

Compressibility effects on sound source distributions in isotropic compressible turbulence

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1 Introduction

Two turbulent mixing noises exist in supersonic jets. One is the Mach waves which are generated from large scale structures, and the other is the acoustic waves from fine scale structures. The two turbulent mixing noises are important, because they appear in ideally expanded jets, under expanded jets and over expanded jets (Tam et al., 1996).

Many researchers have devoted in studying Mach waves. Williams (1963) showed that the far field acoustic power is proportional to the third power of jet velocity. Tam and Burton (1984a,b) developed the model for the mechanism of Mach wave generation in which wall like instability waves moving supersonically are assumed as the sound sources. Lately, the model was validated by comparing with experiments (Tam and Chen, 1992). Recently, in the frame of computational aeroacoustics (CAA) large-eddy simulation (LES) has been one of the strongest tools to predict Mach waves, because Mach waves are generated from large scale structures of turbulence. The prediction accuracy of over all sound pressure levels (OSPL) by LES are within 2dB for moderate Mach number cases ($\mathcal{M}_J \sim 1.5$) (Bodony and Lele, 2005), and within 5dB for high Mach number cases ($\mathcal{M}_J \sim 4.0$) (Nonomura et al., 2014). Note that the errors contain the contributions from acoustic waves from fine scale structures so that the prediction accuracy for Mach waves should be higher than values above.

Acoustic waves from fine scale structures, on the other hand, has been much less understood than those of Mach waves. Only limited knowledge was given by the past studies. Tam et al. (1996) organized a huge amount of experimental data and identified the shape of spectra of acoustic waves from fine scale structures. They showed that the spectra spread wide range of frequency compared with the spectra of Mach waves which have a specific peak. Seror et al. (2000, 2001) investigated the contributions from sub-grid scale to the spectra of SPL. They adopted the idea of sub-grid scale model to the Lighthill’s acoustic analogy for the far field projection coupled with LES, and compared with the results of the filtered direct numerical simulations (DNS) with full-scale Lighthill’s analogy. The model proposed by Seror et al. recoverd the acoustic intensity lost in the filtering procedure and improved the prediction accuracy. Although those studies gave us some useful knowledge to understand the acoustic waves from fine scale structures, the detail mechanisms remain unclarified.

Also, we need to consider compressibility effects, because the effects of compressibility should be large in supersonic flows compared with subsonic and transonic jets. One of the primary effects of compressibility are the reduction of growth rates in free shear layer (Sarkar et al., 1991; Sarkar, 1995; Blaisdell et al., 1993; Vreman et al., 1996; Pantano and Sarkar, 2002). The reason was identified by DNS of compressible shear layer and that is the reduction of pressure-strain correlation (Sarkar, 1995; Vreman et al., 1996; Pantano and Sarkar, 2002). Another example of compressibility effects is the occurrence of eddy shocklets in moderate to high turbulent Mach numbers. They were firstly found in two dimensions by Passot and Pouquet (1987) and later in three dimensions by Lee et al. (1991). The generation of shocklets has large impact on flow fields. The energy dissipation rate in the region of shocklets is more than ten times larger than that of incompressible turbulence (Lee et al., 1991).

In addition, the generation of enstrophy is intensified by the strong compression in shocklets region (Kida and Orzag, 1990a; Wang et al., 2011). Moreover, the compressive energy show Burgers like $k^{-2}$ spectra when the turbulent Mach number or the ratio of compressive kinetic energy to total kinetic energy is high enough for stronger shocklets to appear (Kida and Orzag, 1990b, 1992; Wang et al., 2013). Although the compressibility has large influence on flows, how it affects the flow structures correspond to the sound sources has not been understood.

In this study we conducted the direct numerical simulations of isotropic compressible turbulence and investigated the compressibility effects on the sound source distributions to understand the generation mechanisms of acoustic waves from fine scale structures in supersonic jets. The sound sources were numerically obtained by the Lighthill equation (Lighthill, 1952) using the results of DNS. This kind
of approach which is the combination of DNS of compressible turbulence and the Lighthill equation to investigate the sound sources has not been addressed in the past studies and should provide valuable knowledge. In this paper, as a first step, we focused on the sound sources due to the Reynolds stress term which are the dominant sound sources in low Mach number flows such as subsonic jets and would be important also in supersonic jets.

2 Problem set up and methods

Decaying compressible turbulence was simulated in a cubic box of size $2\pi$ with periodic boundary conditions at the spatial resolution $N = 128^3$. The governing equations were three-dimensional compressible Navier-Stokes equations for an ideal gas in which the viscosity $\mu$ was determined from the Sutherland law, while the Prandtl number was kept constant at $Pr = 0.72$.

The equations were solved by the following hybrid approach to achieve both capturing shocklets and maintaining high accuracy. We adopted the sixth-order skew-symmetric splitting scheme (Pirozzoli, 2011) for the entire regime. In addition, the dissipation term (Nonomura et al., 2014) of the sixth-order weighted essentially non-oscillatory central upwind (WENO-CU6) scheme (Hu et al., 2010) was added for the shocklets region which is identified by the Ducros sensor (Ducros et al., 1999). The time integration was performed by the fourth-order Runge-Kutta method with time step $dt = 5.0 \times 10^{-3}$.

The simulated compressible turbulence was governed by two important parameters: the turbulent Mach number $M_t = \langle u^2 \rangle^{1/2}/\langle c \rangle$, where $c$ is the speed of sound, and the Taylor micro scale Reynolds number $R_\lambda = \langle u'^2/3 \rangle^{1/2} \langle \rho \rangle/\langle \mu \rangle$ where $\lambda = \langle u'^2/3 \rangle^{1/2}/[(\partial u_i/\partial x_j)/3]^{1/2}$ is the Taylor micro-scale. $\langle \cdot \rangle$ denotes the spatial average.

In this simulations the initial velocity field was solenoidal with turbulent Mach number $M_{10} = 0.1 - 1.0$ (see table 1) and the Taylor micro scale Reynolds number $R_\lambda = 72$. The velocity field was determined by the energy spectrum $E(k) \propto k^2 \exp(-2(k/k_0)^2)$ where $k$ is the wave number and $k_0$ is a constant set to be $\sqrt{10}$. For the thermodynamic initial condition, we assume initially homentropic, and then the initial pressure and density were calculated by the following Poisson equation:

$$\nabla^2 p = \frac{1}{\gamma} \frac{\partial u_i}{\partial x_j} \frac{\partial u_j}{\partial x_i}.$$

This equation is derived by taking the divergence of the momentum equation and assuming the solenoidal velocity where the viscous term is ignored. We used a limiter to avoid negative pressures at the highest Mach number of $M_{10} = 1.0$; thus, in that case, the flows were not given exactly by (1). However, the limiter was used with keeping the system homentropic and the

<table>
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<th>Case</th>
<th>$M_{10}$</th>
<th>$k_{max}$</th>
<th>$\eta$</th>
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<td></td>
</tr>
<tr>
<td>B</td>
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<td>2.15</td>
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<tr>
<td>C</td>
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</tr>
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<td>D</td>
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<tr>
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<tr>
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<td>J</td>
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<td>2.02</td>
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Table 1: Parameters used in the direct numerical simulations. Values for $M_{10}$ is the initial values; values for the resolution parameter $k_{max}$ are those at $t/\tau = 1.56$.

3 Results and discussion

For this paper, we conducted the direct numerical simulations with various turbulent Mach numbers while the Reynolds number was set to be the same value for all cases. To compare results for different Mach numbers, we normalize the time by the initial values of the large-eddy turn-over time $\tau$. Furthermore, the flows at $t/\tau = 1.56$ are considered as the statistical equilibrium states because the energy spectrum converges at $t/\tau = 1.56$ and the enstrophy experiences peak values at $t/\tau < 1.56$. In Table 1, we show the resolution parameter $k_{max}$ calculated at this quasi-equilibrium state where $k_{max}$ is the maximum wave number and $\eta$ is the Kolmogorov length scale. In all simulations, the resolution parameter was $k_{max} > 2$; thus the smallest scales of turbulence
were well resolved. In these simulations, the locally supersonic regions occur for $M_{in} \geq 0.5$, and thus the existence of shocklets is expected in those cases. In most discussions in this paper, we use cases at $M_{in} = 0.2$ and 1.0 as representatives of lower and higher Mach numbers, respectively.

**Strength of the sound sources**

Sound source distributions are calculated by the Lighthill equation (Lighthill, 1952) as follows:

$$\frac{\partial^2 \rho}{\partial t^2} - c_0^2 \nabla^2 \rho = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j},$$

where $T_{ij}$ is Lighthill’s turbulent stress tensor written as,

$$T_{ij} = \rho u_i u_j + \delta_{ij}\{p - p_0 - \frac{2}{5}(\rho - \rho_0)\} + \rho \frac{\partial u_i}{\partial x_j} \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \delta_{ij} \frac{\partial u_k}{\partial x_k}.$$  

If a flow at low Mach number and high Reynolds number is considered, the second and third terms in (4) are negligibly small, and thus (2) becomes

$$\frac{\partial^2 \rho}{\partial t^2} - c_0^2 \nabla^2 \rho = S,$$

where $S$ is the following source term,

$$S = \frac{\partial^2 (\rho u_i u_j)}{\partial x_i \partial x_j}.$$  

In this paper, we use the formulation of (4) and (5) as a first step. In nearly incompressible flows density is assumed to be almost uniform $\rho \sim \rho_0$, then the source term (5) is written as $\rho_0 \frac{\partial^2 (u_i u_j)}{\partial x_i \partial x_j} \sim u_{rms}^4$ in which $u_{rms} = \langle u_i^2 \rangle^{1/2}$ is the root mean square (rms) of velocity. If the strength of sound source is defined as $S^2$, the strength of rms sound source $S_{rms}^2 = \langle S^2 \rangle$ is expected to be $S_{rms}^2 \propto u_{rms}^4$ in low Mach number flows. This discussion is equivalent in Lighthill’s $U_j^2$ law (Lighthill, 1952).

Figure 1 shows the time history of the rms sound source strength $S_{rms}^2$ divided by the fourth power of the initial rms velocity $u_{rms}(0)^4$. The differences collapse for the lower Mach number cases ($M_{in} = 0.1 - 0.5$); thus the sound source strengths are almost proportional to $u_{rms}(0)^4$. However, this relation is invalid and high Mach number cases for $M_{in} \leq 0.5$, and (ii) the timing of the (local) maximum depends on the existence of locally supersonic region. (Only the $M_{in} = 0.5$ case has two local maxima. The timing of each maximum agrees with those at the lower and higher Mach numbers.) To summarize, the compressibility primarily weakened the sound source strength; however, once flow fields were locally supersonic, as explained later, shocklets intensified the sound source strength. The probability of the occurrence of shocklets determines the effects that are dominant because the values for $M_{in} = 0.5$ and 0.6 are smaller. This trend differs from that for the time history of the Mach wave, which is monotonically weakened as Mach number increases (Pirozzoli and Grasso, 2004).

For reference we show theoretical predictions of far-field acoustic power in supersonic jets. Note that our results are not for the far field but for the sound source itself. Williams (1963) predicted the acoustic power of supersonic jets is proportional to the third power of the jet velocity $U_j^3$ because the Mach wave radiation is assumed to be the dominant sound source. Our results are not related to the $U_j^3$ law because large structures, Mach waves, are not treated. The present results are related to the acoustic waves from fine scale structures and imply that the velocity dependence on the acoustic power changes in high Mach numbers.

**Sound source and second invariant of the velocity gradient tensor**

In this section, we study sound source distributions by comparing with the second invariants of the veloc-
However, strong sound source levels appear at near mental sound source in nearly incompressible flows. Figure 4 These values are normalized by their rms values. In Q sources exist at very small values of Q same as those for in Figure 3: (i) the distributions of S=S=S=S=S=S=u rms (0) and (b) normalized sound source S=u rms (0) for M10 = 0.2 and M10 = 1.0 at t/τ = 1.56. Q = 0 (Figure 4-b-ii) for M10 = 1.0, in addition to the line that S is highly correlated with Q (Figure 4-b-i) similar to the behaviour for M10 = 0.2. Therefore, another property exists in sound sources of highly compressible turbulence.

Effects of dilatation on sound sources

To clarify the reason why another property of sound source occurs in highly compressible flows, we consider the effects of dilatation on sound source distributions. Lee et al. (2009) and Wang et al. (2012) investigated the effects of local dilation level on statistical properties by conditional sampling on the various levels of dilatation. We follow their analysis and apply the method to sound source distributions. Table 2 shows the percentages of the sound source strength S2 with various dilatation levels for case M10 = 1.0. The fraction of the sound source in the highly compression region (−∞, −2.0]) is the largest; in contrast, sound sources in the expansion region ([1.0, ∞[) make little contribution (below 10%). We now again consider the relation between second invariants of the velocity gradient tensor Q and sound sources S. As shown in Figure 4-(b), two characteristics appear for M10 = 1.0. One is the line (Figure 4-b-i) along which S is highly correlated with Q; this is similar to high correlations in low Mach number flows. The other one is the strong sound source levels near Q = 0 (Figure 4-b-ii). To investigate the dilatation levels at which these two characteristics occur, we show the joint PDFs of S and Q.
at various dilatation levels in Figure 5. The line for the high correlations between \( S \) and \( Q \) (Figure 4-b-i) appears to be almost independent of dilatation levels except in the highest expansion region (\( \theta/\theta_{rms} \geq 2 \)) in which the contribution to the sound source strength is small. In contrast, the line near \( Q = 0 \) (Figure 4-b-ii) strongly appear in the highest compression region (\( \theta/\theta_{rms} \leq 2 \)). In Figure 6 we show the iso-surface of dilatation colour-coded for the sound source \( S \) at \( \theta = -3\theta_{rms} \) and \( \theta = \theta_{rms} \) for \( M_{\infty} = 1.0 \). The iso-surface displays the sheet-like structures at \( \theta = -3\theta_{rms} \) which is similar to shock wave structures shown in Wang et al. (2012) and strong sound source levels appear uniformly on them. In contrast, at \( \theta = \theta_{rms} \), the iso-surface shows rock-like structures and the sound source levels are weaker. Therefore, the existence of the shock waves in the highly compression region should be the cause the strong sound source in high Mach number flows and would lead the exponent of 6 (Figure 2).

\[
\begin{align*}
\theta/\theta_{rms} & \quad [\infty, -2.0] \quad [-2.0, 1.0] \quad [-1.0, 0.0] \\
\text{Fractions} & \quad 29.8 \quad 14.5 \quad 23.1 \\
\theta/\theta_{rms} & \quad [0.0, 1.0] \quad [1.0, 2.0] \quad [2.0, \infty] \\
\text{Fractions} & \quad 23.4 \quad 6.8 \quad 2.4
\end{align*}
\]

Table 2: Percentage of sound source strength \( S^2 \) in flow regions with various dilatation levels for \( M_{\infty} = 1.0 \) at \( t/\tau = 1.56 \).
4 Conclusions

The effects of compressibility on sound source distributions were investigated by the direct numerical simulations of isotropic decaying compressible turbulence with various turbulent Mach numbers. Sound source distributions were obtained numerically from the Lighthill equation; in that equation, the Reynolds stress term which is the dominant sound source in lower Mach number flows, was investigated in detail as a first step. The results show that the compressibility weakens overall sound level; however, once flow fields become locally supersonic, a sound source generation mechanism appears that is associated with shocklets. In the highly compressible flows, this sound generation mechanism strengthens the overall sound level. As the results, in lower Mach number flows, sound source levels are proportional to $u_{rms}^2$, while in highly compressible regime, sound source levels are proportional to $u_{rms}^3$.

The evidence that strong sound generation in higher Mach number flows is associated with shocklets as follows:

1. Strong sound generation occurs in locally high compressible region $\theta/\theta_{rms} \leq -2$.
2. The iso-surface of dilatation at $\theta/\theta_{rms} = -3$ has sheet like structures with strong sources uniformly distributed on them.

This is the first attempt to use DNS of isotropic compressible turbulence to investigate, in detail, the effects of compressibility on sound source distributions. This original approach could provide new insights for aeroacoustic problems such as the noise from supersonic jets and rocket plume acoustics. Thus, we believe this study will be useful for developing the prediction and reduction methods of related aeroacoustic issues.

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References


