1 Introduction

Separation around a rotary wing, curved pipe or other fluidic device, is one of the serious problems. Applying a periodic excitation at the leading edge, or a static vortex-generator device is known to work successfully to suppress separation and expected to be put in practical use (Gad-el-Hak and Bushnell, 1991). These days, many works assuming to use a single dielectric barrier discharge (DBD) actuator as that excitation-control-device have been reported (Visbal, Gaitonde and Roy, 2006, Asada et al., 2009, Sekimoto et al., 2011, Nonomura et al., 2013, Sato et al., 2013, Aono et al., 2014, etc.).

These periodic excitations are known to promote laminar-turbulent transition at low-Reynolds number (Gad-el-Hak and Bushnell, 1991, Talan and Hourmouziadis, 2002 etc.). It results in suppressing separation and causing reattachment. However, it has not been discussed substantially how the periodic excitation affects to the momentum transfer through laminar-turbulent transition. Some effective control frequencies for the re-attachment has been reported (Corke, et al., 2004, etc.).

In the present study, we investigate details of separation and re-attachment caused by the laminar-turbulence transition at two Reynolds numbers around a simple two-dimensional hump, which is immune to geometrical variation of a fluidic device curvature. The objective is to obtain the general knowledge to determine the optimal control parameters, such like excitation frequency and actuator position, or the strategy itself to enhance momentum transfer efficiently at various flow conditions, such like Reynolds numbers.

2 Numerical Procedures

Governing Equations

The governing equations are three-dimensional compressible Navier-Stokes equations (Eq.(3)). All variables are normalized by free stream density, velocity and height of the two-dimensional hump geometry. In equations, $x, y$ and $z$ are the streamwise, spanwise and wall-normal directions, and $t$ is the time.

$$u_i$$ is the velocity components in $i$-th direction. $a$ is sound speed, $\rho, p, e, \tau_{ij}$ are density, pressure, energy and stress tensor per unit volume, respectively. $\delta_{ij}$ is Kronecker delta. $S_i$ is the body force induced by a plasma actuator, described in the other section. Mach number $M$ is 0.2. Prandtl number $Pr$ is 0.72.

$$\frac{\partial p}{\partial t} + \frac{\partial (\rho u_k)}{\partial x_k} = 0,$$  \hspace{1cm} \text{(1)}

$$\frac{\partial (\rho u_i u_k)}{\partial x_k} + \frac{\partial (p u_i)}{\partial x_k} = \frac{1}{Re_h} \frac{\partial \tau_{ik}}{\partial x_k} + S_i,$$  \hspace{1cm} \text{(2)}

$$\frac{\partial c}{\partial t} + \frac{\partial ((e + p) u_k)}{\partial x_k} = \frac{1}{Re_h} \frac{\partial \tau_{ik}}{\partial x_k} + S_k u_k + \frac{1}{(\gamma - 1) Pr Re_h M^2} \frac{\partial^2 a}{\partial x_k^2}.$$  \hspace{1cm} \text{(3)}

Figure 1: Schematic of a hump geometry, computation domain and boundary condition.

Geometry and flow conditions

The hump geometry is analytically-defined semi-circular column, which is joined smoothly with the wall. Schematic of two-dimensional hump geometry, computational domain and boundary condition are shown in Fig.2. Constant Blasius profile is set at the input boundary with the thickness of $\delta_{in} = 0.25$, which is normalized by the hump height. Periodic boundary condition is applied in the spanwise direction. At the out flow boundary, variables are extrapolated from the adjacent nodes, and the second order filtering is applied in the streamwise direction. For the hump surface, no-slip and adiabatic-wall conditions are adopted. Reynolds number $Re_h$, based on the hump height $h$ and the outflow velocity $u_{in,f}$, is set
at 4000 and 16000. The corresponding $Re_h$ based on the minimum boundary layer thickness $\delta$ around the hump, 183.6 and 593.6.

**Body force**

In the study, we apply an electric field model (Suzen et al., 2005) as shown in Fig.3(a). As a periodic excitation, unsteady body-force is formulated as $S_x = D_c \cdot S_{model} \sin(2\pi f_{base} t)$ with sufficiently high base-frequency $f_{base} = 240$, is switched ON/OFF with the bursting frequency $f$, in the present study, the bursting period is set sufficiently as small as 0.01-1/f, and the amplitude $D_c$ is set 100 for all cases. R.m.s. values of the induced streamwise velocities at two Reynolds numbers are shown in Fig.3(b).

![Figure 2: (a) Body force $S_{model}$, which is computed by an electric field model (Suzen et al., 2005). (b) R.m.s. of the induced streamwise velocity $u'_{rms}$ at $x = 0.1$.](image)

**Computation method**

In the study, Fujii-lab in-house solver is employed for the computation. Compact difference scheme-related subroutines in the code have been well tuned for the K computer (Aono et al., 2013). The spatial derivatives of the convective and viscous terms and symmetric conservative metrics and Jacobian (Visbal, M. R. and Gaitonde, D. V., 1999) are evaluated by a sixth-order compact difference scheme (Lele, S. K., 1992). Near the boundary, second-order explicit difference schemes are used. Tenth-order filtering is used with a coefficient of 0.42. For time integration, TVD Runge-Kutta method is used. The computational time step is 0.001 to set Courant-Friedrichs-Levy (CFL) number is less than 1.0.

**Grid convergence study**

Computational grids are fine enough to resolve turbulent structures over the hump. We also note that the current results are based on the complete grid convergence study. $\Delta x_{max} = 11.4$, $\Delta y_{max} = 3.25$ and $\Delta x_{min} = 0.017 \sim 0.337$ in viscous-unit, $\Delta x/\delta = 0.0713$, $\Delta y/\delta = 0.0749$ and $\Delta z/\delta = 0.0078$ to the maximum for the case of $Re_h = 4000$. $\Delta x_{max} = 11.4$, $\Delta y_{max} = 8.37$ and $\Delta x_{min} = 0.0516 \sim 1.42$ in viscous-unit, $\Delta x/\delta = 0.0841$, $\Delta y/\delta = 0.0618$ and $\Delta z/\delta = 0.0105$ to the maximum for the case of $Re_h = 16000$. Total grid numbers are 34 and 118 million for cases at $Re_h = 4000$ and 16000, respectively.

**3 Computational Results**

**Uncontrolled cases**

Snapshots of instantaneous flow fields are shown in Fig.1 for cases at (a) $Re_h = 4000$ and (b) 16000. Iso-surface is that the second invariant of deformation tensor $Q(= \partial u_i/\partial x_j \cdot \partial u_j/\partial x_i) = 0.1$ and 1.0, respectively. Color contour is $u$ (normalized by the outer flow velocity $u_{inf}$) from -1.0 (blue) to 1.0 (red). Two-dimensional roll vortices $(\omega_x, \omega_z)$ appear at both Reynolds numbers. Rolls are due to the K-H instability of a free shear layer.

Time-averaged streamwise-velocity profile $U(z)$ is analyzed in detail. $\partial U^2/\partial z^2$ is plotted in Fig.4. Separation occurs at around $x = 0.0$ (position (a) in Fig.4) at both Reynolds numbers, where $U/\delta z = 0.0$ at the wall. It is noted that the position where an inflection point $(\partial U^2/\partial z^2 = 0.0)$ exists on the surface (position (b) in Fig.4), is slightly upstream from the separation point (a).

Tangential velocities $u_1^+$ at two Reynolds numbers, of three different positions around a hump $(x = 0.0, -0.3$ and $-0.5$) are shown in Fig.5 as dotted lines, open and filled symbols, respectively. At $x = -0.5$, where the boundary layer thickness becomes the minimum around a top of the hump, flow is close to laminar state since they are stabilized by accelerated pressure gradient due to the hump curvature. At $x = 0.0$, flow starts to be separated and is close to be the laminar state too.

![Figure 3: Relation of the separation point (position (a)) and the inflection point of $U$ at the surface (position (b)), where $\partial^2 U/\partial z^2 = 0.0$. Lines are profiles of $\partial^2 U/\partial z^2$, red lines are at $Re_h = 4000$ and blue ones are at $Re_h = 16000$.](image)

**Controlled cases**

In the present paper, the control effect of cases at $Re_h = 4000$ is discussed in detail. Cases at $Re_h = 16000$ is shown the effectiveness of the control. We compare control cases with five different actuator positions, $x_{act} = (-0.7, -0.5, -0.3, 0.0, 0.3)$ and five bursting frequency, $f_h = (0.05, 0.10, 0.50, 1.0, 5.0)$, which corresponds to $\tilde{f}_h = (0.00046, 0.00092, 0.0046, 0.0092, 0.046)$ at $Re_h = 4000$, which depends on the momentum thickness and the mean velocity at the separation point.
Figure 4: Tangential velocity in viscous unit \( u_t^+ \).

\( C_f \) distributions for the controlled cases with different excitation frequencies, and the same actuator position \( x_{act} = -0.5 \) are shown in Fig.6. The position \( x_{act} = -0.5 \) is the inflection point (position (b) in Fig.4). We identify two effective control frequencies to enhance momentum transfer to the wall, at each different positions around a hump. The first is the vortex shedding frequency \( f = 1.0 \) \( (f_h=0.0092) \), which promotes two-dimensional spanwise vortex and \( C_f \) is positive at around \( 0.0 < x < 0.5 \). The second is that with low-frequency excitation \( (f_h=0.10, 0.50) \) to enhance re-attachment at around \( x = 6.0 \). These two control phenomena are apparently different, as shown in instantaneous flow fields in Fig.7 (a) \( f_h=1.0 \) and (b) \( f_h=0.10 \), respectively. Fig.8 show comparison of contour of the streamwise velocity \( U(x, z) \), and fig.9 show comparison of contour of r.m.s. value of the wall-normal velocity fluctuation \( u' \) at the same actuator position \( (x_{act} = -0.5) \). The momentum transfers to the surface are increased by two excitation frequencies at each different hump positions, \( (0 < x < 1.0, x = 4.0) \) and \( (x = 3.0) \), respectively.

High-frequency effect. \( C_f \) and \( C_p \) distributions with the high-frequency excitation \( (f_h = 1.0) \) and

Figure 5: \( C_f \) distributions with variable periodic excitations by the actuator at \( x_{act} = -0.5 \), where the inflection point is at the surface (Fig.4).

Figure 6: Instantaneous flow field with periodic excitations (a) \( f_h = 1.0 \) and (b) \( f_h = 0.1 \) at the actuator position \( x_{act} = -0.5 \). Iso-surface of the second invariant of deformation tensor \( Q = (\partial u_i/\partial x_j, \partial u_j/\partial x_i) = 0.10 \), contour color is \( u \) from \(-1.0\) (blue) to \( 1.0 \) (red).

Figure 7: Comparison of the streamwise velocity \( u \), \(-0.25\) (blue) to \( 1.25 \) (red). (a) without control, (b) with control by the high frequency excitation \( (f_h = 1.0) \) and (c) by the low frequency excitation \( (f_h = 0.10) \) at the same actuator position \( (x_{act} = -0.5) \).
different actuator positions $x_{\text{act}}$ are shown in Fig.10. Cases with the actuator positions $x_{\text{act}} = -0.5, -0.3$ and 0.3 are effective on re-attachment at around $0 < x < 0.5$, where $C_f$ is positive as shown in Fig.11. On the other hand, controls at $x_{\text{act}} = -0.7$ and 0.0 do not work. Comparison of the r.m.s. value of wall-normal velocity fluctuation $w'_{\text{rms}}$ is shown in Fig.12. High-frequency excitation at $x_{\text{act}} = -0.5$ enhances the wall-normal fluctuation at around $x = 1.0$ more than that at $x_{\text{act}} = 0.0$ in Fig.12. With the high-frequency excitation, two-dimensional roll vortices are identified. It is noted that vortex-pairing is not observed very much. These features are shown in Fig.7(a). From the results, one of the significant conclusions is that the high-frequency excitation is sensitive to the actuator position.

**Low-frequency effect.** With the low-frequency, large-scale vortex is generated in the downstream as shown in Fig.7(b). $C_f$ distributions with different actuator positions with the constant low-frequency excitation ($f_h = 0.05$) is shown in Fig.13. Control with the low-frequency excitation is not sensitive to the actuator position. Time history of instantaneous flow field of one cycle for the large-vortex generation with the low-frequency are shown in Fig.18. A bursting of the actuator cause as many as two vortices ($\phi = 0.2\pi$). The first vortex is convected with associated rib structures to the downstream, and the second one stays around a hump ($\phi = 0.4\pi$). The second vortex grows larger ($\phi = 0.8\pi \sim 1.6\pi$), results in becoming chaotic turbulent state ($\phi = 1.8\pi$). In the case that the control frequency is lower than $f_h = 0.05$, the time period of the turbulent state will become longer.
Figure 11: Comparison of the r.m.s. value of the wall-normal velocity fluctuation $w'_{rms}$, 0(blue) to 0.05(red). (a) without control, (b) with control at the inflection point ($x_{act} = -0.5$) and (c) at the top of the hump ($x_{act} = 0.00$), with the high-frequency ($f_h = 1.00$).

Figure 12: $C_f$ distributions with variable actuator positions, and the low frequency $f_h = 0.05$.

Figure 13: Comparison of the streamwise velocity $u$, $-0.25$(blue) to $1.25$(red). (a) without control, (b) with control at the inflection point ($x_{act} = -0.5$) and (c) at the top of the hump ($x_{act} = 0.00$), with the low-frequency ($f_h = 0.05$).

Figure 14: Comparison of the r.m.s. value of the wall-normal velocity fluctuation $w'_{rms}$, 0(blue) to 0.05(red), (a) without control, (b) with control at the inflection point ($x_{act} = -0.5$) and (c) at the top of the hump ($x_{act} = 0.00$), with the high-frequency ($f_h = 0.05$).
High Reynolds number. At higher Reynolds number $R_{ch} = 16000$, we confirm the control effect with the low-frequency excitation $f_h = 0.20$. Comparison of the streamwise velocity $u$ is shown in Fig.16. Instantaneous flow field is in Fig.17. Flow characteristics with the control frequency is almost same as that of low Reynolds number cases.

Figure 15: Comparison of the streamwise velocity $u$, $-0.25$ (blue) to 1.25 (red). (a) without control and (b) with control by the periodic excitation ($f_h = 0.20$) at the actuator position $x_{act} = 0.0$.

Figure 16: Instantaneous flow field with periodic excitations $f_h = 0.2$. Iso-surface of the second invariant of deformation tensor $Q = (\partial u_i/\partial x_j \cdot \partial u_j/\partial x_i) = 1.0$, contour color is $u$ from $-0.5$ (blue) to 1.5 (red).

4 Conclusions

In the present study, we investigated separation and re-attachment caused by the laminar-turbulent transition phenomena around a simple two-dimensional hump, to determine the optimal separation-control condition on various flow conditions. For controlled cases, we identified two effective periodic control frequencies to enhance momentum transfer to the wall. The first is the vortex shedding frequency $f = 1.0$ ($f_{0s} = 0.0092$), which promotes two-dimensional spanwise vortex. This frequency excitation promote re-attachment at around $0 < x < 0.5$. This is sensitive to the actuator position. The second is that with low-frequency excitation ($f_h = 0.10$, 0.50) to cause re-attachment at around $x = 6.0$, which generated a large-vortex around a hump. This control mechanism is not sensitive to actuator positions more than that of high-frequency excitations. At higher Reynolds number, it is confirmed that the low-frequency excitation ($f_h = 0.20$) works well. The control mechanism is close to that at low Reynolds number.

Acknowledgments

The computations in this study are performed by the supercomputer in Advanced Institute of Computational Science, Riken, and by Fujitsu FX10 in Information Technology Center in University of Tokyo. This research is mainly supported by Strategic Programs for Innovative Research (SPIRE) of High Performance Computing Initiative (HPCI). The present authors are grateful for the fruitful discussion with Dr. H. Aono and Dr. M. Sato in JAXA.

References


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Figure 17: Iso-surface of the second invariant of deformation tensor $Q = (\partial u_i/\partial x_j \cdot \partial u_j/\partial x_i) = 0.1$ and 1.0, contour color is $u$ from $-1.0$(blue) to $1.0$(red), at (a) $Re_{\theta} = 4000$ and (b) $16000$, respectively.
Figure 18: Large-vortex generation and turbulence-transition of one cycle with the low-frequency excitation $f_h = 0.05$ at the actuator position $x_{act} = 0.0$. Iso-surface of the second invariant of deformation tensor $Q = (\partial u_i / \partial x_j \cdot \partial u_j / \partial x_i) = 0.10$, contour color is $u$ from $-1.0$ (blue) to $1.0$ (red).