Scaling of the velocity fluctuations in turbulent channels up to $Re_\tau=2003$

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A new numerical simulation of a turbulent channel in a large box at $Re_\tau=2003$ is described and briefly compared with simulations at lower Reynolds numbers and with experiments. Some of the fluctuation intensities, especially the streamwise velocity, do not scale well in wall units, both near and away from the wall. Spectral analysis traces the near-wall scaling failure to the interaction of the logarithmic layer with the wall. The present statistics can be downloaded from http://torroja.dmt.upm.es/ftp/channels. Further ones will be added to the site as they become available.
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A direct simulation of a plane turbulent channel has been performed in a computational box with streamwise and spanwise periodicities $L_x=8\pi h$ and $L_z=3\pi h$, at a Reynolds number $Re_\tau=2003$ based on the friction velocity u_τ and on the channel half-width h. It is therefore comparable with previous simulations at lower Reynolds numbers by our group.[1,2] We integrate evolution equations for the wall-normal vorticity ω_y and for the Laplacian of the wall-normal velocity $\phi=\nabla^2 v$, as in Ref. 3. The streamwise and spanwise coordinates are x and z, and the corresponding velocity components are u and w. The spatial discretization uses dealiased Fourier expansions in x and z, and seven-point compact finite differences in y, with fourth-order consistency and extended spectral-like resolution.[4] The temporal discretization is third-order semi-implicit Runge-Kutta.[5] The details of the code and of the parallelization strategy, as well as a fuller analysis of the results, will be the subject of future publications. Here we restrict ourselves to the description of the low-order statistics and to the discussion of the scaling of the velocity fluctuation intensities in the newly extended range of Reynolds numbers.

Table I summarizes the parameters of the present simulation as well as those of the previous ones used for comparison. It uses $N_x=6144$, $N_y=633$, $N_z=4608$ collocation points. The simulation ran for about 6×10^6 processor hours on 2048 processors of the Marenostrum computer at the Barcelona Supercomputing Center and generated approximately 25 TB of raw data. The wall-normal grid spacing is adjusted to keep the resolution, $\Delta y=1.5\eta$, approximately constant in terms of the local isotropic Kolmogorov scale $\eta=(\nu^3/\varepsilon)^{1/4}$. It is slightly better in that respect than those of the lower Reynolds numbers. At the center of the channel the resolutions along the three coordinates are approximately equal, $\eta-1.8\eta$, comparable to those of well-resolved simulations of isotropic turbulence.[6] The microscale Reynolds number at that location is $Re_\lambda\approx94$. The running times are given in terms of turnover periods for eddies of size h and of velocity u_τ. For the present box they are roughly equivalent to flowthroughs. The rms differences between one-point statistics over the first and second halves of the run are of the order of 1%, and the asymmetry of the profiles is of the same order.

The mean velocity profile is shown in Fig. 1(a) in terms of the inverse Kármán "constant" $y\partial_y U^+$, where U^+ is the mean velocity in wall units. This parameter is nowhere constant, but that is also the case for the high-Reynolds-number pipe data[7] included for comparison, and for other experimental data not included in the figure. This is obviously the effect of the higher-order terms usually included in the "wake" component of the profile, but the minimum is reached in all the numerical cases around $y^+=50$, calling into question the possibility, or even the relevance, of estimating the extent of the logarithmic layer in this way. If we take the usual estimate that $y^+\gtrsim100$ and $y/h\lesssim0.2$, the present simulation has a logarithmic region extending over a factor of 4. We will see below that there is a sizable part of the channel over which some length scales are approximately proportional to the wall distance y.

The rms velocity fluctuation profiles are shown in Fig. 1(b). None of them collapses exactly in wall units.

In the case of the transverse components, v' and w', the effect is weak. The three cases collapse well near the wall except within the buffer layer, where viscous effects are important. To our knowledge, the presence of a peak in the spanwise component had not been described before, and it is unclear whether it would tend to a finite limit at infinite Reynolds numbers. Away from the wall there is a weak tendency for v' and w' to increase with increasing Reynolds numbers, most noticeably at the center of the channel. Experimental data in pipes and channels, not shown, lend some support to this trend, although the scatter is large and values tend to cluster by facility rather than by Reynolds number.

The clearest effect is in the streamwise intensity u', which collapses neither near the wall nor far from it. The scaling failure near the wall was first reported in boundary layers,[8] but there is no general agreement on its causes, and the authors of that article made its explanation a challenge for theoreticians. The available data are collected in Fig. 1(c). The intensity in the near-wall peak, which is always achieved around $y^+=15$, increases steadily with the Reynolds number. The numerical cases confirm that trend. Any failure

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TABLE I. Summary of cases. The resolution is measured in collocation points, and the symbols are used in the figures.

	Re_τ	Δx^+	Δz^+	$\Delta y^+_{\max}$	L_x/h	L_z/h	tu_τ/h
▽, Ref. 1	547	8.9	4.5	6.7	8π	4π	12.0
△, Ref. 2	934	9.2	3.8	7.6	8π	3π	8.5
○, Present	2003	8.2	4.1	8.9	8π	3π	10.3

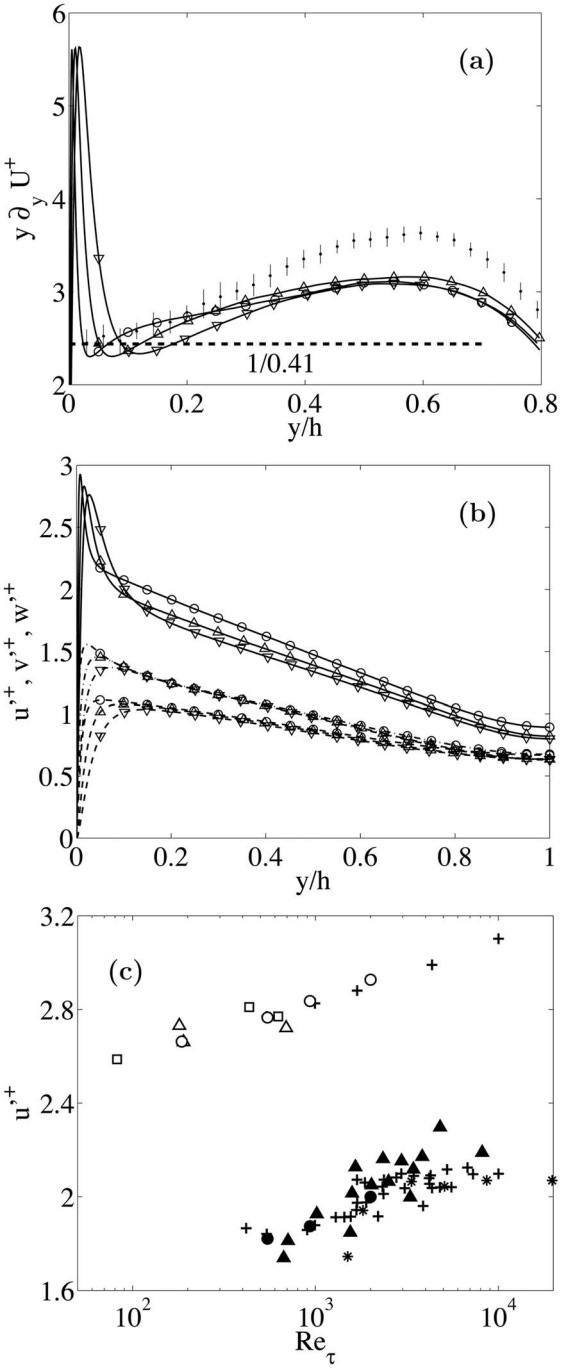

FIG. 1. (a) Inverse Kármán constant. Symbols as in Table I. Dots with error bars are pipes (Ref. 7) with $\mathrm{Re}_\tau > 2000$. (b) Velocity fluctuation intensities, in wall units. —, u'; – – v'; –·–, w'. (c) Streamwise fluctuation intensities from various sources. Open symbols, near-wall maximum; closed symbols, $y/h=0.15$, $y^+>60$. □, Couette flows; △, pipes and channels; *, pipes from Ref. 9; +, boundary layers; ○, present numerical channels.

of near-wall scaling must come from the interaction of near-wall modes that scale in wall units with modes residing away from the wall, for it is only in that way that the Reynolds number is defined. The best-known proposal involves wall-parallel large-scale modes that reach the wall without being constrained by impermeability.[10] It was suggested in Ref. 1, on the basis of lower-Reynolds-number simulations, that the important outer modes were global ones spanning the whole channel and scaling with the boundary layer thickness, and others have also described the near-wall spectrum in terms of the superposition of "inner" and "outer" energy peaks.[11] The subject of most of the rest of this paper is the discussion of whether the data sustain such "two-scale" models.

The behavior of the intensities in the logarithmic layer is also included in Fig. 1(c), but it is less clear. There is a general growing trend, also shared by the numerics, but it could be argued that boundary layers saturate above $\mathrm{Re}_\tau \approx 3000$, and at least one experiment[9] has concluded the same for pipes. It is plotted separately for emphasis. The subject has been discussed, for example, in Ref. 2, but, except to present the additional data point in this figure, it will not be further discussed here.

Two-dimensional (2D) spectral energy densities at the height of the near-wall kinetic-energy maximum are shown in Fig. 2. Two isolines are given for each case, representing the high-intensity core of the spectrum, and its outer border. They confirm the results of simulations at lower Reynolds numbers.[1] The core isolines scale well in wall units. Because the kinetic energy is the integral of the spectrum, this part of the energy also scales in wall units. The scaling failure appears for u and w in the upper right-hand corner, where a spectral ridge extends roughly along $\lambda_z = 0.15\lambda_x$. The ridge gets longer as the Reynolds number increases, reaching up to $\lambda_x \approx 10h$ in the case of ϕ_{uu}. The eddies in this ridge are inactive in the sense of Townsend.[10] They are neither found in the v spectrum, nor in the Reynolds stress cospectra in Fig. 2(c).

The vertical structure of the eddies responsible for the inactive ridge is shown in Fig. 3(a), which shows contours of u'^2 integrated over logarithmic boxes of wavelengths centered along the ridge, with a fixed ratio between their longest (widest) and shortest (narrowest) wavelengths [see Fig. 2(c) for an example]. The energy at the shortest boxes is concentrated near the wall. They fall in the core part of the spectrum, and they correspond to the classical buffer-layer streaks. It follows from Fig. 2(c) that these structures are active and carry Reynolds shear stress. It was hypothesized in Ref. 12, on the basis of lower Reynolds number simula-

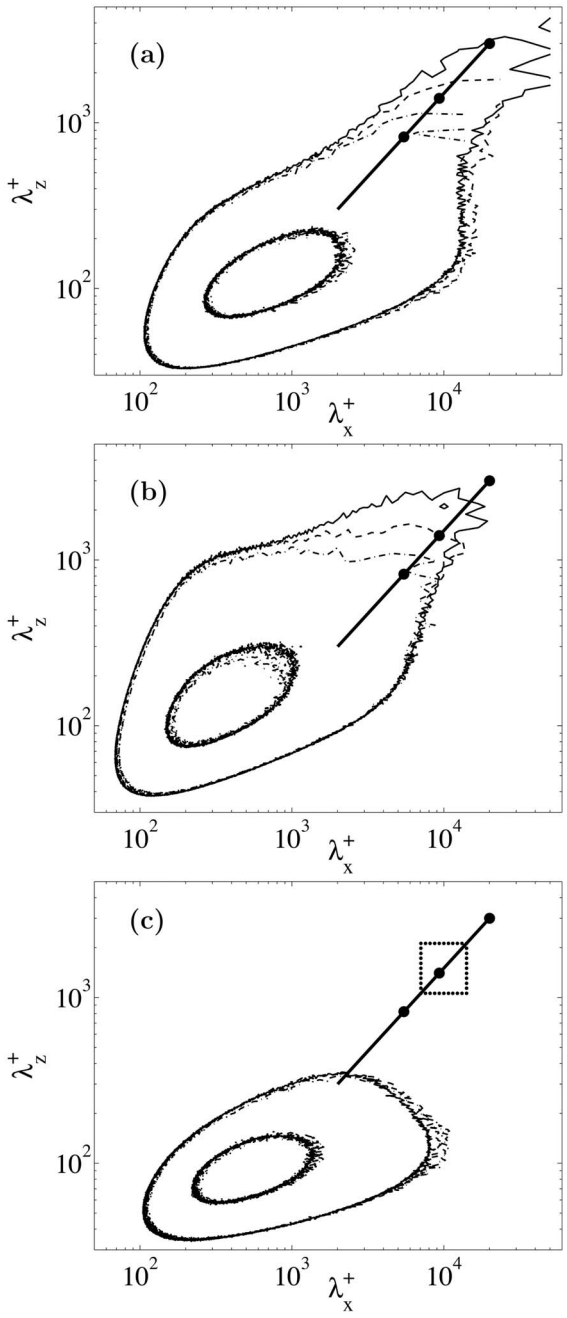

FIG. 2. Spectral energy densities $\phi=k_x k_z E(k_x,k_z)$ at $y^+=15$, in terms of the wavelengths $\lambda=2\pi/k$. (a) ϕ_{uu}. (b) ϕ_{ww}. (c) Cospectrum $-\phi_{uv}$. –·–, $\mathrm{Re}_\tau=547$; – – , 934; —, 2003. Spectra are normalized in wall units, and the two contours for each spectrum are 0.125 and 0.625 times the maximum of the spectrum for the highest Reynolds number. The heavy straight line is $\lambda_z=0.15\lambda_x$, and the heavy dots are $\lambda_x=10h$ for the three cases. The dotted rectangle in (c) is an example of the filtering boxes used to isolate the energies in Fig. 3.

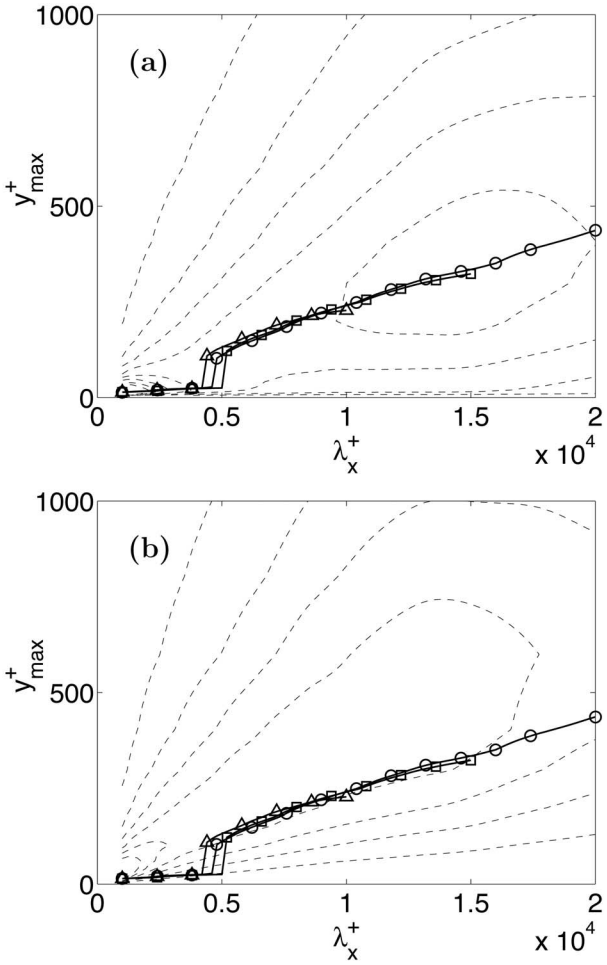

FIG. 3. Contour plots of the spectral energy summed over logarithmic boxes of width α^2, centered along $\lambda_z=0.15\lambda_x$, such as the one in Fig. 2(c), as a function of y and of λ_x at the box center. $\mathrm{Re}_\tau=2003$. (a) Streamwise velocity fluctuations. Contours are 0.04(0.04)0.32. (b) Shear stress cospectrum. Contours are 0.01(0.01)0.06. In both case the solid lines with symbols are the y position of the maxima of the profiles in (a). ○, △, as in Table I with $\alpha=\sqrt{2}$; □, $\mathrm{Re}_\tau=2003, \alpha=2$.

tions, that the streaks are intrinsically infinitely long but that they are truncated by the action of outer layer turbulence. This is now seen not to be the case. The "wall modes" have a definite length of the order of 10^4 wall units, which is independent of the Reynolds number. The ridge modes form a continuum that links those inner structures with the ones scaling in outer units. Figure 3(a) also contains a line joining the position of the maxima of the individual filtered energy profiles. The longer modes peak farther from the wall, and the locations of their peaks grow linearly as $y_{\max}\approx\lambda_x/40=\lambda_z/6$. Note that the locations of the peaks are within the nominal logarithmic layer and that they agree for the two Reynolds numbers for which a comparison is possible. This picture is sensitive to the path in the $(\lambda_x-\lambda_z)$ plane that is used to generate it, but depends little on the size of the filtering box. Figure 3 contains two lines of maxima computed with boxes of different sizes. They agree almost exactly.

Even if the eddies along the spectral ridge are inactive near the wall, the same is not true at the levels where their intensities peak. That is seen in Fig. 3(b), which shows isocontours of the box-filtered Reynolds stress $-u'v'$. The ridge modes become more intense and more active as they grow taller, and they actually constitute the dominant modes within the logarithmic layer. The highest contours in the outer part of both Figs. 3(a) and 3(b) are comparable to the corresponding intensities in the buffer layer at the core of the u spectrum.

If we interpret λ_z as a width and $y_{\max}$ as a vertical semiaxis, the cross sections of the eddies would be oblate ellipses

with aspect ratios of 3:1. Because of the linear growth of both quantities, the eddies can be visualized as conical. In that interpretation, the inclination of their axes would be $\arctan(1/40) \approx 1.5^{\circ}$, although it is clear from Fig. 3(a) that the eddies are not symmetric and that any identification method based on integral quantities would result in somewhat higher angles. The inclination of the upper branches of the isocontours in both figures is approximately 10°–12°, which is of the order of the angles measured for the velocity correlation in experimental boundary layers.[13,14]

In summary, we have presented a new simulation of a turbulent channel containing an appreciable logarithmic layer, at a Reynolds number higher than those currently available. Several features in the logarithmic layer have been shown to grow linearly with wall distance, as in the classical scaling of the overlap region. The scaling failure of the intensity at the near-wall kinetic-energy peak has been shown to be the result of eddies conforming to Townsend's inactive model, restricted at that wall distance to a spectral ridge of long and wide structures that are clearly distinguishable from the near-wall streaks. The latter scale well in wall units. The ridge modes are the near-wall footprints of eddies whose maximum intensities lie in the logarithmic region, and they link the near-wall streaks with the global modes identified elsewhere. They are inactive at the wall, but they support most of the Reynolds shear stresses at the location of their peaks.

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