CAUSALITY IN ISOTROPIC TURBULENCE AT LOW REYNOLDS NUMBER

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What kind of turbulent structures most affect the flow in homogeneous isotropic turbulence (HIT) at low Reynolds number? How do they affect it? Even though these questions have been discussed for years, they remain open. Here, we address whether causality analysis may provide us with new insights on them.

For our analysis, we perform a large number of three-dimensional direct numerical simulations of HIT in a 64^3 cubical grid, at Reynolds number $Re_\lambda \approx 50$. Following previous work in 2-D turbulence [1, 2], we modify small parts of the flow (cells) at $t = 0$, monitor the effect on the flow, and define as casually significant cells those whose modification leads to a large overall change in the flow at some later measurement time, $t = t_m$. Our goal is to first determine whether especially significant flow regions exist, and, second, whether they share some common characteristic. To do this, 65 basic initial turbulent flows are created, and each of them is divided into 10^3 cubical cells, about 12η on the side. The flow within each cell is modified in eight different ways (4 modifications of the vorticity field, and 4 of the velocity field), and the simulations are continued for one or two turnover times. Several definitions of 'significance' are used, based on the normalized L_2 and L_∞ norms of the velocity and vorticity difference between the modified and unmodified flows at $t = t_m$: i.e., $\|\Delta \boldsymbol{u}\|_2$, $\|\Delta \boldsymbol{u}\|_\infty$, $\|\Delta \boldsymbol{\omega}\|_2$, and $\|\Delta \boldsymbol{\omega}\|_\infty$. They all measure the importance of the original cell in determining the future behaviour of the flow as a whole. This is repeated for each cell and each kind of experimental modification, for a total of $65 \times 1000 \times 8 = 5.2 \times 10^5$ simulations. In addition, two normalizations are used for the significance: 'absolute significance', defined as in $\epsilon_a(L_2 u) = \|\Delta \boldsymbol{u}\|_2 / \|\boldsymbol{u}\|_2^{t=0}$, and 'relative significance', as in $\epsilon_r(L_2 u) = \|\Delta \boldsymbol{u}\|_2 / \|\Delta \boldsymbol{u}\|_2^{t=0}$. Norms without superscript are evaluated at $t = t_m$. The final step is to find which 'diagnostic' cell properties, defined as cell averages $\langle \bullet \rangle_c$ at $t = 0$, can best be used to predict whether cells will turn out to be significant or not.

Our results indicate that especially significant regions exist in HIT at low Reynolds number. Figure 1 shows the classification score of three different diagnostic quantities, where unity means 100% classification success, zero means 100% failure, and 0.5 is akin to a random guess. Figure 1(a), for a vorticity experiment, shows that the most sensitive cells are in that case those with initially high enstrophy, and that their effect peaks at about half a turnover time. Figure 1(b), for a velocity experiment, shows that the diagnostic quantity in this case is high initial kinetic energy. Note that these are not trivial results, such as that strong perturbations have strong effects, because the diagnostic ability applies to both absolute and relative significance, especially for velocity perturbations. In general, high enstrophy and high-energy cells do not coincide, suggesting two different types of significant flow configurations.

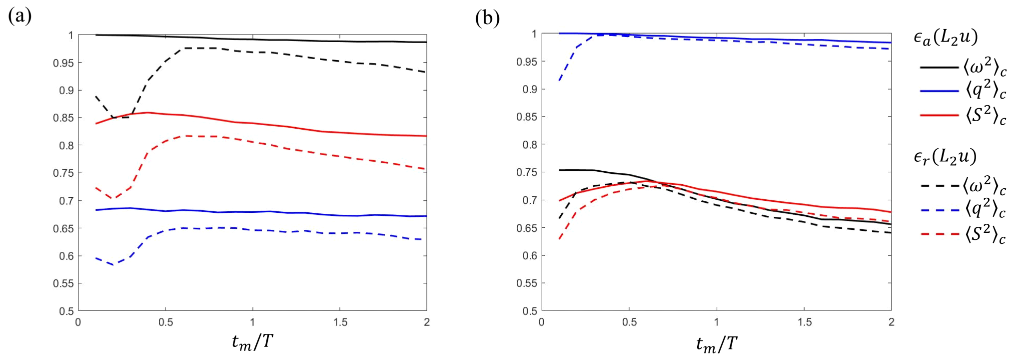

Figure 1. Score for the classification of cells as significant or insignificant, using three different diagnostic quantities, $\langle \omega^2 \rangle_c$, $\langle q^2 \rangle_c$, and $\langle S^2 \rangle_c$. For perturbations (a) $\boldsymbol{\omega} \to 0$. (b) $\boldsymbol{u} \to 0$. Significant and insignificant regions are defined by the L_2 norm of the velocity difference, $\|\Delta \boldsymbol{u}\|_2$, both as 'absolute significance', $\epsilon_a(L_2 u)$, or as 'relative significance', $\epsilon_r(L_2 u)$.

References

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