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On instability characteristics of isolated vortices and models of trailing-vortex systems

S. Hein [a,*], V. Theofilis [b]

[a] DLR, Institute of Aerodynamics and Flow Technology, Bunsenstraße 10, D-37073 Göttingen, Germany
[b] Escuela Técnica Superior Ingenieros Aeronáuticos, Universidad Politécnica de Madrid, Plaza Cardenal Cisneros 3, E-28040 Madrid, Spain

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Abstract

This paper demonstrates the applicability of a two-dimensional eigenvalue problem approach to the study of linear instability of analytically constructed and numerically calculated models of trailing-vortex systems. Chebyshev collocation is used in the 2D eigenvalue problem solution in order to discretize two spatial directions on which non-axisymmetric vorticity distributions are defined, while the third, axial spatial direction is taken to be homogeneous and is resolved by a Fourier expansion. The leading eigenvalues of the matrix discretizing the equations which govern small-amplitude perturbations superimposed upon such a vorticity distribution are obtained by Arnoldi iteration. The present approach has been validated by comparison of its results on the problem of instability of an isolated Batchelor vortex. Here benchmark computations exist, employing classic instability analysis, in which the azimuthal direction is also treated as homogeneous. Subsequently, the proposed methodology has been shown to be able to recover the classic long- (Crow) and short-wavelength instabilities of a counter-rotating vortex-pair basic flow obtained by direct numerical simulation. Finally, the effect on the eigenspectrum of the isolated Batchelor vortex is documented, when the basic flow consists of a linear superposition of such vortices. The modifications of the eigenspectrum of a single vortex point to the potential pitfalls of drawing conclusions on the instability characteristics of a trailing-vortex system by monitoring the constituent vortices in isolation.

1. Introduction

Renewed industrial interest in control of wakes behind commercial airliners, in particular those of the ICAO 'heavy' class prompted by the introduction of the A380, has sparked a large-scale

* Corresponding author.
E-mail address: stefan.hein@dlr.de (S. Hein).

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European activity in this field, analogous with efforts around the B747 in the US in the late 1970s. Amongst the broad spectrum of flow control methodologies currently examined from the point of view of their technological feasibility, control of vortex instability has been identified as one option which could potentially be exploited in order to arrive at physically based criteria to minimize aircraft separation [4]. Besides this technological driver, vortex instability and breakdown is of practical interest in other areas of external aerodynamics. Flight characteristics of delta wings at high angle of attack are to a large extent determined by vortex instability and breakdown. While several idealized models to describe vortex instability and breakdown in either the wake-vortex system or a delta wing configuration have been used (and hotly debated), experimental evidence exists that minimally two or more vortices (and their symmetric/antisymmetric counterparts), each with distinct core structures, are involved in these processes. Moreover, core radii and core structures, as well as spacing between the individual vortices are slowly changing in the axial/downstream direction.

A plethora of models of isolated vortices exist in the literature. Of these, the best known model which incorporates an axial velocity component in the analysis is the Batchelor [2] vortex. Mayer and Powell [15] were the first to employ numerical solution of ODE-based eigenvalue problems to study both the inviscid and viscous instability properties of an axisymmetric Batchelor vortex model flow. This approach is restrictive as regards modelling the trailing-vortex system behind aircraft, since it assumes an axisymmetric basic state, thus neglecting in principle the influence of any neighbouring vortices. Nevertheless, it provides detailed instability characteristics for comparisons with results of more elaborate, partial-differential-equation-based, instability analyses. Crow [5], Jiménez [13], Crouch [3] and Fabre and Jacquin [10], amongst others, have used vortex filament methods to analyze the instabilities triggered by interaction between neighbouring vortices. However, vortex core instabilities [15] or absolute instability [6] are beyond the scope of a vortex filament approach. In addition, the vortex filament method has limitations (besides that of strictly being applicable to inviscid flow) in the permissible distributions of vorticity in the wake, in terms of both strength and placement of the constitutive vortices. Motivation thus exists to expand the scope of available analysis methodologies in order to address viscous instability of arbitrary azimuthally inhomogeneous (non-axisymmetric) vorticity *distributions* on a plane normal to the axial flow direction, where all three two-dimensional velocity components may exist.

BiGlobal instability analysis [21], based on the numerical solution of partial-derivative eigenvalue problems, is such a methodology. The goal of the present work is to investigate its applicability to non-axisymmetric open vortical flows. The ambiguity of choice of a basic state to analyze is circumvented here by focussing on wake-vortex models constructed by linear superposition of analytical Batchelor vortices. Further, in view of the fact that linear superposition of such vortices does not satisfy the equations of motion, a counter-rotating vortex pair is obtained by direct numerical simulation and analyzed with respect to its linear instability. Noteably, in the BiGlobal instability analysis methodology, unlike that based on the classic vortex filament method (e.g. Saffman [18]), no restrictions are imposed on the core size and location of the vortices constituting the basic flow. In Section 2 the Batchelor model is introduced, some details of the direct numerical simulation approach for the recovery of a basic state are discussed and the equations governing the BiGlobal instability problem are presented. In Section 3 the methods utilized for the numerical solution of the two-dimensional eigenvalue problem are briefly exposed. Results are presented in Section 4, where first the partial-derivative eigenvalue problem is validated on a single Batchelor vortex. This aids identification of the limitations of the present numerical approach and proposal of alternatives. Subsequently, long- and short-wavelength instabilities of a counter-rotating vortex pair are recovered by employing BiGlobal instability analysis. Finally, eigenspectrum results pertaining to systems of two and four Batchelor vortices are presented. Concluding remarks are provided in Section 5.

2. Theory

2.1. Analytical models for the basic flow

Here the basic flow is taken to be described by linear superposition of Batchelor vortices. A single such vortex comprises three velocity components $\bar{\mathbf{q}}(x,y,z) = (\bar{u},\bar{v},\bar{w})^{\mathrm{T}}$ in a Cartesian coordinate system (x,y,z) in which x denotes the axial flow direction. Taking U_∞ to denote the freestream velocity in the axial direction of the vortex, $\Delta U = U_{\mathrm{c}} - U_\infty$, with U_{c} the axial core velocity, Ω_{c} the rotation rate on the axis and R a measure of the core radius, the following dimensionless parameters [6] may be defined

$$a = \frac{U_\infty}{\Delta U}, \qquad q_n = \frac{\Omega_{\mathrm{c},n} R}{\Delta U}, \qquad Re = \frac{\Delta U R}{\nu}, \tag{1}$$

where ν denotes the kinematic viscosity of the incompressible fluid. In terms of this non-dimensionalisation the basic Batchelor vortex flowfield in Cartesian coordinates is

$$\bar{u}(y,z) = a + \exp(-r^2), \tag{2}$$

$$\bar{v}(y,z) = -q_n(z - z_n)\{1 - \exp(-r^2)\}/r^2, \tag{3}$$

$$\bar{w}(y,z) = q_n(y - y_n)\{1 - \exp(-r^2)\}/r^2, \tag{4}$$

where $r^2 = (y - y_n)^2 + (z - z_n)^2$ and (y_n, z_n) denotes the centre of vortex n.

2.2. A basic flow model obtained numerically

It can be argued that the far-field of a vortex will deform in a non-linear manner if an additional vortex is introduced in the flow. Consequently, instability results obtained using a basic flow composed of a system of linearly superimposed vortices may differ from those of a basic state which satisfies the two-dimensional equations of motion. A basic flow of the latter class is calculated here by incompressible direct numerical simulation; numerical details of the spectrally accurate methodology are presented elsewhere [21].

The initial condition is analytically prescribed as linear superposition of the elliptic vorticity distributions

$$\Omega(y,z;t=0) = q_+ \exp\left\{-\frac{(y-y_+)^2 + (z-z_+)^2/16}{r_0^2}\right\} + q_- \exp\left\{-\frac{(y-y_-)^2 + (z-z_-)^2/16}{r_0^2}\right\}. \tag{5}$$

Two sets of simulations have been performed at $Re = 10^2$ and 10^3. In the first case a square computational domain $[-12, 12]^2$ comprising 400^2 equidistributed spectral collocation points has been considered. The parameters $q_\pm = \pm\pi$, $r_0 = \frac{1}{4}$, $y_\pm = 4$, $z_\pm = \pm 2$ have been used in the initial condition (5) and the equations of motion have been integrated in time until $t_\infty = 50$, when a single pair of vortices has developed and descends with a constant speed. In the second set of simulations, the respective parameters are a computational domain $[-18, 18]^2$ resolved by 600^2 points, with $q_\pm = \pm\pi$, $r_0 = \frac{1}{4}$, $y_\pm = 6$, $z_\pm = \pm 0.5$ and $t_\infty = 65$. Considering a frame of reference moving with the respective descent speed, quasi-steady states satisfying the incompressible Navier–Stokes and continuity equations are obtained; the axial velocity component in either case is taken to be zero.

2.3. The disturbance flow

Separability of the time and space coordinates in the governing equations of motion and introduction of harmonic time-dependence of the disturbance quantities results in a three-dimensional eigenvalue problem which is currently not tractable numerically at Reynolds numbers of relevance to external aerodynamics. Hence, the dependence of the basic state on one spatial direction must be neglected on grounds of numerical feasibility. Taking this direction to be that along the axial coordinate, x, restricts the class of flows that may be studied to vortical systems in which

$$\bar{\mathbf{q}}_x \ll \bar{\mathbf{q}}_y, \quad \text{and} \quad \bar{\mathbf{q}}_x \ll \bar{\mathbf{q}}_z. \tag{6}$$

Note that this assumption is more general than that leading to the classic Orr–Sommerfeld type of ordinary-differential-equation-based eigenvalue problems [6,15], where in addition to (6) spatial homogeneity in the azimuthal direction is assumed. By contrast, (6) permits introducing into the linearized disturbance equations small-amplitude perturbations $\hat{\mathbf{q}}$ of the form

$$\hat{\mathbf{q}}(x, y, z, t) = \tilde{\mathbf{q}}(y, z)\exp(\mathrm{i}\Theta_{2D}) + \text{c.c.}, \tag{7}$$

where

$$\Theta_{2D} = \alpha x - \omega t. \tag{8}$$

This results in the following partial-derivative eigenvalue problem

$$\mathrm{i}\alpha\hat{u} + \mathscr{D}_y\hat{v} + \mathscr{D}_z\hat{w} = 0, \tag{9}$$

$$\mathscr{L}\hat{u} - (\mathscr{D}_y\bar{u})\hat{v} - (\mathscr{D}_z\bar{u})\hat{w} - \mathrm{i}\alpha\hat{p} = -\mathrm{i}\omega\hat{u}, \tag{10}$$

$$[\mathscr{L} - (\mathscr{D}_y\bar{v})]\hat{v} - (\mathscr{D}_z\bar{v})\hat{w} - \mathscr{D}_y\hat{p} = -\mathrm{i}\omega\hat{v}, \tag{11}$$

$$-(\mathscr{D}_y\bar{w})\hat{v} + [\mathscr{L} - (\mathscr{D}_z\bar{w})]\hat{w} - \mathscr{D}_z\hat{p} = -\mathrm{i}\omega\hat{w}, \tag{12}$$

where the linear operator is

$$\mathscr{L} = (1/Re)(-\alpha^2 + \mathscr{D}_y^2 + \mathscr{D}_z^2) - \mathrm{i}\alpha\bar{u} - \bar{v}\mathscr{D}_y - \bar{w}\mathscr{D}_z. \tag{13}$$

Here $\mathscr{D}_y \equiv \partial_y$, $\mathscr{D}_z \equiv \partial_z$ and α is a wavenumber in the direction of the aircraft motion, x, which defines a periodicity length $L_x = 2\pi/\alpha$. In the temporal framework chosen in the present work, ω is a complex eigenvalue the real part of which, $\omega_r \equiv \Re\{\omega\}$, is related with the frequency of an eigenmode $\hat{\mathbf{q}}$ while the imaginary part, $\omega_i \equiv \Im\{\omega\}$, is its growth/damping rate; a positive value of ω_i indicates exponential growth of $\hat{\mathbf{q}}$ in time, while $\omega_i < 0$ denotes decay of the eigenmode in time. The objective of the analysis becomes the identification of unstable eigenvalues ω_i and associated eigenvector amplitude functions $\tilde{\mathbf{q}}$ for a given basic state $\bar{\mathbf{q}}$ describing the wake-vortex system.

Straightforward extensions of the eigenvalue problem (9)–(12) employing Floquet theory [1,12] may relax the assumption of a steady to that of a time-periodic basic state. Further, potentially existing downwash of the wake-vortex system can be formally addressed by solving (9)–(12) on a coordinate system which moves downwards by the same constant speed of the wake-vortex system basic flow; neither of these refinements will be considered here. On the other hand, it should also be noted that solution of one of the alternative simplified (and computationally less demanding) forms of the partial derivative eigenvalue problem (9)–(12), valid in case of a single velocity component in either viscous [19] or inviscid flow [11], is not permissible in the wake-vortex stability problem.

3. Numerical methods

A general discussion of methods appropriate for the numerical solution of the partial-derivative eigenvalue problem is presented elsewhere [21]; we concentrate here on the particular requirements of vortical systems. Numerical methods of high formal order of accuracy are necessary since the coupled spatial discretization in the numerical solution of the eigenvalue problem (9)–(12) cannot be increased at will in order for convergence to be achieved; here Chebyshev polynomials have been used to discretize both spatial directions. Use of an analytic basic state eliminates concerns regarding the influence of inadequate resolution of the basic state on the instability results and attention can be exclusively focussed on the numerical solution of the eigenvalue problem. On the other hand, convergence of the numerically obtained basic states has been ensured in the spectrally accurate direct numerical simulation.

The resolution requirements for the eigenvalue problem may be inferred from the structure of typical azimuthally inhomogeneous eigendisturbances of the Batchelor vortex [9,15,16], one of which is shown in Fig. 1. The domain boundary shown is approximately one-third of that corresponding to the radius value r_b at which the azimuthal velocity of the Batchelor vortex attains a maximum. In the specific example, where $q_n = q_0 = 2$, $y_n = z_n = 0$, r_b can be calculated by combining (3) and (4) to give $V = q_0/r[1 - e^{-r^2}]$ and solving the transcendental equation $(dV/dr)_{r=r_b} = -(q_0/r_b^2)(1 - e^{-r_b^2}) + 2q_0 e^{-r_b^2} = 0$, which results in $r_b \approx 1.12091$. In other words, eigenmodes of reasonably complex structure are confined within a radius approximately $r_b/3$, which is more than an order of magnitude smaller than the radius value at which the basic flow itself decays to machine-zero level. A corollary of this observation is that vortex centres need to be resolved using substantially higher-density grids than those required for the accurate description of the basic flow. On the other hand, in order to reduce the integration domain in the far-field, asymptotic boundary conditions can be derived and imposed. However, such boundary conditions are particular to the single-vortex basic flow model and this lack of generality makes homogeneous Dirichlet boundary conditions on disturbance components the first candidate to be imposed. This results in the need to address rather wide integration domains and the issue of

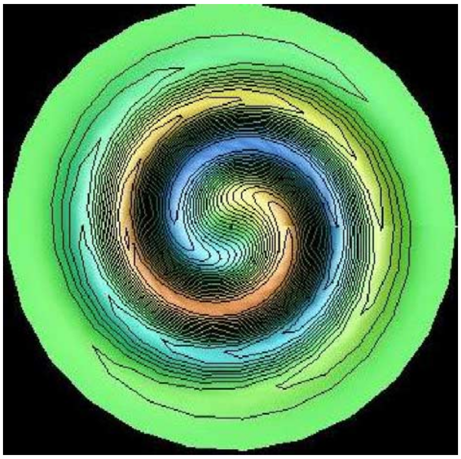

Fig. 1. A non-axisymmetric eigenmode of the Batchelor vortex [9,15,16]. The figure boundary approximately corresponds to $\frac{1}{3}r_b$.

appropriate mappings between the standard Chebyshev grid $x \in [-1, 1]$ and the actual calculation domain becomes critical to the success of the overall algorithm. One mapping used, which satisfies the requirements outlined is

$$\eta = \eta_0 + \eta_\infty \frac{\tan \frac{c\pi}{2} x}{\tan \frac{c\pi}{2}}, \tag{14}$$

where η is either of the discretized spatial directions y or z, while η_0 and η_∞ are the respective centrepoint and farfield truncation locations. A typical grid for the calculations in shown in Fig. 2.

In all but the lowest Reynolds number calculations, where a QZ algorithm can be used (such calculations are not presented here), a shift-and-invert Arnoldi algorithm has been employed for the recovery of the eigenvalues. The difference between the QZ and the Arnoldi algorithm is that

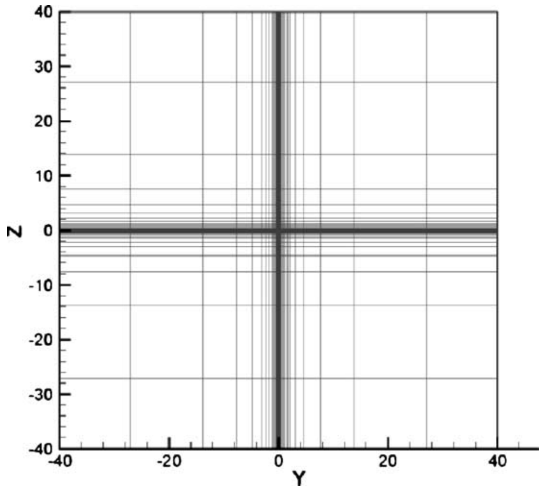

Fig. 2. A typical grid, generated by (14), for BiGlobal instability analysis of an isolated or a system of vortices.

in the former the entire spectrum of the complex non-symmetric eigenvalue problem (9)–(12) can be recovered, while the latter algorithm delivers a window of eigenvalues around a prescribed estimate. Specifically, the Arnoldi algorithm solves the standard eigenvalue problem

$$\hat{\mathbf{A}}X = \mu X, \qquad \hat{\mathbf{A}} = (\mathbf{A} - \sigma\mathbf{B})^{-1}\mathbf{B}, \qquad \mu = \frac{1}{\omega - \sigma}, \tag{15}$$

where $\mathbf{A}$ and $\mathbf{B}$ are the matrices discretizing the original system (9)–(12) and σ is a shift parameter. All calculations presented in what follows have been performed using 64-bit arithmetic.

4. Results

Indicative of successful applications of BiGlobal instability analysis based on (9)–(12) are the studies of instability in a pressure-gradient-driven rectangular duct [20], where the symmetries of the respective basic states were not *a priori* imposed in order to reduce the computing effort but were recovered as part of the computations [19]. This is conceptually relevant to the present computations of instability of isolated vortices, where symmetries such as those shown in Fig. 1 are also present but are not explicitly imposed on the expected solutions. Further successful applications of the present methodology to vortical flows are the analyses of lid-driven- and open-cavity configurations [22,25], that of boundary-layer flow which encompasses a closed recirculation bubble [24] and the swept attachment-line boundary layer [23].

Here, numerical solutions of the partial-differential-equation eigenvalue problem have been obtained using $c \in [0.8, 1)$ in (14), leading (complex) matrix dimension in (15) of up to 4×72^2, where the two factors respectively reflect the number of equations in (9)–(12) and the degrees of freedom used to discretized domain $y \in [-y_\infty, y_\infty] \times z \in [-z_\infty, z_\infty]$ with $y_\infty, z_\infty = 40$ in the single Batchelor vortex case, $y_\infty, z_\infty = 60$ in the case of the numerically obtained basic flow, and $y_\infty = 45$, $z_\infty = 40$ in the four-vortex case.

We discuss first the instability analysis results of the basic flow obtained by direct numerical simulation. The counter-rotating vortex systems are expected to support both long-wavelength (Crow) and short-wavelength instabilities. In Fig. 3(a) and (c) we respectively present the vorticity $\Omega(y, z; Re = 100, t = 50)$ and $\Omega(y, z; Re = 1000, t = 65)$ of the basic states analyzed at $Re = 10^2$ and 10^3. In both results the non-linear deformation of the counter-rotating vortices can be seen and an estimate of the distance between the centres of the vortex cores can be obtained $d_{100} \approx 4.6$ and $d_{1000} \approx 2.0$, respectively. Fig. 3(b) shows the dependence of the amplification rate ω_i on the axial wavenumber α in the low-Reynolds number case. Only long-wavelength instabilities are monitored here; it can be seen that the instability recovered by the present BiGlobal analysis peaks at $L_x \approx 2\pi/0.17 = 37.0$, an axial wavelength corresponding to the classic Crow instability, the amplification of which peaks at around $L_x \approx 10d_{100}$. At $Re = 10^3$, on the other hand, short-wavelength instabilities have been monitored and the amplitude functions of the most amplified eigenmode at $\alpha = \pi$ are shown in Fig. 3(d). The eigenvalue [1] is $(\pm0.0305, 0.0125)$, corresponding to two eigendisturbances travelling in opposite directions along the x-axis. At the highest

[1] The convergence of which has been ensured by grid analysis studies comprising up to 72^2 collocation points, distributed according to (14) with $c = 0.975$.

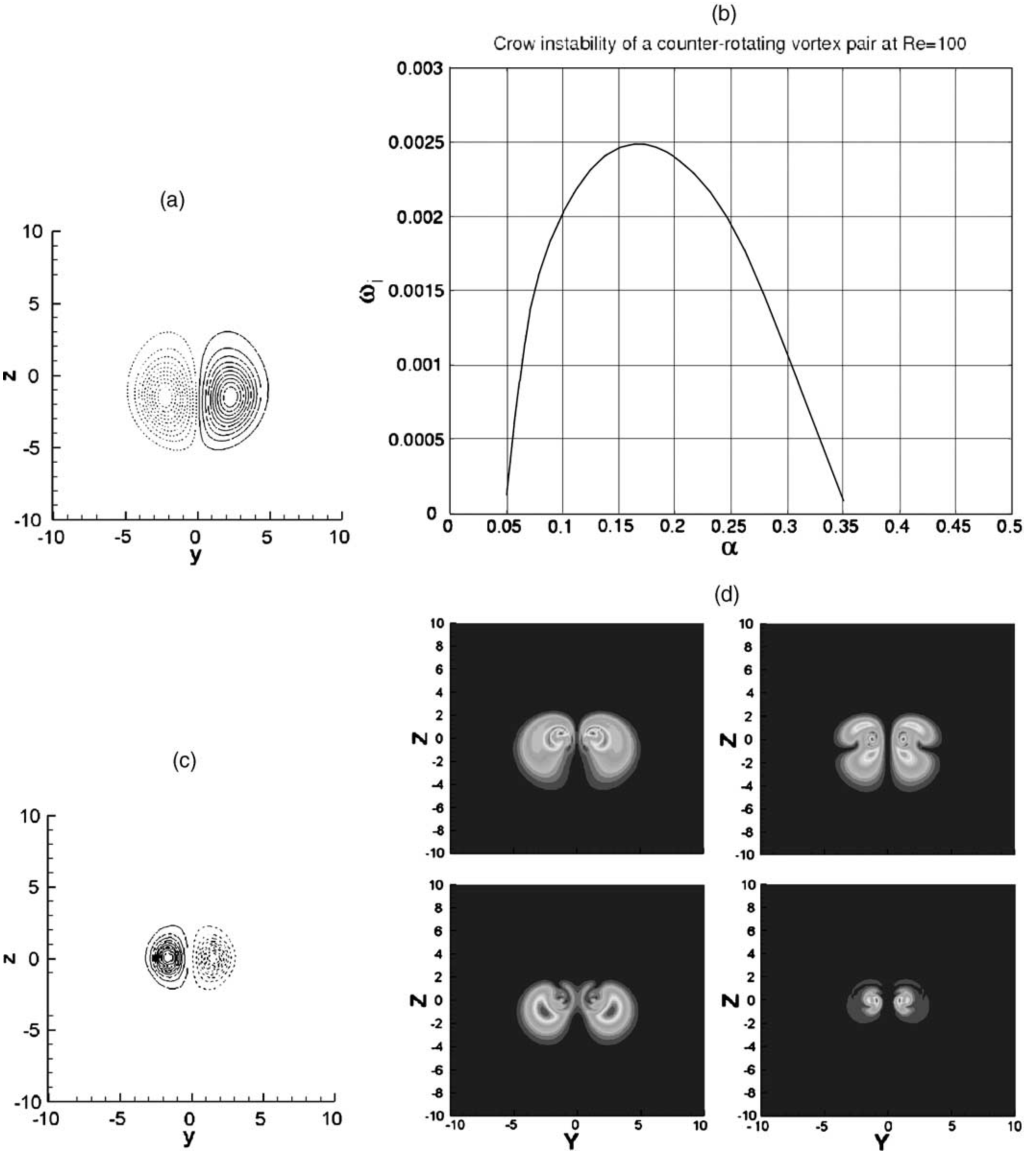

Fig. 3. (a) Vorticity $\Omega(y,z;t=50)$ of the field evolving from the initial condition (5) at $Re=10^2$. Ten equidistributed contours respectively between the maximum and zero (solid) and the zero and minimum (dashed) values are drawn. (b) Long-wavelength (Crow) instability developing upon the basic state shown in (a). (c) Vorticity $\Omega(y,z;t=100)$ of the field evolving from the initial condition (5) at $Re=10^3$. (d) Amplitude functions of the short-wavelength instability developing upon the basic state shown in (c); upper left: $|\hat{u}|$, upper right: $|\hat{v}|$, lower left: $|\hat{w}|$, lower right: $|\hat{p}|$.

resolution these computations demand $\sim$7 GB of main memory, whereby a large portion of the discretization points is wasted in the farfield. In this respect, an alternative spatial discretization scheme, e.g. based on spectral elements [7,8,14], could become worthy of investigation in this problem.

Subsequently attention is focussed exclusively on the analytically constructed basic state. Fig. 4 shows a summary of calculations performed for different vortical flowfields in terms of the respective axial component of the basic flow and the obtained eigenspectrum. As a means of further validation of the numerical approach on vortical flows, our first concern has been with reproduction of the benchmark calculation results of Mayer and Powell [15] at several combinations of the parameters (q_n, Re, α). Of these, Fig. 4(a), shows the eigenspectrum obtained at $a = 0$, $q_n = 0.475$, $Re = 100$, $\alpha = 0.418$. This set of parameters according to Mayer and Powell [15] pertains to the most amplified viscous mode at $Re = 100$, which is an asymmetric mode with an azimuthal wavenumber (in the notation of [15]) of $m = 1$. Unfortunately, at these conditions these authors provide growth rate information only. Hence, this value is marked in the presently obtained eigenvalue spectrum as a horizontal dashed line at a positive value of ω_r.

Of prime interest in this result is the agreement between the eigenvalues obtained by the two alternative descriptions, that based on numerical solution of the partial-derivative eigenvalue problem being orders of magnitude more intensive computationally than numerical solution of the appropriate Orr–Sommerfeld equation. However, it should be pointed out that in the approach of Mayer and Powell [15] the azimuthal wavenumber is an input parameter in the numerical solution of the appropriate one-dimensional eigenvalue problem, whereas the present BiGlobal instability analysis, when applied to an axisymmetric case, provides solutions of different azimuthal periodicities in a single run. However, the resolution utilized in the latter approach cannot be increased at will for reasons which have been discussed earlier and, consequently, only non-axisymmetric modes of low m can be expected to be resolved. Note also that, despite the (characteristic of viscous flow instability) smallness of the amplification rates at these parameters, the numerical algorithm outlined in the previous section is capable of successfully addressing this class of problems. Results showing analogously good agreement at several different parameter values have been obtained but are not shown here due to space limitations; for the same reason the respective convergence histories and studies of influence of the mapping parameter on the results are omitted.

Instead, attention is focussed on the issue of modelling the trailing-vortex system from a physical point of view. The main concern in this respect is the potential modification of the well-understood (single) Batchelor vortex eigenspectrum [6,9,15] on account of the linear superposition of a second such vortex in the flowfield, taken to be counter-rotating with respect to the first. Such a system delivers initial conditions for a quasi-steady solution of the equations of motion in a frame of reference moving at a given downwash speed. At the parameters chosen, the downwash speed is in the range of one tenth of ΔU, which has been taken to be zero in this simplified linear superposition model. Compared with the grid used for the single Batchelor vortex instability calculations, the clustering of grid points in y has been relaxed by choosing a mapping parameter value $c = 0.93$. All other parameters, in particular q_n, α and Re remained unchanged. The resulting eigenspectrum is shown in Fig. 4(b).

The most interesting point which can be made when comparing this spectrum against that of the single Batchelor vortex is the strong modification of the instability characteristics of the latter vortex. Both the amplification rates and the frequencies of the two systems are practically unrelated to one-another. This result is qualitatively in line with that obtained in a variety of flows using classical linear theory based on numerical solution of variants of the Orr–Sommerfeld equation, where the precise characteristics of the basic flowfield strongly influence the instability

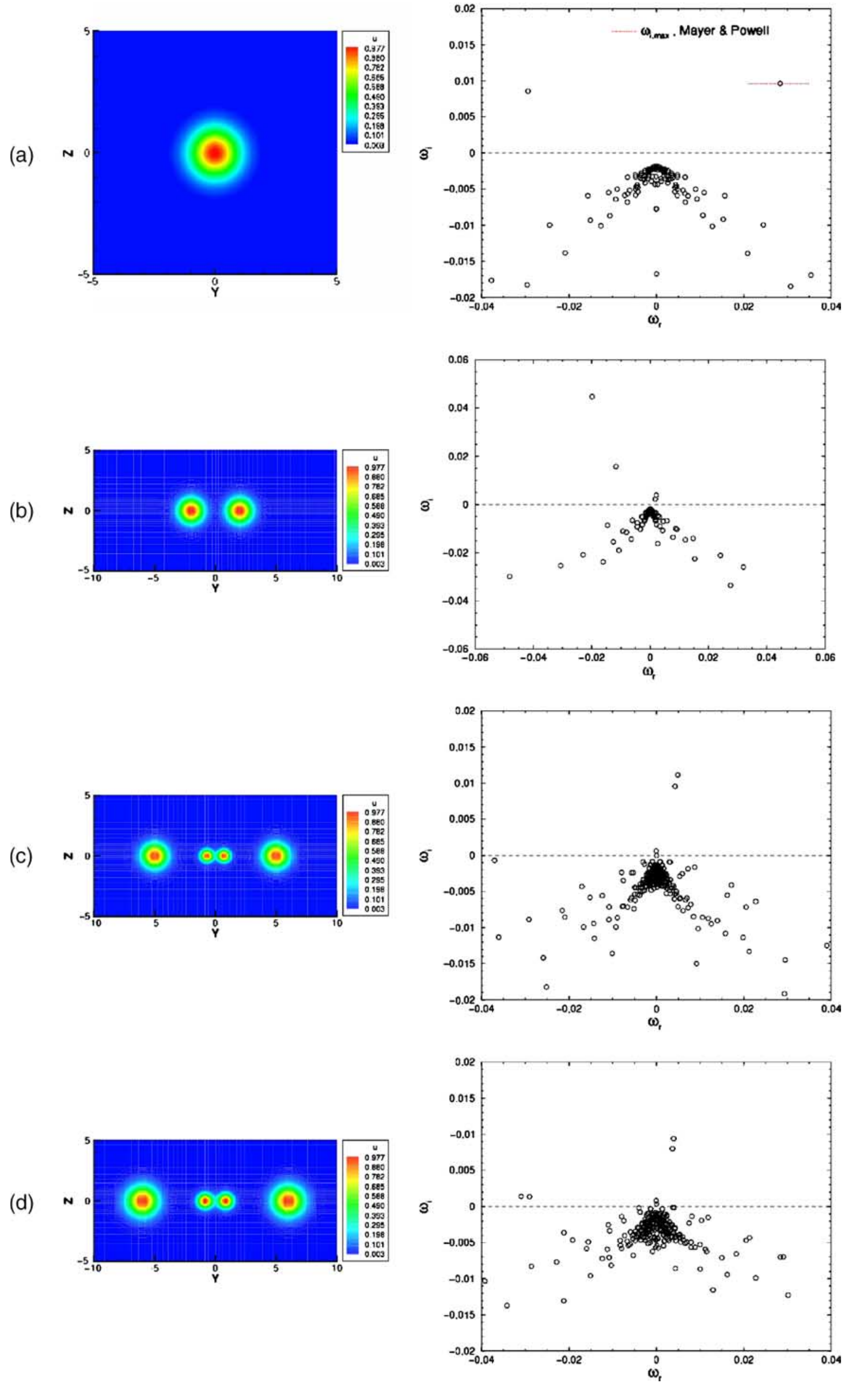

Fig. 4. Axial basic flow velocities and eigenspectra of (a) an isolated Batchelor vortex at $a = 0$, $q_n = 0.475$, $Re = 100$, $\alpha = 0.418$, (b) pair of counter-rotating Batchelor vortices at the same parameters as (a), (c) four counter-rotating Batchelor vortices satisfying the Rennich and Lele [17] condition at $Re = 100$, (d) the same as (c) at $Re = 120$.

properties of the flow. This result is especially significant in terms of proposing flow control methodologies that are based on frequency information of the most unstable flow eigenmode. For such reliable methodologies to be devised, the detailed properties of the appropriate basic flow need be known. Given that the trailing-vortex system of a particular aircraft is a consequence of the roll-up process of the particular circulation/vorticity distribution on the aircraft wings, it is unlikely that a unique flow-control methodology can be proposed by monitoring the instability characteristics of generic vortices in isolation.

As a next step a basic state is constructed by introducing a second pair of vortices in the previous basic flow, as a model of the vortical system generated at the tips of the wing and the horizontal stabilizer. Such a four-vortex system also corresponds to a quasi-steady solution of the equations of motion when the Rennich and Lele [17] criterion is satisfied. The eigenspectrum resulting at the parameters $Re = 100$, $\alpha = 0.30$, and ratios of circulation, radii and core distance of the inner and outer pairs of −0.4, 0.5 and 0.14, respectively, [10,17] is presented in Fig. 4(c). Note that the ratio of vortex core radii to distance between the vortices has been imposed by applicability of the Rennich–Lele criterion and the vortex filament method. Such a restriction can in principle be relaxed by the present BiGlobal analysis. On the other hand, the eigenspectrum result (c) (being typical of those obtained at different parameters) further strengthens the assertion that frequency information to be used for flow-control purposes strongly depends on the structure of the basic flow.

Finally, the question is posed what the modifications of the spectrum are when small parameter changes are considered in structurally identical flows. Fig. 4(d) demonstrates the effect on the BiGlobal eigenspectrum of an increase of the Reynolds number considered in (c) to $Re = 120$ by increasing the core-radius of the outer vortices by 20%. This results in an increase of the vortex distances as described by the Rennich–Lele condition, while all other parameters cited in case (c) remain the same. Compared with case (c) two points can be made. While the frequencies of the modes amplifying at both $Re = 100$ and 120 remain practically unchanged, two new eigenmodes have crossed the axis $\omega_i = 0$ and are amplified at $Re = 120$. Their frequencies are unrelated with those of the modes amplifying at both $Re = 100$ and 120. [2] The implication is that all eigenfrequencies must be taken into consideration if a methodology based on control of flow instability is to be devised at the higher Reynolds number value. This result too points to the direction of the conclusion reached on account of considering different basic states. It further underlines the fact that flow control methodologies based on exploitation of unstable eigenmode frequencies, although presumably effective at the parameter range around which they have been designed, cannot be extrapolated too far off their respective design points.

5. Discussion

This work has addressed the question of applicability of BiGlobal linear instability analysis to study viscous instability properties of trailing-vortex models flows. Results delivered by the present approach have been validated against a known viscous core instability of an isolated

[2] Although the modes amplified at $Re = 120$ can be identified as damped eigendisturbances at the lower Reynolds number value.

vortex and both short- and long-wavelength instability mechanisms in a single counter-rotating vortex pair. Subsequently, focus has been placed on non-axisymmetric configurations modelling the wake by a single and two pairs of counter-rotating Batchelor vortices. It has been demonstrated that the instability characteristics of the systems differ significantly from those of a single Batchelor vortex. Resolution requirements have not been prohibitive for the analysis of multi-vortex configurations at the modest Reynolds numbers monitored, although an alternative numerical methodology to improve efficiency has been proposed.

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