Kinematic alignment effects in turbulent flows

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It is shown that, in the neighborhood of a vortex whose maximum vorticity is large with respect to that in the surrounding flow, the two principal strains with the largest absolute values lie in the equatorial plane, so that the vorticity is automatically aligned to the intermediate eigenvector. This purely kinematic effect is offered as an explanation for the alignment properties recently reported for the velocity and pressure derivatives in turbulent flows. The model is compared with experimental evidence from numerical simulations. The observed ratio between the principal strains is also related to the properties of a two-dimensional Burgers' vortex.

It has been known for a long time that the small scales of turbulent flows are relatively organized. It was first shown in Ref. 1 that the statistics of the velocity derivatives are incompatible with an uncorrelated random behavior of the velocity field at high wave numbers. It was Kuo and Corrsin[2] that first presented suggestive evidence that the high vorticity loci were vortex tubes or, at most, ribbons, but it was necessary to wait for the advent of direct visualizations of numerically simulated flows before these more or less flattened vortex tubes were shown to be the dominant structures of isotropic turbulent flows at high vorticity amplitudes,[3–6] at least at moderate Reynolds numbers. A reexamination of older data fields in numerically simulated turbulent shear layers, homogeneous shear flows, and the wall region of turbulent channels, shows that compact vortices of roughly similar characteristics are present in all of them.[7] A summary of those data is presented below. More recently the existence of strong concentrated persistent vortices in homogeneous turbulence has been proved in the laboratory by direct flow visualization.[8] Organized strain structures of comparable dimensions were published in Ref. 9.

There seems to be reasonable agreement in that the tubes, defined as the regions in which the vorticity magnitude is larger than $1/e$ times its maximum values at the axes of the cores, have diameters intermediate between the Taylor microscale λ and the Kolmogorov scale η, in the range of 4–10η. However, since all estimates refer to flows in which $\mathrm{Re}_\lambda = u'\lambda/\nu \approx 100$, and $\lambda/\eta \approx 15$, it is difficult to distinguish between low multiples of η and high fractions of λ. Their length is quoted as being of the order of either a small multiple of λ or of the integral scale of the flow L. Again, since $L/\lambda \approx \mathrm{Re}_\lambda/5 \approx 20$, for $\mathrm{Re}_\lambda \approx 100$, it is difficult to distinguish both possibilities.

The peak vorticity at the axis of the vortex cores $\omega_{\max}$ is always several times higher than the rms vorticity for the flow field ω', in the range $\omega_{\max} \approx 5$–$10\omega'$. This means that the flow in the immediate neighborhood of the vortices is dominated by them and is relatively independent of the influence of other structures. We will show below that this can be used to understand some of the alignment properties that have been reported in recent years between the different velocity derivatives.

The total circulation γ in each vortex tube lies in the range $\mathrm{Re}_\gamma = \gamma/\nu \approx 200$–$400$ (± 100). This is true even for flows, like the wall region of the channel, or like the turbulent mixing layer, in which the detailed dynamics is presumably quite different from that of homogeneous turbulence.[7] Once more, all the observations come from numerical simulations at low Reynolds numbers, and it is difficult to know whether this result should be interpreted as an absolute range for Re_γ, or as evidence for a dependence on Re_λ. There is some weak evidence for the latter interpretation.

On the other hand, $\mathrm{Re}_\gamma \approx 150$ is at least a plausible value for the circulation of the weakest observable vortices. Since the cores are defined as loci of very high vorticity, they have to be produced by stretching of previous structures. As long as a vortex is being stretched, it can be maintained indefinitely but, if it is going to survive after the strain ceases to act, its Reynolds number has to be high enough. Vortices that survive for very short times would be difficult to identify as different from the background turbulent motion, and their dynamical significance would be small. All the available observations indicate that the lifetime of the compact vortex tubes is long with respect to their inertial time scale.

It can easily be shown that the peak vorticity of a two-dimensional axisymmetric, unstrained, self-similar vortex, diffusing under the effect of viscosity, decays by a factor of 2 in a time, $\tau = \mathrm{Re}_\gamma T_E/16\pi^2$, where $T_E = 4\pi^2\rho^2/\gamma$ is the circulation time at the $1/e$ radius ρ. Because of the large denominator in τ, vortices with Re_γ much smaller than 150 decay too fast, and would not be identified as coherent.

It was first suggested in Ref. 4 and shown in Ref. 10 that the vorticity in homogeneous turbulent flows is preferentially aligned to the eigenvector corresponding to the intermediate eigenvalue of the strain tensor S_{ij}, especially at high values of the enstrophy. This was confirmed later in

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other cases, both in numerical simulations[6] and in laboratory experiments.[11] That observation was considered surprising since it had been assumed that the vorticity vector would be stretched along the direction of any eigenvector of the strain tensor with a positive eigenvalue, and that it would eventually be aligned to the eigenvector of the most positive one. In fact, the existence of tubes was also considered initially controversial, because it can be shown that the most probable state for the strain tensor is to have two extensional eigenvalues,[12] and it was felt that this should give rise preferentially to vortex sheets.

That this is not necessarily so can be seen by considering *two-dimensional* vortices and vortex sheets. In those cases the vorticity is normal to the x_1-x_2 plane, and generates a strain tensor in which the only two nonzero eigenvalues are equal in magnitude and opposite in sign, with eigenvectors normal to the vorticity. Thus the vorticity is aligned with the eigenvector of the intermediate (zero) eigenvalue, and is not stretched by any of the other two. Since the picture that emerges from the previous survey of experimental results, at least at high enstrophy values, is one of elongated, essentially two-dimensional, compact vortices, it is not surprising that the strain produced by them is normal to their axes, and that it dominates the strain tensor, so that any residual eigenvector is, by the orthogonality property, aligned to the vorticity. It is only the residual axial eigenvalue that does the stretching or compression of the vortex tube, while the two equatorial ones are local effects of the vorticity and do not participate in its dynamics. Note that this arrangement automatically satisfies the requirement that two eigenvalues of the strain tensor be predominantly positive, while being consistent with the observation of a prevalence of tubes. Vortex tubes, formed by randomly occurring axial strains, would automatically provide a second extensional eigenvalue to the average, while weak compressed vortices would not add any special bias to the general random strain distribution.

This interpretation is reinforced by Fig. 1, which is adapted from Ref. 13, and which shows that the pressure gradient, conditioned on the angle that it forms with each of the three eigenvectors, is maximum when it is aligned normal to the intermediate one, and 45° away from either the most compressive or extensional eigenvector. It is easy to recognize in this arrangement the strain generated around a two-dimensional vortex, in which the pressure gradient is directed away from its axis, while the two principal strains are normal to it, and aligned 45° away from the radial direction.

In a turbulent field, Fig. 2 shows a normal cross section through a compact core, corresponding to the head of a hairpin vortex in the simulation of a homogeneous turbulent shear flow in.[14] The vorticity is aligned to the axis of the core and normal to the plane of the page. The figure also shows the eigenvectors corresponding to the most compressive and to the most extensional eigenvalues, projected on the plane of the section. It is clear that, while outside the vortex the orientations of the vectors are irregular, inside it they correspond essentially to that of a simple plane shear.

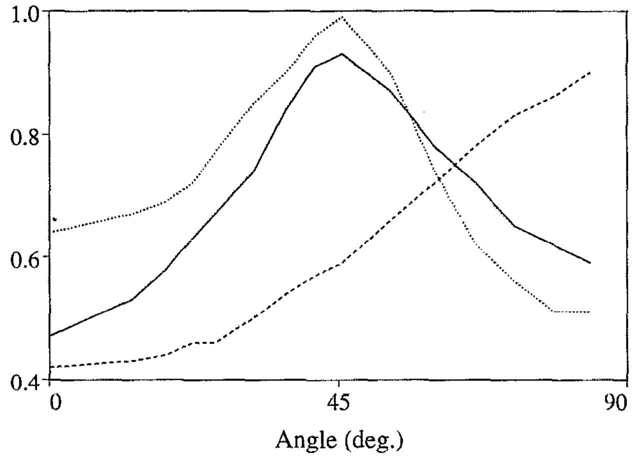

FIG. 1. Averaged, squared pressure gradients in a turbulent field, measured along lines forming a given angle to each principal strain axis. Solid line: angle measured to most extensional eigenvector. Dotted: most compressive. Dashed: intermediate. Adapted from Ref. 13.

The argument can be made a little more general. Consider a vorticity distribution, $\mathbf{\Omega}(\mathbf{x})$, which in some neighborhood can be written as

$$\mathbf{\Omega}(\mathbf{x}) = \omega_0(\mathbf{x})\mathbf{v}_3 + O(\omega'), \tag{1}$$

where boldfaced quantities are three-dimensional vectors, $\mathbf{v}_3$ is the unit vector along the x_3 axis, and $\omega_0 \gg \omega'$. Assume moreover that $\partial\omega_0/\partial x_1$ and $\partial\omega_0/\partial x_2$ are $O(\omega_0)$, but note that the solenoidal character of the vorticity implies that $\partial\omega_0/\partial x_3 = O(\omega')$. We can express the velocity $\mathbf{u}$ in general as

$$\mathbf{u}(\mathbf{x}) = -\frac{1}{4\pi}\int \frac{(\mathbf{x}-\mathbf{x}')\times\mathbf{\Omega}(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^3}\,\mathrm{d}^3\mathbf{x}', \tag{2}$$

plus a potential part that is independent of the vorticity and that will be assumed to be $O(\omega')$. Differentiating with respect to x_i, and integrating by parts we obtain the velocity gradient as

$$\frac{\partial u_j}{\partial x_i} = -\frac{1}{4\pi}\int \frac{1}{|\mathbf{x}-\mathbf{x}'|^3}\left((\mathbf{x}-\mathbf{x}')\times\frac{\partial\mathbf{\Omega}}{\partial x_i'}\right)_j \mathrm{d}^3\mathbf{x}', \tag{3}$$

plus small terms coming from the potential. It can be seen after some reflection that the part of this deformation tensor that is $O(\omega_0)$ is confined to the top 2×2 diagonal submatrix. Under those conditions the generic eigenvalue structure of the strain tensor is formed by two dominant eigenvalues, $O(\omega_0)$, whose eigenvectors form at most a small angle $O(\omega'/\omega_0)$ with the equatorial plane (x_1,x_2), and by a third eigenvalue, $O(\omega')$, whose eigenvector is similarly aligned to $\mathbf{v}_3$, and to the dominant vorticity. The strain structure discussed above for a two-dimensional vortex is a particular case of this arrangement, but the argument is more general and should apply to other situations. It is important to realize that Eqs. (2) and (3) are completely kinematic, and that the alignment of the strains with respect to the vorticity is independent of the particu-

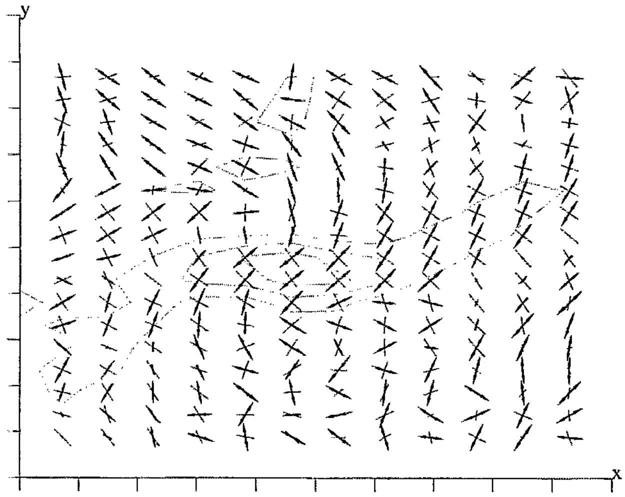

FIG. 2. Section normal to a strong vortex core in the turbulent field in Ref. 14. Dotted lines are isovorticity contours. Heavy arrowed lines are most extensional eigenvectors, projected in the plane of the figure. Light lines, without arrows, are most compressive eigenvectors. Eigenvectors are not scaled with eigenvalues. Length variation is due solely to projection.

lar dynamical mechanism involved in the generation of the vorticity concentration (1).

In particular, it was suggested in Refs. 4 and 10 that the alignment of the vorticity to the intermediate eigenvector is due to the tendency of the vorticity vector to rotate in that direction, as a consequence of the asymptotic behavior of the solutions of a truncated local approximation to the Navier–Stokes equations. While those equations might still be useful in explaining the formation of the vorticity concentrations, the present discussion suggests that the explanation of the alignment is simpler, and that it is the strain tensor that rotates toward the vorticity, once the latter becomes strong enough.

The same model can be used to explain some of the quantitative information available on the magnitude of the strain eigenvalues at the points of maximum dissipation, which has been measured from numerical simulations[10] to be approximately in the ratio $(1{:}3{:}-4)$. From the model that we have developed here, we may visualize those structures as stretched vortices in which the straining and the viscous diffusion are roughly in equilibrium. This would at least be true during the formation stage, at which dissipation is maximum.

The vorticity distribution for an equilibrium Burgers' vortex, whose circulation is $\nu\,\mathrm{Re}_\gamma$, subject to an extensional axial strain $u_3 = \alpha x_3$, is Gaussian.[15] The corresponding strain tensor has three eigenvalues. The first one is $\sigma_0 = \alpha$, and its eigenvector is aligned with x_3. It corresponds to the driving strain. One of the other two is extensional, and the other compressional, and their eigenvectors are in the equatorial plane, oriented roughly $\pm 45°$ away from the radial direction. Their magnitude with respect to the driving strain can be characterized by the ratio $(\sigma_+ - \sigma_-)/2\sigma_0$, which vanishes at the center of the vortex and at infinity, where the driving strain prevails, but is maximum just outside the $1/e$ radius, where the three eigenvalues are in the ratio $(1{:}\pm 0.012\,\mathrm{Re}_\gamma - 0.5)$. This is also the point of maximum energy dissipation.

The annular dissipation distribution has been documented in Ref. 5 for numerical isotropic turbulence. For $\mathrm{Re}_\gamma \approx 200$–$400$, corresponding to the range of experimental values, the predicted eigenvalue ratio varies between $(1{:}2{:}-3)$ and $(1{:}3{:}-5)$. These values are in rough agreement with the numerical ones quoted above, but they increase with vortex intensity. It would be interesting to check whether flow with larger measured values of $\omega_{\max}/\omega'$ also have a larger ratio of the principal strains, but the Reynolds numbers of the present numerical simulations are too low for that purpose.

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