VARIABILITY OF THE FLOW AROUND AN IMPULSIVELY STARTED PLATE

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The Navier Stokes- and continuity equations describe a deterministic dynamical system in which the current state uniquely determines the future. However, the system has positive Lyapunov exponents, which renders the flow unpredictable. In our experiment a vortical flow turns into turbulence. We exactly repeat this experiment 42 times, and wonder whether the variation between the 42 time-dependent flow fields can be predicted.

An industrial robot [1, 2] accelerates a plate ($0.1 \times 0.2\,\mathrm{m}^2$) in an initial still fluid to a velocity of 0.4 m/s. We measure the 2D projection of the velocity field in a plane using particle image velocimetry (PIV). A single run consists of 1495 PIV frames, sampled every 2 ms. We visualize the wake through the vorticity field, which evolves from two isolated counter-rotating vortices that merge and break up, into a fully turbulent flow.

We gauge the similarity between the flows using the dimensionless error energy, $\Delta u^2(t) = \langle |\boldsymbol{u}_i - \boldsymbol{u}_j|^2 \rangle_{i \neq j} / 2 \langle |\boldsymbol{u}_i|^2 \rangle_i$, where $i, j = 1, \ldots, 42$ are the experiment realizations. For completely uncorrelated flows $\Delta u^2(t) = 1$ [3]. The error rises from $\Delta u^2(t) \approx 10^{-2}$ at the end of the acceleration phase ($t = 0.7\,\mathrm{s}$) approximately as $\Delta u^2(t) \propto \mathrm{e}^{\lambda t}$, with $\lambda \approx 1.1\,\mathrm{s}^{-1}$. While the energy rises to $\Delta u^2(t) \approx 0.2$, the corresponding enstrophy error rises to $\Delta \omega^2(t) \approx 0.9$ at the end of a run ($t = 3\,\mathrm{s}$), indicating that the small scales loose similarity first.

The question is whether observation of a single experiment can predict the variability between experiments. From a single experiment we determine the Finite-time Lyapunov field $\Lambda(\boldsymbol{x})$ that gauges the exponentially fast separation of two nearby fluid parcels [4]. In figure 1(a) we show a snapshot of $\Lambda(\boldsymbol{x})$ in a single run.

We measure the variability between experiments through the difference between trajectories of fluid parcels that were started at the same $\boldsymbol{x}$ in all experiments. For both fields the integration time was $T = 0.128\,\mathrm{s}$. As the figure shows, both fields are strikingly similar, which we view as a demonstration of ergodicity

The symmetry of this flow is characterized by the generated circulation in the top and bottom half of the domain, Γ_1 and Γ_2, such that, averaged over realizations, $\Gamma_1 = -\Gamma_2$. Turbulence induces fluctuations of $\Gamma_{1,2}$. Surprisingly, these fluctuations preserve symmetry.

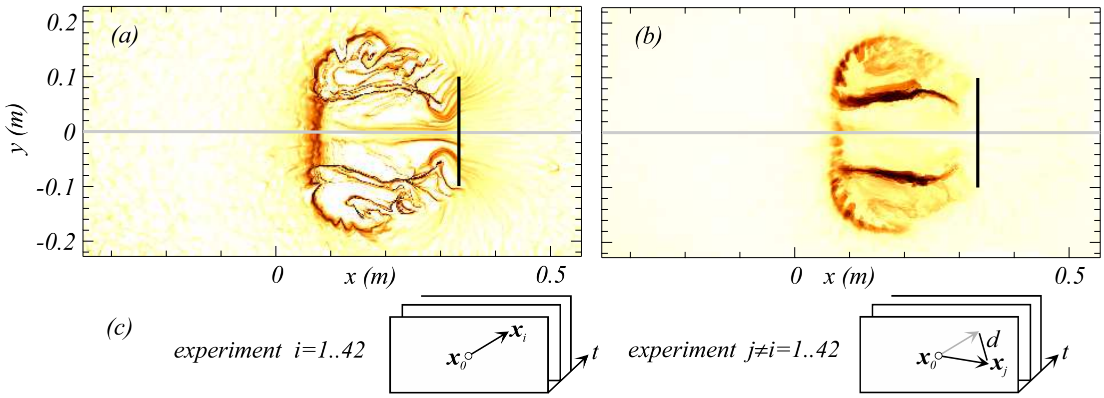

Figure 1. (a) The finite-time Lyapunov field $\Lambda(\boldsymbol{x})$ of the wake of an accelerated plate. At $x = -0.2$ m the plate reaches a constant velocity of 0.4 m/s. The Lyapunov field was computed from a single run (the first of 42 repeated experiments). (b) The variability between 42 repeated experiments, averaged over all 882 pairs. (c) Illustrates the procedure to compute this variability. It is defined as the difference between the final location of fluid parcels that were started exactly the same in each experiment; this final location is interrogated after 0.128 s. The gray lines indicate the symmetry plane.

References

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