non-zero limit at the wall for the outer flow dimensionless groups, agrees with experimental evidence from velocity fluctuations in high Reynolds number flows, making the existence of anomalous power laws unlikely.

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