On global instabilities of separated bubble flows and their control in external and internal aerodynamics applications

Vassilis Theofilis
School of Aeronautics, Universidad Politécnica de Madrid
Pza. Cardenal Cisneros 3, E-28040 Madrid, Spain

vassilis@torroja.dmt.upm.es

Spencer J Sherwin and Nadir Abdessemed
Department of Aeronautics, South Kensington Campus, Imperial College London
London, SW7 2AZ, UK

s.sherwin@imperial.ac.uk, n.abdessemed@imperial.ac.uk

ABSTRACT

BiGlobal linear analysis provides a unified means of addressing instability phenomena of separated flows in complex configurations. It has been demonstrated that closed separation bubbles sustain global eigenmodes distinct from the known inflectional instability of the shear-layer in three case-studies of canonical flat-plate and complex airfoil and low-pressure turbine flows. In all three applications and parameter ranges studied, a consistent picture emerges regarding the characteristics of the respective most amplified / least damped global mode; the latter is found to have analogous characteristics in terms of frequency and spatial structure of the respective disturbance eigenfunctions in the different applications. The dominant global mode of laminar separation remains less significant than other types of linear instabilities in all three applications, but its omission would result in an incomplete description of the phenomenon of laminar separated flow instability or efforts to reconstruct and control the respective flowfields using reduced-order models.

1 INTRODUCTION

Theoretical flow control methodologies, which aim at utilizing the interaction of separation-bubble flow instability and transition in order to modify a given flow state, rely on modifying the instability characteristics of such separated flow. Short of employing large-scale computation, precise knowledge of the two- and three-dimensional eigenspectra of a given separated flow requires analyzing such flow with respect to both the local and global instabilities it may sustain. In particular, the potential of a laminar separation bubble to become self-excited through BiGlobal linear instability [19], besides being susceptible to the well-known inviscid instability mechanism of linear amplification of incoming disturbances at the shear-layer, has been recognized in the last decade, both theoretically [3, 11, 2, 22] and experimentally [10].

From a theoretical point of view, Allen and Riley [3], Hammond and Redekopp [11] and Alam and Sandham [2] have considered different models of laminar separation bubbles and, assuming incompressible laminar quasi-parallel flow on a flat plate, applied linear local analysis based on numerical solutions of one-dimensional eigenvalue problems (EVP) of the Orr-Sommerfeld class in order to determine the conditions for absolute instability of the respective separation bubbles. It should be noted that different means of introduction of an adverse pressure gradient, resulting in topologically and qualitatively analogous but quantitatively different closed separation bubble flows have been considered in these works. On the other hand, Theofilis, Hein and Dallmann [22] performed incompressible 2D direct numerical simulations (DNS) to recover the steady laminar separated-bubble boundary-layer flows described by Briley [5] and used such (essentially nonparallel) steady flows as basic states in 3D BiGlobal linear instability analyses. The appropriate partial-derivative EVP was solved numerically and the potential of separated flow to support global instability, without having to resort to restrictive assumptions on the basic flow, was demonstrated for the first time.

In parallel, investigations of global instability of separation bubbles in more complex configurations have successfully been pursued. In two papers Theofilis and Sherwin [23] and Theofilis, Barkley and Sherwin [20] addressed three-dimensional BiGlobal instability of incompressible steady laminar separated flow at the trailing edge of a NACA 0012 airfoil. Crouch, Garbaruk, Schur and Strelets [7] have solved as a global instability problem the buffeting phenomenon of compressible turbulent flow on the same configuration. Another (related) complex configuration, the flowfield around a model of a low-pressure turbine (LPT) passage, has also received renewed attention from the point of view of analyzing and understanding instability mechanisms, also in the trailing-edge separation region of the blade, using experiments [17] and three-dimensional DNS [25, 24, 9, 15], the latter approach modelling the actual flow around LPT blades at realistic Reynolds numbers of $O(10^5)$. However, the computational effort that underlies three-dimensional DNS renders this numerical approach accessible only to large-scale facilities and certainly inappropriate for detailed parametric studies. Abdessemed, Sherwin and Theofilis [1] have recently performed 2- and 3D BiGlobal instability analyses of the 2D steady laminar basic states around a row of T-106/300 LPT blades; of particular interest here are results pertinent to the trailing-edge separation region.

Global instability of separated flow, treated as an absolute instability of a weakly nonparallel basic state is reviewed by Chomaz [6]. The same problem, addressed from the point of view of solution of the partial derivative EVP, is currently being addressed in order to provide systematic criteria for identification of global instabilities [16], using the model flat-plate configuration and scanning the multi-parametric space in which laminar separated flows may be found. In the present contribution we attempt to present emerging patterns on the instability characteristics of the most significant eigenmodes in the three case studies to which the BiGlobal analysis concept has been applied so far, namely the flat plate, the NACA airfoil and the LPT blade. In section 2 aspects of the numerical methodologies utilized are highlighted, followed in Section 3 by results on the basic flows and the leading global eigenmodes; discussion of the results is offered in Section 4.

2 NUMERICAL APPROACHES

The case for accuracy of both the basic state and the numerical approach for the solution of the eigenvalue problem is even stronger in the context of BiGlobal, compared with classic instability theory [19]. The reason is the coupled resolution of two spatial directions and the memory/runtime requirements for the solution of the associated partial-derivative EVP. The superior resolution properties of spectral methods, compared with standard finite-difference approaches, are thus put to optimal use in this context. The spatial discretization of two-dimensional domains encompassing a laminar separation bubble in canonical rectangular or curvilinear geometries has been accomplished using two-dimensional collocation grids based on Jacobi polynomials, Gauss-Lobatto points and, when necessary, conformal mappings; the pressure perturbation is either collocated on the (Jacobi) Gauss-Lobatto or staggered on the associated Gauss grid. For complex geometries, the spectral/*hp* element method is used [12]; the underlying space tessellation is performed using unstructured mesh generators, which construct triangular or rectangular elements in order to resolve boundary layers adjacent to solid surfaces and triangular elements in the remaining part of the flow field. The flexibility of the spectral/*hp* element method to work with either type of meshes is thus fully exploited. An example of the spatial discretizations utilized in the numerically challenging cases of the NACA 0012 airfoil and the T-106/300 LPT blade is shown in Figure 1.

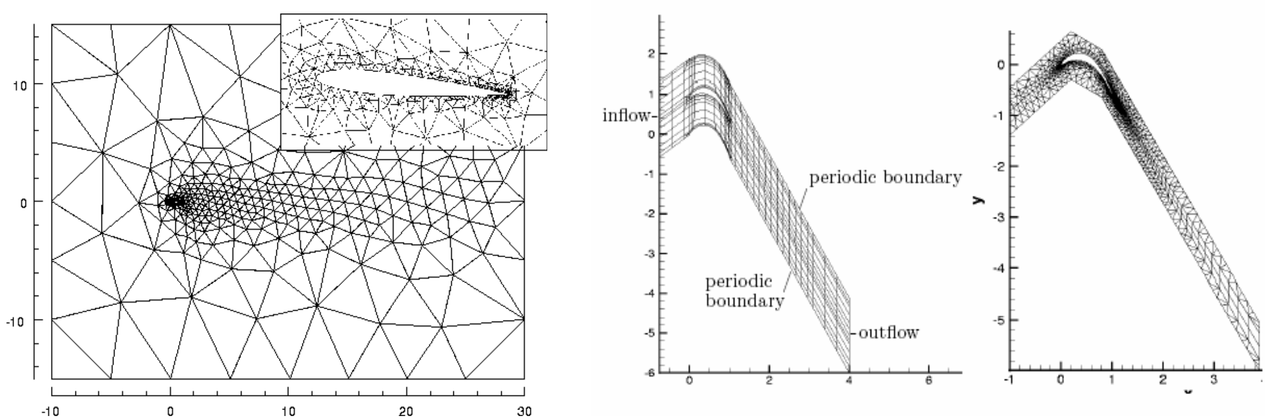

Figure 1: Left: Space triangulation using the spectral/*hp* element method and an unstructured mesh around a NACA 0012 airfoil, alongside details of the mesh around the airfoil [23]. Right: Structured and unstructured grids used for the discretization of the flowfield around the T-106/300 LPT blade; also shown are the boundary conditions imposed in this application [1].

Regarding boundary conditions for separated bubble flow instability analyses, at solid walls no-slip is imposed on the disturbance velocity components and compatibility conditions derived from the governing equations are used for the pressure perturbation, if the pressure is collocated on the same grid as the velocity components. If otherwise dictated (e.g. by periodicity of the domain, as in the case of a row of LPT blades) in the free-stream homogeneous Dirichlet conditions are imposed on all disturbance components. In the other two spatial directions the boundary conditions of *choice*, which have been successfully tested so far, are homogeneous Dirichlet at inflow- and variable-order extrapolation from the interior of the domain at the outflow boundaries. The first boundary condition is artificial, from a physical point of view, and serves to ensure that no perturbations other than potentially self-excited global eigenmodes (e.g. no Tollmien-Schlichting waves) enter the separated flow region. The second boundary condition has been imposed far downstream of closed recirculation bubbles, after it has been shown to perform well in open global instability problems in which the spatial structure of the eigenmodes is analytically known [21]. Once spatial discretization and boundary conditions have been implemented, the leading eigenvalues of the resulting large matrix eigenvalue problem are recovered using Krylov subspace iteration. In all applications tackled so far the Reynolds numbers, at which the respective two-dimensional steady separated basic state bifurcates to an unsteady two- or three-dimensional flow, are such that accurate results can be obtained at modest levels of computing effort.

3 RESULTS

3.1 Basic states

Results obtained in case studies of separated flows of academic interest and industrial significance are now discussed in some detail, starting with a description of the respective basic states. Figure 2 shows the steady basic flow vorticity in three laminar separated flows; in Figure 2 (a) a flat plate boundary layer is shown, obtained by imposing a free-stream velocity U_e composed of three parts [5, 23]; first U_e is kept constant (unity) up to a point $x = x_i$, where Howarth's deceleration is imposed, second the linear profile $U_e = 1 - \alpha x$ is imposed in $x \in [x_i, x_o]$, which is long enough along the downstream direction for a closed recirculation bubble to be formed and, third, the value attained by U_e at x_o is maintained constant up to the end of the calculation domain. Figure 2 (b) shows extracts of the flow vorticity around a NACA 0012 airfoil at chord Reynolds number $Re = 1000$ and angle of attack $\alpha = 5^o$ [23]. Although the entire domain shown in the left part of Figure 1 has been computed, only the immediate vicinity of the airfoil is shown and the part of the domain which contains the steady trailing edge separation region, as has been analyzed in what follows, is highlighted. In Figure 2 (c) results for a T-106/300 LPT blade at chord Re = 750 [1] analogous to those in Figure 2 (b) are presented. Despite the different means of producing separation, in all three cases steady closed separation bubbles have been obtained.

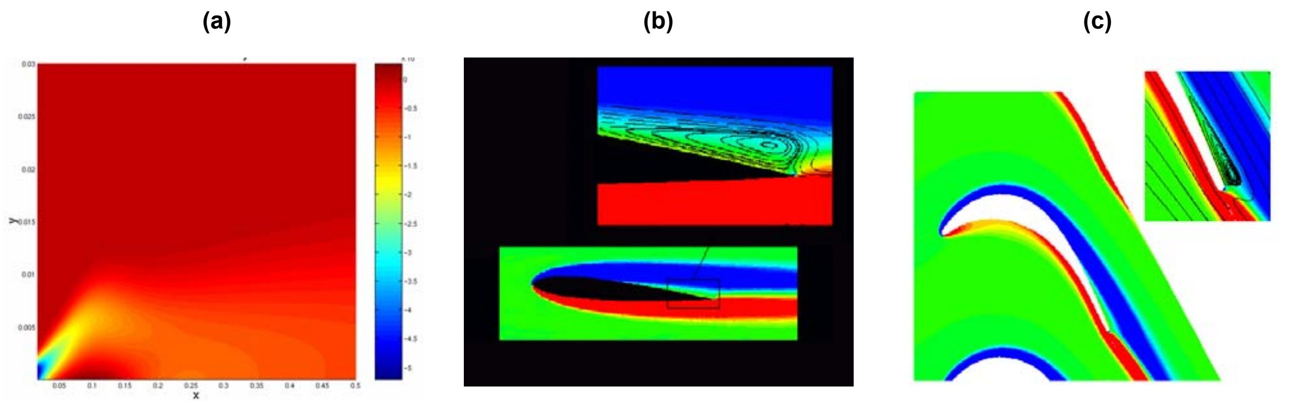

Figure 2: Basic flow vorticity in (a) the flat plate [22], (b) the NACA 0012 airfoil at an angle of attach [23, 20] and (c) the T-106/300 Low Pressure Turbine blade [1].

3.2 Instability analyses

A full discussion of the instability characteristics of the respective configurations, including details on the different scalings and parameter values at which global instability of the separated region is to be expected, may be found in the original references. Attention here is focused on the single most unstable or least damped BiGlobal eigenmodes in the respective applications, at the (different, three-dimensional) conditions at which this mode attains maximum amplification (or minimum damping) in an attempt to highlight the relationship of these modes amongst themselves and with other potentially existing instabilities in the respective flowfields.

One of the intriguing results in the work of Theofilis *et al.* [22] has been the spatial structure of the most amplified three-dimensional (stationary) BiGlobal eigenmode of the Briley bubble. The dominant disturbance velocity component is that along the streamwise direction; it peaks in the neighbourhood of primary reattachment and gives rise to a sequence of secondary reattachment/separation/reattachment. The latter, on account of the three-dimensional nature of this instability, results in a highly convoluted imprint of the primary reattachment line on the flat plate surface, while the primary separation line remains unaffected by the amplification of this instability. The agreement between the conjectured scenario of topological changes in the reattachment zone of a laminar separation bubble during vortex shedding, alongside the associated necessary wall-shear distribution, put forward by Dallmann, Vollmers, Su and Zhang [8] and the computed wall-shear distribution ensuing instability of the flat-plate global mode computed by Theofilis *et al.* [23] is shown in Figure 3, where $|\varepsilon|$ indicates percentage magnitude of the basic flow maximum. The spatial structure of the

BiGlobal eigenmode is shown, in terms of the spanwise disturbance velocity component in Figure 5 (a).

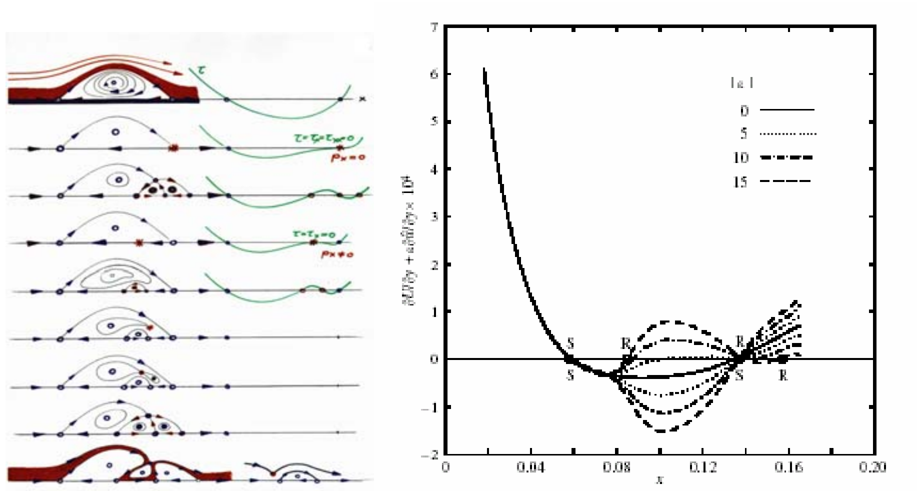

Figure 3: Left: Conjectured vortex-shedding mechanism and associated wall-shear distribution on account of a globally-unstable laminar separation bubble [8]. *Right*: computed wall-shear distribution of the most unstable BiGlobal eigenmode superimposed upon τ_w of the Briley bubble, indicating the generation of secondary reattachment/separation/reattachment in the primary reattachment region [22].

The existence of the BiGlobal mode of the separation bubble and the generality of the conclusions put forward in [22] was examined in the two subsequent case studies concerning more complex geometries, an airfoil and a turbine blade. Two related works [23, 20], which used independent numerical approaches, also detected a BiGlobal eigenmode in the trailing-edge separation region of a NACA 0012 airfoil at an angle of attack $\alpha = 5^o$. The spatial structure of the disturbance eigenfunctions of this mode is analogous to that of the flat-plate mode. In line with the result on the flat plate, the disturbance velocity component along the streamwise direction dominates over that along the normal and the spanwise disturbance velocity component, the latter shown in Figure 5(b). In contrast to the flat-plate disturbance, the global mode of the airfoil trailing-edge separation is damped at the Reynolds number and all (three-dimensional) spanwise wavenumbers investigated. It is to be expected that a further increase of the Reynolds number will result in the eigenvalue of this mode changing sign, rendering the results of the flat-plate and NACA airfoil analyses completely analogous as far as separated flow global instability is concerned. However, a different point of view was taken in the analysis of Theofilis, Barkley and Sherwin [20]. While in [23] only the portion of the flowfield around the trailing edge separation was analyzed, in [20] the instability of the entire basic state developing upon the domain shown in the left part of Figure 1 was investigated. The latter generalization permitted the identification of a *different* instability mode developing in the wake of the airfoil, which is amplified at the same set of parameters. It could thus be concluded that the first bifurcation in the NACA 0012 application will result on account of unsteadiness in the wake, much like in the classic circular cylinder flow [4].

The same scenario as in the NACA airfoil, i.e. unsteadiness of the wake preceding instability of the trailing-edge separation, was shown to occur in the flowfield around the LPT blade model investigated by Abdessemed *et al.* [1]. The latter work utilized exclusively the spectral/*hp* element methodology, which permitted direct comparisons between the different instability modes. Three-dimensional BiGlobal analysis has been performed using a spanwise wavenumber, β, associated with the extent of the domain in that direction, L_z, by $L_z = 2\pi / \beta$.

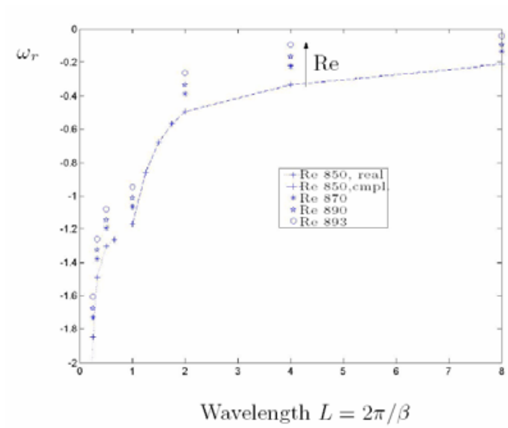

Figure 4: Three-dimensional instability analysis of flow around the T-106/300 low pressure turbine blade model; plotted are the damping rate ω_r as a function of the spanwise wavelength L_z [1].

Parametric studies were performed by changing β at fixed *Re*, in a range of subcritical Reynolds numbers. Results are shown in Figure 4, where each symbol represents a constant Reynolds number and two bands of BiGlobal eigenmodes, overlapping around $L_z \approx 1$, may be identified. All real-parts of the eigenvalues remain negative, with a tendency toward the positive complex plane for increasing Reynolds number and decreasing β, suggesting that three-dimensional linear instability of the trailing-edge separation bubble does not occur below the critical Reynolds number for unsteadiness of the two-dimensional base state shown in Figure 1(c). The eigenmodes obtained for $L_z \leq 2/3$ correspond to global eigenmodes of the trailing-edge separation region, are stationary and have the same qualitative characteristics with those of the eigenmodes of the flat-plate and the NACA 0012 airfoil, in terms of frequency (stationary) and spatial distribution of their amplitude functions; that of the spanwise disturbance component is shown in Figure 5 (c). On the other hand, eigenmodes obtained at $L_z > 1$ are distinct from the separation bubble disturbance and are damped less strongly than that mode, except in the immediate vicinity of $L_z = 1$. The spatial structure of the large wavelength disturbances reveals that these modes develop in the LPT blade wake, in a manner analogous with instability of the airfoil and, in turn, the circular cylinder applications.

(a) **(b)** **(c)**

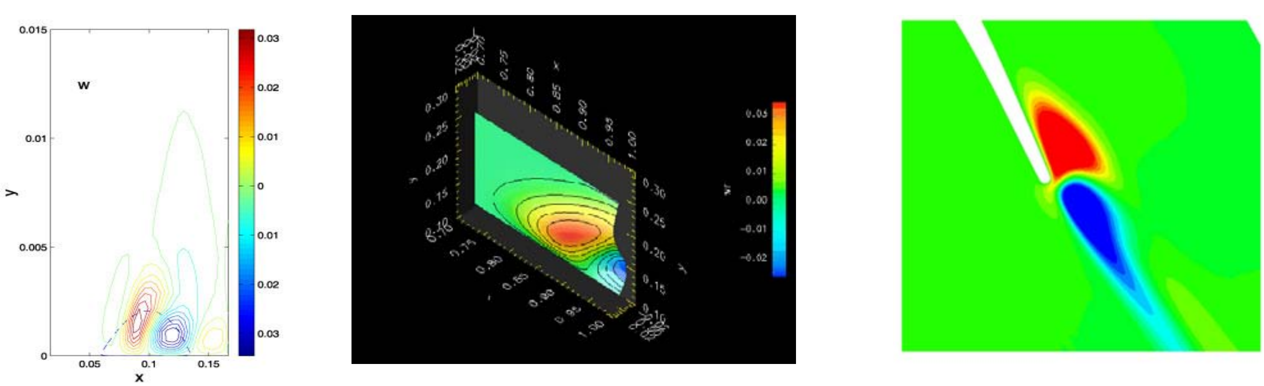

Figure 5: The spatial structure of the most unstable / least damped BiGlobal eigenmode of the basic flows of Figure 2, visualized by isosurfaces of the spanwise disturbance velocity component, in the cases of (a) the flat plate [22], (b) the NACA 0012 airfoil [23] and (c) the LPT blade [1].

4 DISCUSSION

Comparing the respective analyses, probably the most interesting instability result in the separation region on the flat-plate, the NACA airfoil and the LPT blade, is the very existence of BiGlobal eigenmodes pertinent to the separated flow region in all three flows. Despite the quantitative differences in the scalings and parameter values at which the respective eigenmodes attain their maxima in the three applications, striking similarities in the frequency and spatial structure of the disturbance velocity components of the most amplified/least damped global eigenmode in the three applications has been documented.

However, when comparing these modes with other types of instabilities, one observes that in all three cases the BiGlobal eigenmodes of the separated region are less significant (in terms of magnitude of their amplification/damping rates) than the inviscid instabilities of the separated shear layer in the flat plate and, in addition, those of the wakes in both the airfoil and the turbine blade. Interestingly, in all three applications the instability characteristics of the dominant disturbances can be approximated by application of classic linear theory, simplifying the respective two dimensional basic states by one-dimensional profiles at successive downstream distances, namely the shear layer on the flat plate and the wake profiles in the other two applications. This fact may have contributed to the global eigenmodes on the applications in questions having gone unnoticed until recently, despite extensive efforts having been devoted to the problem of instability of separated flows over several decades.

Two final comments are in order here, first regarding the potential relevance of weakly amplified (or damped) eigenmodes to the problem of (reduced) flow modelling and control. Recent efforts to reconstruct turbulent flowfields using (empirical Galerkin) reduced order models [14, 18] have demonstrated that inclusion of instability information (in the cases studied in [14, 18] the most unstable BiGlobal eigenmode) is essential for the accurate reproduction of the time-signal, as obtained in experiment or DNS [13]. The ability to address global instability of separated flow in an accurate manner should pave the way for new activities in this direction of flow modelling. Second, the same capability to solve the BiGlobal EVP, in order to identify the full spectrum of eigendisturbances of nonparallel flows, could be exploited in the context of solution of the associated singular value decomposition problems, which lie at the heart of transient growth studies. Both avenues are completely open, in the authors' view, worthy of pursuing in future studies.

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