COMMUTATION ERRORS IN PITM SIMULATION

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1 Introduction

Large eddy simulation is a promising route. This approach has been largely developed in the past two decades for simulating unsteady flows. But up to now, even with the increase of super-computer power, this method is not affordable in term of computational resources for industrial applications and especially for flows performed at high Reynolds numbers. This is the reason which has led researchers to develop hybrid RANS-LES methods that combine the advantages of both RANS and LES methods, see Fröhlich and Von Terzi (2008). Among these hybrid RANS-LES methods, we focus interest in the partially integrated transport modeling (PITM) method introduced by Schiestel and Dejoan (2005) using a two equation subfilter scale viscosity model and by Chaouat and Schiestel (2005) for the extension to stress transport subfilter scale models in the framework of second moment closure (SMC). The PITM method used for hybrid RANS-LES simulations with seamless coupling between RANS and LES regions proved a promising route, as shown by Chaouat (2012), Chaouat and Schiestel (2013). This method was initially developed for constant filter width or at least filter width slowly varying in time and space. In the present work, we examine the effect of variable filter width in the PITM model equations, the commutation errors arising from the non-commutativity of the filtering process with temporal or spatial derivatives, see Ghosal and Moin (1995), van der Boss and Geurts (2005), Geurts and Holm (2006), Hamba (2011), and the way to account for this effect in numerical simulations. We perform PITM numerical simulations of isotropic decaying turbulence on an expanding grid for the case where the commutation errors are distributed in the whole field as well as the fully developed turbulent channel flow on several grids to investigate the effect of a sudden step refinement and enlargement in the streamwise grid size.

2 Commutation terms

Turbulent flow of a viscous incompressible flow is considered. In large eddy simulation, the variable \( \phi \) is decomposed into a large scale (or resolved part) \( \bar{\phi} \) and a subfilter-scale fluctuating part \( \phi^* \) or modeled part such that \( \phi = \bar{\phi} + \phi^* \). The filtered variable \( \bar{\phi} \) is defined by the filtering operation as the convolution

with a filter \( G \) in space \( \bar{\phi} = G \ast \phi \) that leads to the computation of a variable convolution integral

\[
\bar{\phi}(x, t) = \int_{\mathbb{R}^3} G[x - \xi, \Delta(x, t)] \phi(\xi, t) d\xi
\]  (1)

where in this expression, \( \Delta \) denotes the filter-width that varies in time and space. Due to the fact that the filtering operation does not commute with the space derivative, a commutation term appears in the derivative as

\[
\frac{\partial \bar{\phi}}{\partial x_j}(x, t) = \frac{\partial \bar{\phi}}{\partial x_j}(x, t) + \frac{\partial \Delta}{\partial x_j} \frac{\partial \bar{\phi}}{\partial \Delta}(x, t) \tag{2}
\]

and equivalently, if transposing Equation (2) in time.

Then, if using Equation (2) and the corresponding equation in time, we get

\[
\frac{\partial \bar{\phi}}{\partial t} = \frac{\partial \phi}{\partial t} + \frac{\partial (u_j \phi)}{\partial x_j} \tag{3}
\]

The correlation \( u_j \phi \) appearing in Equation (5) can be developed in a more explicit form as

\[
u_j \phi = u_j \bar{\phi} + [u_j \phi - \bar{u}_j \bar{\phi}] = \bar{u}_j \bar{\phi} + \tau(u_j, \phi) \tag{6}
\]

where \( \tau(u_j, \phi) \) is defined as \( \tau(u_j, \phi) = u_j \bar{\phi} - \bar{u}_j \bar{\phi} \). So that Equation (5) becomes

\[
\frac{\partial \bar{\phi}}{\partial t} = \frac{\partial \bar{\phi}}{\partial t} + \frac{\partial (u_j \bar{\phi})}{\partial x_j} + \frac{\partial \tau(u_j, \phi)}{\partial x_j} - \beta_T(\phi) \tag{7}
\]

where \( \beta_T(\phi) = \beta_1(\phi) + \beta_{x_j}(u_j \phi) \) with

\[
\beta_1(\phi) = \frac{\partial \Delta}{\partial t} \frac{\partial \bar{\phi}}{\partial \Delta} \tag{8}
\]

\[
\beta_{x_j}(u_j \phi) = \frac{\partial \Delta}{\partial x_j} \frac{\partial}{\partial \Delta} \left( u_j \bar{\phi} + \tau(u_j, \phi) \right) \tag{9}
\]
As a result, Equation (7) including $\beta_T$ will be the main functional operator that will be used as a base throughout the following work.

3 Filtered Navier-Stokes equation

Using the functional operator (7), the exact filtered equation of motion conservation takes the form as

$$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial (\bar{u}_i \bar{u}_j)}{\partial x_j} - \beta_T(u_i) = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 \bar{u}_i}{\partial x_j \partial x_j} \frac{\partial^2 \bar{u}_i}{\partial \Delta^2}$$

and the filtered Navier-Stokes equation for the motion is

$$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial (\bar{u}_i \bar{u}_j)}{\partial x_j} - \beta_T(u_i) = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 \bar{u}_i}{\partial x_j \partial x_j} \frac{\partial^2 \bar{u}_i}{\partial \Delta^2}$$

with the general definition $\tau(f, g) = \int g - f\, \hat{g}$ and $\tau(f, g, h) = \int g \, \tau(g, h) - \int f \, \hat{g} \, (h - \hat{h}) \, f - \hat{f} \, \hat{g}$ for any turbulent quantities $f, g, h$ and where $S_{ij}$ denotes the strain deformation

$$S_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$$

As indicated by Chaouat and Schiestel (2013), the commutation terms in Equation (16) takes the very complex form as

$$\beta_T(u_i u_j) - \bar{u}_i \beta_T(u_j) - \bar{u}_j \beta_T(u_i) = \frac{\partial \bar{u}_i}{\partial \Delta} \frac{\partial \bar{u}_j}{\partial \Delta} + \frac{\partial \bar{u}_j}{\partial \Delta} \frac{\partial \bar{u}_i}{\partial \Delta} + \frac{\partial \bar{u}_i}{\partial \Delta} \frac{\partial \bar{u}_j}{\partial \Delta} + \frac{\partial \bar{u}_j}{\partial \Delta} \frac{\partial \bar{u}_i}{\partial \Delta}$$

but in practice, it is reduced in a first approximation to

$$\beta_T(u_i u_j) - \bar{u}_i \beta_T(u_j) - \bar{u}_j \beta_T(u_i) \approx \frac{D \Delta \partial \bar{u}_i}{D \Delta}$$

The subfilter turbulent energy is obtained by the tensor contraction of the subfilter scale stress tensor $\tau(u_i, u_j)$ as

$$k_{sfs} = \frac{1}{2} \tau(u_i u_j)$$

leading to its transport equation deduced from Equation (16) as

$$\frac{\partial k_{sfs}}{\partial t} + \frac{\partial (k_{sfs} \bar{u}_k)}{\partial x_k} - \frac{1}{2} \frac{\partial \tau(u_i, u_j)}{\partial x_k} \frac{\partial \bar{u}_k}{\partial x_k} = \frac{1}{2} \frac{\partial \bar{u}_k}{\partial x_k} \frac{\partial \bar{u}_k}{\partial x_k} - \frac{\partial \bar{u}_i}{\partial x_k} \frac{\partial \bar{u}_j}{\partial x_k}$$

with also for the turbulence energy commutation terms, the simplified and more tractable form as

$$\beta_T(u_i u_j/2) - \bar{u}_i \beta_T(u_j) \approx \frac{\partial k_{sfs}}{\partial \Delta} \frac{D \Delta}{D t}$$
5 Modeled PITM equations

The modeled subfilter scale stress transport equation deduced from Equation (16) is derived as

\[
\frac{\partial \tau_{ij}}{\partial t} + \frac{\partial}{\partial x_k} \left( \tau_{ij} \bar{u}_k \right) = P_{ij} + \frac{D \Delta}{2} \frac{\partial \tau_{ij}}{\partial \Delta} + \Pi_{ij} - \frac{2}{3} \delta_{ij} \epsilon_{sfs} + J_{ij}
\]

where \( P_{ij} \) denotes the exact production term,

\[
P_{ij} = -\tau_{jk} \frac{\partial \bar{u}_j}{\partial x_k} - \tau_{jk} \frac{\partial \bar{u}_i}{\partial x_k}
\]

the redistribution term \( \Pi_{ij} \) is usually decomposed into a slow part \( \Pi^1_{ij} \) which characterizes the return to isotropy due to the action of turbulence on itself and a rapid part \( \Pi^2_{ij} \) which describes the return to isotropy by action of the filtered velocity gradient,

\[
\Pi^1_{ij} = -c_1 \frac{\epsilon_{sfs}}{k_{sfs}} \left( \tau_{ij} - \frac{2}{3} k_{sfs} \delta_{ij} \right)
\]

and

\[
\Pi^2_{ij} = -c_2 \left( P_{ij} - \frac{1}{3} P_{mm} \delta_{ij} \right)
\]

where \( c_1 \) is the Rotta coefficient modified to account for the spectrum splitting whereas \( c_2 \) remains the same as in statistical modeling. The diffusion term \( J_{ij} \) is modeled assuming a well known gradient law hypothesis

\[
J_{ij} = \frac{\partial}{\partial x_k} \left( \nu \frac{\partial \tau_{ij}}{\partial x_k} + c_s \frac{k_{sfs}}{\epsilon_{sfs}} \tau_{kl} \frac{\partial \tau_{ij}}{\partial x_l} \right)
\]

where \( c_s \) is a constant numerical coefficient. The subfilter dissipation-rate \( \epsilon_{sfs} \) equation including the commutation terms is modeled as

\[
\frac{\partial \epsilon_{sfs}}{\partial t} + \frac{\partial}{\partial x_k} \left( \epsilon_{sfs} \bar{u}_k \right) = J_\epsilon + \frac{c_{sfs}}{k_{sfs}} \left[ P + \frac{D \Delta}{2} \frac{\partial k_{sfs}}{\partial \Delta} \right] - c_{sfs} \frac{\epsilon_{sfs}^2}{k_{sfs}}
\]

where \( J_\epsilon \) denotes the diffusion term modeled as

\[
J_\epsilon = \frac{\partial}{\partial x_j} \left( \nu \frac{\partial \epsilon_{sfs}}{\partial x_j} + c_s \frac{k_{sfs}}{\epsilon_{sfs}} \tau_{jm} \frac{\partial \epsilon_{sfs}}{\partial x_m} \right)
\]

including a constant coefficient \( c_s \). As mentioned by Chaouat and Schiestel (2012), the coefficient \( c_{sfs} \) is now a linear function of the ratio of the subfilter energy to the total energy \( k_{sfs}/k \) as follows

\[
c_{sfs} = c_{e1} + \frac{(k_{sfs})}{k} \left( c_{e2} - \frac{3}{2} \right)
\]

whatever the \( c_{e1} \) and \( c_{e2} \) values used in RANS and where \(< . > \) denotes the statistical average. The ratio \( (k_{sfs})/k \) appearing in Equation (30) is evaluated by reference to an analytical energy spectrum \( E(\kappa) \) inspired from a Von Kármán spectrum considered as a limiting equilibrium distribution. Analytical developments made by Chaouat and Schiestel (2009) lead to the final result

\[
c_{sfs} (\eta_c) = c_{e1} + \frac{c_{e2} - c_{e1}}{[1 + \beta \eta_c^{1/2}]}\(31\)
\]

Equation (31) indicates that the parameter \( \eta_c \) acts like a dynamic parameter which controls the location of the cutoff within the energy spectrum and the value of the function \( c_{sfs} \) then controls the relative amount of turbulence energy contained in the subfilter range. The theoretical value of the coefficient \( \beta \) in Equation (31) has been found to be \( \beta \approx 2/(3C_K) \) where \( C_K \) is the Kolmogorov constant. In order to account in a simple way for the anisotropy of the grid near the walls, the effective filter \( \Delta(\Omega) \) around the cell \( \Omega \) is defined by

\[
\Delta(\Omega) = \Delta_n \left( \zeta + 1 - \zeta \frac{\Delta_n}{\Delta_9} \right)
\]

where the filters lengths \( \Delta_n \) and \( \Delta_9 \) are defined by \( \Delta_n = \left( \Delta_1 \Delta_2 \Delta_3 \right)^{1/3} \) and \( \Delta_9 = (\Delta_1^2 + \Delta_2^2 + \Delta_3^2)/3 \) and where \( \zeta \) is a parameter set to 0.8. As usually made in LES simulations, the resolved stresses is defined by the relation

\[
\langle \tau_{ij} \rangle_{res} = \bar{u}_i \bar{u}_j - \bar{u}_i \bar{u}_j
\]

The full Reynolds stress tensor \( R_{ij} \) including the small and large scale fluctuating velocities is computed as the sum of the subfilter and resolved stresses.

6 Numerical estimate of extra terms arising from the non-commutativity

For any quantity \( \phi \), Equation (13) can be written as

\[
\beta_T(\phi) \approx \frac{\partial \Delta}{\partial \Delta} \frac{\partial \phi}{\partial \Delta} + \frac{\partial \phi}{\partial \tau_{ij}} \frac{\partial \Delta}{\partial \tau_{ij}} = \frac{D \Delta}{\partial \phi} \frac{\partial \Delta}{\partial \phi}
\]

where in this expression, \( \partial \phi / \partial \Delta \) must be evaluated. In the present case, as proposed by Iovieno and Tordella (2003), this term is determined by applying a second filtering operation, with a larger width. As a result, whatever the variable \( \phi \), the gradient \( \partial \phi / \partial \Delta \) is computed as

\[
\frac{\partial \phi}{\partial \Delta} \approx \frac{\tilde{\phi} - \phi}{\tilde{\Delta} - \Delta}
\]

where \( \tilde{\Phi} \) denotes the twice filtered variable and \( \tilde{\Delta} \) the width of the superfilter. In practice \( \Delta = 2 \Delta \). The corresponding filtered velocity \( \tilde{\bar{u}}_i \) is obtained by applying the superfilter to the currently calculated resolved solution at each time step of the calculation.

7 Numerical schemes

The numerical simulations are performed using the numerical code developed by Chaouat (2011) based
on a finite volume technique including a Runge-Kutta scheme of fourth-order accuracy in time with a combination of a quasi-centered scheme of fourth-order accuracy in space which has shown good numerical properties.

8 Decay of isotropic turbulence

With the aim to illustrate the theoretical development of the effect of varying filter width in time and space on the governing equations, we perform numerical simulations of isotropic decaying turbulence on fixed and expanding grids as shown on Figure 1. This test case is well appropriate for studying turbulence models in their capacity to mimic the Kolmogorov cascade process. The box is of dimension \( L = 1.25 \) m and the mesh accounts for \( 80^3 \) grid points for a medium wave number \( \kappa_c = 2 \) cm\(^{-1}\). The grid size \( \Delta(\Omega) \)

associated with the cell \( \Omega \) increases in time in order to account for the increase of turbulence scales during decay according to the law

\[
\Delta(\Omega)(t) = \Delta(\Omega)(0)(1 + \alpha t)^p
\]

where \( p \) is a constant coefficient and \( \alpha \), a parameter depending on the initial turbulence. Figure 2 displays the decay of the three-dimensional spectra starting from the initial time for the PITM simulations performed both on the fixed and moving meshes at different time advancements, denoted PITM1 and PITM2, respectively. The simulation performed on the moving mesh consists in solving the filtered transport equations including here the commutation terms. As it can be seen from this Figure, all simulations are able to reproduce the evolution of the spectrum at different times in accordance with the Kolmogorov law. The inertial transfer zone for the energy cascade computed initially at the Reynolds number \( Re \approx 5000 \) is well visible. As expected, the PITM2 performed on the moving mesh allows a better description of the energetic region of the spectrum than the PITM1 simulation, especially in the zone at low wave numbers. Figure 3 displays the decay of the spectra, for both PITM simulations performed on the moving mesh, with and without the commutation terms, denoted PITM2 and PITM3, respectively. As a result, it is found that the PITM2 curves are slightly more dissipative than the PITM3 curves which are located slightly above the corresponding PITM2 spectrum. This outcome can be physically explained. In the case where the grid size increases with time

\[
\Delta(\Omega)(t) = \Delta(\Omega)(t)/\partial t > 0 \text{ or } E(\kappa_c)\partial \kappa_c/\partial t < 0,
\]

then a part of the energy contained into the resolved scales is removed and fed into the modeled spectral zone, whereas on the contrary, when
$\partial \Delta(t)/\partial t < 0$ or $E(\kappa)\partial \kappa_c/\partial t > 0$, a part of energy coming from the modeled zone is injected into the resolved scales.

9 Fully developed turbulent channel flow

This flow is chosen to determine the effect of a varying grid size on the statistical data, see Ghosal and Moin (1995), Fröhlich et al. (2007), Cubero and Piomelli (2006). To investigate this effect, the mesh in the streamwise direction $x_1$ includes a sudden step refinement and enlargement in the grid size. The dimensions of the channel in the streamwise, spanwise and normal directions along the axes $x_1$, $x_2$, $x_3$ are $L_1 = 6\delta$, $L_2 = 2\delta$ and $L_3 = \delta$, respectively. Accordingly, the grid size of the mesh is defined by $\Delta_1 = f(x_1)\Delta_0$ where $\Delta_0$ is the uniform grid size and $f$ a given function. If $x_1 < 3\delta$, $f(x_1) = 1.5 + 0.5 \tanh(\alpha(x_3/\delta - 1.5))$ for the increasing and if $x_1 > 3\delta$, $f(x_1) = 1.5 - 0.5 \tanh(\alpha(x_3/\delta - 4.5))$ for the decreasing grid size ratio where $\alpha = 5$. The mesh is uniform in the spanwise direction. In the normal direction to the wall, the grid points are distributed in different spacings with refinement near the wall according to the transformation $x_{3j} = \frac{1}{2} \tanh[\xi_j \tanh(\eta)]$ where $\xi_j = -1 + 2(j - 1)/(N_3 - 1)$ ($j = 1, 2, \cdots, N_3$) $\eta = 0.98346$ and $N_3 = 84$. Figure 4 shows the cross section of the mesh. Figure 5 describes the evolution of the grid-size ratio $\Delta_1/\Delta_0$ in the streamwise direction. Although the grid size is increased two-fold in the streamwise direction, it can be mentioned however that the effective filter $\Delta_1(x_1)$ defined by Equation (32) is here increased by a factor of 1.35. Numerical simulations of the spatially developing channel flow are performed on coarse meshes requiring $48 \times 24 \times 84$ grid points and the results are compared with data of direct numerical simulation worked out by Moser et al. (1999) for the Reynolds number $Re_p = \rho_u u_c \delta/2\mu = 357$, based on the averaged friction density $\rho_u$, the averaged friction velocity $u_c$, and the channel half width $\delta/2$. Several PITM simulations are carried out to assess the effect of the filter width on the solutions. More precisely, PITM1 is performed on the uniform mesh whereas PITM2 and PITM3 are performed on the non-uniform mesh in the streamwise direction without and with the commutation terms appearing in Equations (10), (12), (23) and (28). Periodic boundary conditions are applied in the streamwise and spanwise directions whereas a no slip velocity condition is imposed at the walls. The statistics of the fluctuating velocity correlations are achieved both in space in the spanwise homogeneous direction $x_2$ and also in time. In the present case, it is found that that the filter derivative $D\Delta/Dt$ involved in the commutation term reduces to

$$\frac{D\Delta}{Dt} \approx \bar{u}_1 \frac{\partial \Delta}{\partial x_1}$$

(37)

because $\partial \Delta/\partial x_2 = 0$ and $\bar{u}_1 \partial \Delta/\partial x_1 \gg \bar{u}_3 \partial \Delta/\partial x_3$.

Figure 6 displays the evolution of the instantaneous dimensionless filtering term $C_1 = (\partial \Delta_1/\partial t)/(\bar{u}_1/\Delta)$ at $x_3/\delta = 1/2$ versus the dimensionless distance $x_1/\delta$ at a given time. If using Equation (35), this term can be rewritten as $C_1 = (\bar{u}_1 - \bar{u}_i)/\bar{u}_1$. As a result, one can see that the signal is characterized by high frequencies, particularly in the zones including sharp variations of the grid size. Figure 7 depicts the evolution of the commutation term $\beta_i(u_i)$ for $i=1,2,3$ defined by Equation (13) at $x_3/\delta = 1/2$ and versus the dimensionless distance $x_1/\delta$ at a given time. As expected, $\beta_i(u_i)$ is non-zero only in the zone including the sharp variation of the grid size at $x_1/\delta = 1.5$ and $x_1/\delta = 4.5$ according to Figure 5. In the following, we analyze the PITM results performed on several grids by considering first the PITM1 simulation for comparison purpose. Figure 8 describes the mean velocity profile $\langle u_1 \rangle/\bar{u}_1$ in logarithmic coordinate. Overall, one can see that the profile compares very well with the DNS data except however in the channel core where the velocity is slightly underpredicted. Figure 9 displays the evolution of the streamwise, spanwise and normal turbulence intensities normalized by the wall friction velocity $u_c$. One can observe that the shape of the profiles is well recovered but the peaks of turbulence in the boundary layer are slightly overpredicted in comparison with DNS. This result must be attributed to the grid size which is very coarse both in the streamwise and spanwise directions, $\Delta_1^+ = 100$ and $\Delta_3^+ = 66$.

For PITM2 and PITM3 simulations, velocity and stress profiles are investigated at four different locations in the channel at $x_1/\delta = 1, 1.5, 3$ and 4.5. As an interesting result, Figure 10 shows that all mean velocity profiles returned by PITM2 and PITM3 are practically identical in term of accuracy although the grid size is suddenly coarsened and afterward refined in the streamwise direction. In the present case, the accounting for the commutation terms due to the commutation errors seems not crucial for reproducing the mean velocity profile that agrees well with DNS. This outcome is quite different from the one obtained for zonal RANS-LES methods where it is commonly known that the neglect of the commutation terms is responsible for the log-layer mismatch (Hamba, 2009). Figure 11 depicts the profile of the turbulent energy $k/\bar{u}_c^2$ at the four locations $x_1/\delta = 1, 1.5, 3$ and 4.5 First at all, one can see that the profiles differ from one location to another. As a result, it appears that PITM3 returns a better level of the turbulent energy than PITM2, especially at $x_1 = \delta$ and $x_1 = 1.5\delta$, but for both simulations, one can see that the turbulence intensity is overpredicted in the near wall region. Figure 12 and 13 describe the streamwise development of the subfilter scale turbulent energy $k_{sfs}/\bar{u}_c^2$ and resolved scale turbulent energy $k_{sfs}/\bar{u}_c^2$, respectively, for all PITM simulations at the wall distance $x_3^+ = 10$. As also observed by Cubero and Piomelli (2006) considering in their study the use of a filter width that is twice
Figure 4: Illustration of the grid variation $\Delta$ in the streamwise direction $x_1$.

Figure 5: Evolution of the grid size ratio $\Delta/\Delta_0$ in the streamwise direction $x_1$.

Figure 6: Evolution of the instantaneous dimensionless filtering term $(\partial \bar{u}_1/\partial \Delta)/(\bar{u}_1/\Delta)$ at $x_3/\delta = 1/2$ versus the dimensionless distance $x_1/\delta$.

Figure 7: Evolution of the commutation term $\beta_i(u_i)$ for $i=1,2,3$ defined by Equation (13) versus the dimensionless distance $x_1/\delta$ at $x_3/\delta = 1/2$. △: $i=1$; ◇: $i=2$; ◇: $i=3$.

Figure 8: Mean velocity profile $\langle u_1 \rangle / u_\tau$ in logarithmic coordinate. PITM1 ◆; DNS : —. $R_\tau = 395$.

Figure 9: Turbulence intensities in wall unit $R_{ij}^{1/2}/u_\tau$. PITM1: △: $i=1$; ◇: $i=2$; ◇: $i=3$. DNS : —. $R_\tau = 395$. 
Figure 10: Mean velocity profile $\langle u_1 \rangle / u_\tau$ in logarithmic coordinate at various locations. $x_1/\delta = 1$: PITM2 ▼; PITM3 ▲; $x_1/\delta = 1.5$: PITM2 — ; PITM3 ▼; $x_1/\delta = 3$: PITM2 — ; PITM3 ▼; $x_1/\delta = 4.5$: PITM2 — ; PITM3 ▼. DNS : — . $R_e = 395$.

Figure 11: Turbulent energy $k/u_\tau^2$ at various locations. $x_1/\delta = 1$: PITM2 ▼; PITM3 ▲; $x_1/\delta = 1.5$: PITM2 — ; PITM3 ▼; $x_1/\delta = 3$: PITM2 — ; PITM3 ▼; $x_1/\delta = 4.5$: PITM2 — ; PITM3 ▼. DNS : — . $R_e = 395$.

Figure 12: Evolution of the subfilter scale turbulent energy $k_{sfs}/u_\tau^2$ in the streamwise direction $x_1$ at the dimensionless wall distance $x_3^+ = 10$. PITM1 — ; PITM2 ▼; PITM3 ▲.

Figure 13: Evolution of the resolved scale turbulent energy $k_{les}/u_\tau^2$ in the streamwise direction $x_1$ at the dimensionless wall distance $x_3^+ = 10$. PITM1 — ; PITM2 ▼; PITM3 ▲.
larger than the grid size, it is found in the present case that the subfilter energy increases (or decreases) as the grid size increases (or decreases) since larger (smaller) scales must be modeled, the filter width is identified to the grid size in the present case. The evolution of the resolved scale energy is more difficult to explain because of the reincrease of energy in the zone of the channel where the grid size is constant.

10 Conclusion

The complex expressions of the commutation terms appearing in the filtered turbulence equations have been analytically derived using the rules of convection operators with variable kernels. A mathematical physics formalism has been then proposed in the case of the PITM method considering that the commutation effect in the material derivative is of primary importance. In this work, the noncommutation terms have been computed by means of a superfilter that is applied on the instantaneous equations of mass, momentum and subfilter turbulent stresses. We have then considered different but complementary applications to illustrate the role of commutation errors in PITM simulations. Firstly, we have simulated the decaying turbulence on fixed and expanding grids without and with the correction terms arising from the commutation errors in the equations of mass, momentum and stresses. As a result of interest, it has been shown that the PITM1 simulation taking into account the commutation errors is able to accurately reproduce the decay of the spectrum in the region of the large wave numbers characterized by the Kolmogorov law as well as in the region of slow wave numbers dominated by large scales of energy. Secondly, we have simulated the fully developed turbulent channel flow on several grids including a sudden step refinement and enlargement in the grid size in the streamwise direction, without and with the commutation terms in the equations. Computations of the commutation errors have been carried out. The effect of using a variable grid size ratio has been investigated from the velocity and turbulent stresses of the flow. As a result, it has been found that the PITM simulations performed on varying grid size return almost the same mean velocity profile and that the accounting for the commutation terms in the filtered LES equations is beneficial to get a better level of turbulence energy. Hence, the impact of the commutation terms on the solution in PITM seems not as large as that observed for usual hybrid zonal RANS/LES methods, but this result must be confirmed by more various extensive applications.

References