ULTIMATE HEAT TRANSFER IN COHERENT THERMAL CONVECTION

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Rayleigh–Bénard convection is one of the most canonical flows and the fundamental problem in hydrodynamic stability theory. When the Rayleigh number Ra (dimensionless temperature difference) reaches a certain critical value, convection occurs and becomes turbulent eventually as Ra increases. One of the primary interests in convective turbulence is the scaling law of the Nusselt number Nu (dimensionless vertical heat flux) with Ra, and two power laws for very high Ra, the 'classical' scaling $Nu \sim Ra^{1/3}$ and the 'ultimate' scaling $Nu \sim Pr^{1/2}Ra^{1/2}$ (where Pr is the Prandtl number), have been intensively discussed (see e.g. Ref. [1]). The classical scaling has been reported in many experiments and numerical simulations, and recently found for nonlinear steady solutions to the Oberbeck–Boussinesq equations [2, 3, 4]. On the other hand, the ultimate scaling has not been observed yet in conventional Rayleigh–Bénard convection.

Recently, it has been numerically found that the ultimate scaling can be achieved in thermal convection between permeable walls [5]. In the simulation, the vertical transpiration velocity on the walls is assumed to be proportional to the local pressure fluctuation (Jiménez *et al.*, 2001 [6]). In the present work, we have employed the permeable-wall condition and obtained steady solutions for thermal convection between horizontal, no-slip and permeable walls with a distance H and a constant temperature difference ΔT by using the Newton–Krylov iteration (for details, see Ref. [7]).

Figure 1(a) shows Nu of the two-dimensional (2-D) and three-dimensional (3-D) steady solutions and the turbulent state in the thermal convection between permeable walls as a function of Ra. The ultimate scaling $Nu \sim Ra^{1/2}$ can be observed at high Ra in both steady and turbulent states. It is also confirmed that the steady solutions exhibit higher heat transfer than those in the turbulent state. Thermal and flow structures of the 2-D and 3-D steady solutions at $Ra = 10^7$ are shown in figures 1(b,c). In the steady ultimate states, the buoyancy induces remarkable large-scale thermal plumes fully extending from one wall to the other, leading to strong vertical velocity of the order of the terminal velocity U as well as intense temperature fluctuation of $O(\Delta T)$. Consequently, the wall-to-wall heat flux scales with $U\Delta T$ independent of thermal diffusivity, although the heat transfer on the walls is dominated by thermal conduction.

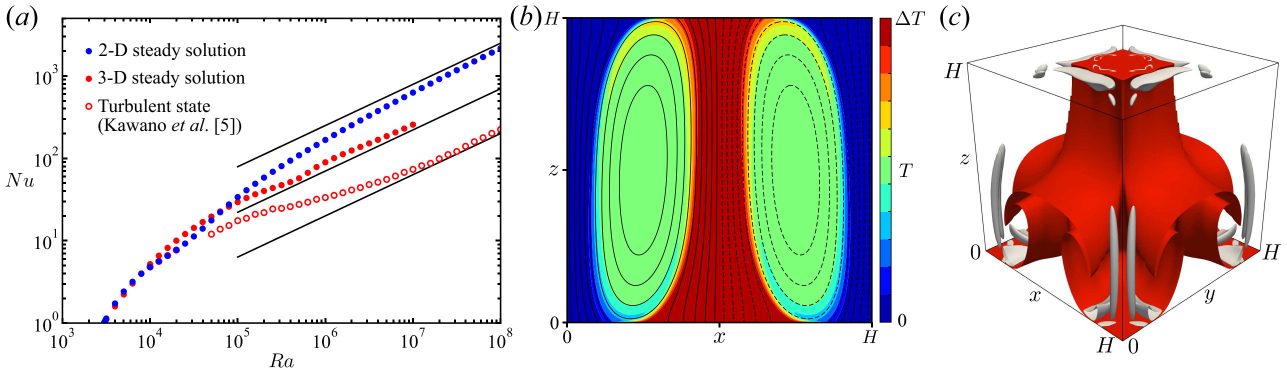

Figure 1. Steady and turbulent thermal convection between permeable walls with the permeability parameter $\beta U = 3$ for the horizontal period $L/H = 1$ and $Pr = 1$. (a) Nu as a function of Ra. The black lines indicate the ultimate scaling $Nu \sim Ra^{1/2}$. (b) Temperature and flow fields of the 2-D steady solution at $Ra = 10^7$. Large-scale counter-clockwise (or clockwise) rolls are visualised by the black solid (or dashed) streamlines. (c) Thermal and vortical structures of the 3-D steady solution at $Ra = 10^7$. The red and grey objects, respectively, represent the isosurfaces of the high temperature and of the positive second invariant of the velocity gradient tensor.

References

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