Comparative Computational Study of Turbulent Flow in a 90° Pipe Elbow

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Abstract

Turbulent flow through a 90° pipe elbow in a range of moderately high Reynolds numbers between 14000 and 34000 is studied computationally using wall-resolved Large Eddy Simulation (LES) as well as various RANS (Reynolds Averaged Navier-Stokes) models aiming at a comparative assessment to illustrate benefits and drawbacks of different computational approaches for the considered case. Accordingly, characteristic secondary motions resembling the axially oriented counter-rotating Dean vortices, see Bradshaw (1987), are further investigated and their predictability using LES is assessed.

1 Introduction

Fully turbulent flow through curved conduits and channels have been studied intensively in the past by applying theoretical and experimental approaches. Berger et al. (1983) and the earlier publication by Smith (1976) provide a broad range of theoretical investigations regarding curved pipe flow. However, with the scenario of curved flows being considerably more complex and resource-demanding than flows through straight domains, only recently, in the past few decades numerical investigations have been added to the range of scientific tools assessing curved pipe flows, see e.g. Rütten et al. (2005). With increasing computing capacities, the applicable methods to predict curved pipe flows are now shifting from simpler, steady-state approaches to more elaborate, temporally resolving techniques.

The present study is thus dedicated to the investigation of turbulent flow through 90° pipe bends using state-of-the-art numerical models of different complexity, accuracy and computational cost. Prior to simulating the 90° pipe elbow configuration, fully-developed flow through a straight pipe at different Reynolds numbers is computed in order to validate the employed computational procedures and to approve the adopted mesh properties. The results for the case of a 90° elbow are subsequently evaluated regarding mean velocity and Reynolds stress fields and are compared to experimentally obtained data from Kalpakli and Örlü (2013). The RANS models applied in the framework of this study include both the basic low Reynolds number \( k-\epsilon \)-model, see Launder and Sharma (1974), as well as a near-wall second moment closure model as proposed by Jakirlić and Hanjalić (2002). In the following these two RANS approaches are referred to as Eddy Viscosity Model (EVM) and Reynolds Stress Model (RSM), respectively.

The simulations performed within this framework essentially represent preliminary investigations concerning specific inlet conditions for the erosion process of a Helium stratification by an air jet within the containment of a nuclear reactor as motivated by the THAI-26 program of the Reactor Safety Research Project 150 1455, see Fig. 1. The eventual goal is to demonstrate LES’ uprising capabilities for handling relevant engineering applications with significant streamline curvature such as flows through bends, manifolds or helically coiled pipes, as they appear in...
2 Numerical Methodology

The equations governing the mass and momentum balance for a flow in a bent pipe are given by the Navier-Stokes equations for incompressible Newtonian fluids as shown in Eq. (1) and (2), respectively.

\[
\frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 U_i}{\partial x_j \partial x_j} \tag{1}
\]

\[
\frac{\partial \tilde{U}_i}{\partial t} + \frac{\partial (\tilde{U}_j \tilde{U}_i)}{\partial x_j} = -\frac{1}{\rho} \frac{\partial \tilde{p}}{\partial x_i} + 2\nu \frac{\partial}{\partial x_j}(\tilde{S}_{ij}) - \frac{\partial \tau_{ij}}{\partial x_j}. \tag{2}
\]

In order to reduce the computational effort in the sense of LES, i.e. to explicitly resolve the larger scales while modelling the fairly universal smaller ones by means of approximative assumptions, the above equations are spatially filtered yielding

\[
\frac{\partial \tilde{U}_i}{\partial t} = 0 \tag{3}
\]

\[
\frac{\partial \tilde{U}_i}{\partial t} + \frac{\partial (\tilde{U}_j \tilde{U}_i)}{\partial x_j} = -\frac{1}{\rho} \frac{\partial \tilde{p}}{\partial x_i} + 2\nu \frac{\partial}{\partial x_j}(\tilde{S}_{ij}) - \frac{\partial \tau_{ij}}{\partial x_j}. \tag{4}
\]

In the above expressions the tilde-operator denotes a three-dimensional filter operation in space with a filter width of \( \Delta \). In Eq. (4), \( \tilde{S}_{ij} \) is the filtered strain rate tensor, whereas \( \tau_{ij} = \tilde{U}_j \tilde{U}_i - \tilde{U}_i \tilde{U}_j \) represents the part of the non-resolved (i.e. subgrid) motions in the governing equation for the resolved momentum, which needs to be modelled in order to close the equation. The common standard nomenclature for velocity, pressure, density and kinematic viscosity being \( U_i, p, \rho \), \( \nu \), respectively, is employed.

In this study, the well established approach according to Boussinesq is utilized, which assumes linearity between the subgrid-scale (SGS) tensor \( \tau_{ij} \) and the mean strain rate tensor \( \tilde{S}_{ij} \) according to

\[
\tau_{ij} = \frac{1}{3} \delta_{ij} \tau_{kk} - 2\nu_t \tilde{S}_{ij}, \tag{5}
\]

with \( \delta_{ij} \) being Kronecker’s delta. In Eq. (5), the artificial SGS viscosity \( \nu_t \) is approximated according to the assumptions stated by Smagorinsky (1963) as

\[
\nu_t = C_S \Delta^3 |\tilde{S}_{ij}|, \tag{6}
\]

where the model parameter \( C_S \) is dynamically evaluated as a function of the smallest resolved scales as first proposed by Germano et al. (1991). However, in order to allow for backscatter, i.e. energy transfer from subgrid to resolved scales, the modification according to Lilly (1991) is employed throughout this study, thus yielding

\[
\nu_t = C_D \Delta^2 |\tilde{S}_{ij}|. \tag{7}
\]

In Eq. (7), the dynamically determined coefficient \( C_D \) can locally become negative (backscattering). We refer to Fröhlich (2006) and Lilly (1991) regarding further details for the different dynamical Smagorinsky models as well as specific equations for \( C_D \).

To account for statistically steady, turbulent inlet conditions, a periodic inlet section is used which has a driving pressure gradient imposed to the momentum balance in order to maintain a constant mass flow rate. The viscous sublayer of the pipe’s near-wall region is resolved, thus essentially resulting in a wall-resolved LES. The dimensionless grid spacings in azimuthal and streamwise direction are chosen to be in the order of \( \Delta_z \approx 15 \) and \( \Delta_\theta \approx 30 \), respectively, while the first near-wall cell centers are located at \( y^+ \approx 1 \).

3 Numerical Validation

In order to validate the employed numerical procedures, fully developed turbulent pipe flows at various bulk Reynolds numbers are considered prior to simulating the case of a 90° bend, see Fig. 2. The diameter-based bulk Reynolds number is hereby defined as \( Re_b = U_b D/\nu \), where \( U_b, D = 2R \) and \( \nu \) are the pipe bulk velocity, the pipe diameter and the fluid kinematic viscosity, respectively. The computational setup is hence validated for pipe flow in the Reynolds number range from \( Re_b = 11700 \) to \( Re_b = 34000 \).

Fig. 3 shows the comparison of LES, RSM, EVM and DNS pipe flow results for the non-dimensional streamwise velocity \( U^+ = (U_z)/u_\tau \) as function of the normalised wall distance \( r^+ = u_\tau (R - r)/\nu \) at \( Re_b = 11700 \). Here \( u_\tau \) is the wall friction velocity and \( r \) the radial coordinate in a cylindrical coordinate system, while \( \langle . \rangle \) denotes the temporal averaging over a sufficiently long interval. The DNS data are taken from El Khoury et al. (2013) and refer to a friction Reynolds number \( Re_\tau = u_\tau (D/2)/\nu \) of \( Re_\tau = 360 \). The RANS results are obtained using the near-wall Reynolds stress model according to Jakirlić and Hanjalić (2002). Without any exceptions, all three approaches yield satisfying results with respect to mean velocity prediction.
The statistics of the mean velocity fluctuations corresponding to the afore-mentioned turbulent pipe flow at $Re_D = 11700$ ($Re_T = 360$). Results from LES are compared against RSM, EVM, DNS and the universal equilibrium laws of the wall, assuming $\kappa = 0.41$ and $B = 5.1$; DNS data taken from El Khoury et al. (2013).

Figure 3: Non-dimensional streamwise velocity for a fully developed, turbulent pipe flow at a bulk Reynolds number of $Re_b = 11700$ ($Re_T = 360$). Results from LES are compared against RSM, EVM, DNS and the universal equilibrium laws of the wall, assuming $\kappa = 0.41$ and $B = 5.1$; DNS data taken from El Khoury et al. (2013).

In analogy to the above case of $Re_b = 11700$ pipe flow, the applied procedures of LES, RSM and EVM are also validated for the upper limit of the presently considered Reynolds number range. Hence, Fig. 5 shows the non-dimensional streamwise velocity for a fully developed, turbulent pipe flow at $Re_b = 34000$ ($Re_T = 910$). The graph shows that the LES can return the mean flow velocity in a fairly good agreement with the DNS data, whereas both RANS approaches result in a slight underprediction within the logarithmic region. It should be mentioned that although the computations are performed for a slightly lower value of the friction velocity than the DNS of $Re_T = 1000$, the comparison can be regarded a proper validation since no major differences in the velocity profile are to be expected resulting from a mismatch of this order in the considered Reynolds number range, see also El Khoury et al. (2013).

Figure 4: Non-dimensional turbulent intensities in streamwise ($z$), azimuthal ($\Theta$) and wall-normal ($r$) directions, complementing Fig. 3. DNS data taken from El Khoury et al. (2013).

The statistics of the mean velocity fluctuations corresponding to the afore-mentioned turbulent pipe flow at $Re_D = 34000$ ($Re_T = 910$). Results from LES are compared against RSM, EVM, DNS and the universal equilibrium laws of the wall, assuming $\kappa = 0.41$ and $B = 5.1$; DNS data taken from El Khoury et al. (2013).

Figure 5: Non-dimensional streamwise velocity for a fully developed, turbulent pipe flow at a bulk Reynolds number of $Re_b = 34000$ ($Re_T = 910$). Results from LES are compared against RSM, EVM, DNS and the universal equilibrium laws of the wall, assuming $\kappa = 0.41$ and $B = 5.1$; DNS data taken from El Khoury et al. (2013).

The objective regarding resolution requirements of the performed Large Eddy Simulations is to resolve a minimum of 80% of the total turbulent kinetic energy, i.e. keeping the part of the modelled subgrid-scales below 20%, see e.g. Pope (2011). To evaluate whether these requirements are fulfilled, the above pipe flow simulations are compared against DNS turbulent kinetic energy spectra from El Khoury et al. (2013) yielding a satisfying fraction of about 85% totally re-
solved scales. The ratio $\Delta/\eta$ between Kolmogorov scales $\eta$ and mean LES filter width $\Delta$ is located in the order of $\Delta/\eta \approx 6$ in near-wall regions while it drops down to $\Delta/\eta \approx 2$ in the pipe core region.

4 Pipe Bend Configuration

After preliminary validation by computing the fully developed pipe flow, the methods of Large Eddy Simulation as well as the RANS approaches using a Reynolds Stress and an Eddy Viscosity Model are subsequently applied to the case of a $90^\circ$ pipe bend. Results of the different approaches are compared to each other and their engineering applicability is assessed regarding computational effort, accuracy and usability.

The investigated flow configuration represents a pipe of diameter $D$ which is turned around $\pi/2$ with a distance between the center line and the pivot of $R_c = 1.58D$. With the curvature parameter $\kappa = D/(2R_c)$, a non-dimensional group according to

$$De = Re_b\sqrt{\kappa} = \frac{U_bD}{\nu} \sqrt{\frac{D}{2R_c}}$$  \hspace{1cm} (8)

is built which is referred to as the Dean number $De$. The Dean number can be used as an indication for whether a flow develops secondary motions past a bend, the so-called Dean vortices. With the investigated bulk Reynolds numbers of $Re_b = 14000$ and 34000 and the according Dean numbers of $De = 7900$ and 19000, secondary vortices are expected to be found for both cases.

5 Results

To start with the qualitative behavior of the flow through a bent pipe, Fig. 7 exemplarily illustrates the normalised mean velocity magnitude inside the elbow at $Re_b = 34000$ as obtained by the present LES. Although it is difficult to judge from the shown cross-section view, it is interesting to notice that the flow does not separate from the inner wall of the bend within the symmetry plane of the setup, despite a substantial momentum deficit in this region.

In accordance with the above illustration, Fig. 8 shows various mean streamwise velocity profiles at different positions along the pipe elbow as they are computed using LES. The local acceleration of the fluid flow along the inner wall associated with the velocity magnitude increase at the outer wall behind the bend is clearly visible.
Quantitative comparison of simulation results for this configuration are specifically shown in Fig. 9 where the mean streamwise velocity is plotted at a certain location behind the pipe elbow. It is observed that the LES yields notably good results regarding the overall velocity profile development. Although the results obtained by the RSM deviate noticeably from the experimental data on the inner side of the elbow, the prediction of the velocity gradient at the wall agrees well with the LES results. The standard $k-\varepsilon$-model, however, fails entirely with respect to the mean velocity in the inner region of the bend and thus yields a wrong prediction of the wall shear stress.

Figure 9: Comparison of the non-dimensional mean streamwise velocity evaluated in the domain’s symmetry plane at a position of $0.67D$ behind the pipe elbow for $Re_b = 34000$. The position of $(r/R = -1)$ refers to the outer wall of the bend, while $(r/R = 1)$ is the inner one.

Fig. 10 shows an exemplary comparison of contour plots for the time-averaged streamwise velocity across the pipe’s cross-section at a position of $0.67D$ behind the pipe bend, coinciding with the plane evaluated in Fig. 9. Illustrated are the experimentally obtained results using PIV (left) as performed by Kalpakli and Örlü (2013) and the numerical results of the present LES. The comparison is supposed to underline the capability of LES to qualitatively capture the three-dimensional behaviour of the flow behind the bend since, unfortunately, no relevant quantitative data sets are available.

In analogy to Fig. 10, the qualitative behavior of the mean in-plane velocity field is illustrated in Fig. 11 as a comparison between PIV and LES results. The Large Eddy Simulation predicts the in-plane velocities similar as in the case of the streamwise velocity.

In order to assess the predicted velocity deficiency occurring on the inner wall of the elbow using the Reynolds Stress Model, the computed turbulence intensity components are compared against the results obtained by LES, see Fig. 12. The graph reveals that the overall slopes of the Reynolds stress intensities are captured correctly, however exhibiting a substantial magnitude underprediction. This observation is consistent with the velocity deficiency in the mean streamwise profile since the underprediction of turbulence leads to a lower degree of turbulent mixing, thus resulting in a delayed recovery of the flow.

Figure 10: (Left) Mean streamwise velocity contours behind the elbow obtained from PIV by Kalpakli and Örlü (2013). (Right) Corresponding numerical results reproduced by the present LES at $Re_D = 34000$.

Figure 11: (Left) Mean in-plane velocity contours at $0.67D$ behind the elbow obtained from PIV by Kalpakli and Örlü (2013). (Right) Corresponding numerical results reproduced by the present LES at $Re_D = 34000$.

Figure 12: Non-dimensional turbulent intensities in streamwise ($u$), azimuthal ($\Theta$) and wall-normal directions ($r$) in the inner section of the bend, complementing Fig. 9.
The analysis of the instantaneous flow field shows that in-plane oscillations, the so-called Dean vortices, can be observed using LES. These vortices rotate with alternating clockwise and counter-clockwise direction in the region past the pipe elbow. Such an exemplary counter-clockwise Dean cell is illustrated in Fig. 13 in comparison to relevant experimental observations.

6 Conclusions

The present study embodies a comparative assessment of how the employed computational methods of LES and RANS can handle turbulent flows with significant streamline curvature and potential separation regions. It reveals the superiority of LES over the applied RANS approaches, however at the cost of significant increase in computational effort. It is shown that the mean velocity field of the flow through a pipe elbow can be predicted accurately and that occurring secondary vortices are precisely captured.

As addressed above, a future goal is to perform a highly transient, multi-species LES of the erosion process of a Helium stratification by an entraining jet using the inlet conditions obtained within this framework. Hence, this study additionally yields a database of turbulent inlet conditions for this specific case.

The computations of the pipe elbow configuration at $Re_b = 14000$ and $Re_b = 24000$ are currently in progress and will be presented at the conference.

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References


