1 Introduction

In engineering and environmental applications, it is frequent to encounter turbulent mixing of scalars released from multiple sources. For example, in the combustion chamber, the reaction rates depend on the rate of mixing of multiple scalars. In environmental pollution, hazardous materials are often released from multiple sources that are close to each other. In these cases, the first-order statistics of the scalar (i.e., mean concentration) can be simply calculated as a sum of two independent releases. However, the second- and higher-order statistics of the scalar (e.g., variance of concentration fluctuations) cannot be calculated as a sum of two independent releases, as the interaction of higher-order statistics of two scalar fields is typically nonlinear. Despite the importance of the interaction of passive scalars emitted from multiple concentrated sources, studies on this subject are still limited in literature.

Warhaft (1984) experimentally studied the interference of passive thermal fields produced by two line sources in decaying grid turbulence. He observed that the interference between the two thermal plumes may alter total temperature variance level, and this effect varies with the distance between the two line sources, source position from the grid, and downstream location where the measurements were taken. Yee et al. (2003) developed an analytical model for evaluation of joint concentration statistics released from two separated continuous point sources. Their results of the second-order correlation function produced by the model were in good agreement with experimental data for a two point source release in grid turbulence. Vrieling and Nieuwstadt (2003) performed direct numerical simulation (DNS) to study the interference of two nearby sources in a channel flow. They placed their line sources in the center of the channel to emulate concentration release in homogeneous turbulence, and observed that the covariance between two plumes depends on the spacing between the sources and the downstream distance from the sources. Costa-Patry and Mydlarski (2008) conducted a wind-tunnel experiment to quantify the interaction of two scalar fields generated by line sources in a fully-developed high-aspect-ratio turbulent channel flow. The traverse profiles of the second-order correlation function exhibited that generally the quality of mixing of two scalars is the largest at the edges of the thermal plume and smallest in its core region.

In spite of these experimental, numerical, and theoretical investigations, high-quality data sets of concentration statistics on two or more passive scalars are still rather limited. In this paper, we use DNS to study the interference effect on second- and third-order concentration moments generated by releasing of passive material from two point sources (located in the vicinity of the wall) into an inhomogeneous neutrally-stratified wall-bounded shear flow. We also aim at analyzing the mixing of two plumes using probability density functions (PDF). In general, the PDF method has not been well-established towards this type of applications. In this research, we will use this method to provide a better understanding of the mixing of two plumes.

The paper is organized as follows. In section 2, the numerical procedure is described. In section 3, results of numerical simulations are presented and analyzed. Finally, in section 4, major conclusions of this research are summarized.

2 Numerical Method

In this study, we considered an open channel configuration. Figure 1 shows the schematic of the computational domain. The computational domain sizes are $2\pi\delta \times \delta \times \pi\delta$ in the streamwise ($x$), vertical ($y$) and spanwise ($z$) directions, respectively, where $\delta = 0.04$ m is the height of the open channel. The Reynolds number is set to $Re_r \equiv u_r \delta / \nu = 395$ (where $u_r$ is the friction velocity and $\nu$ is the kinematic viscosity of the fluid). The passive scalar fields are produced by two concentrated point sources, namely, concentration ‘A’ and concentration ‘B’, which are symmetrically separated in the spanwise direction and are located in the vicinity of the lower wall at the same height $y_{\text{m}}^+ \equiv y_{\text{m}}u_r / \nu = 5$ to emulate ground level source releases. The sources are of equal strength and the size (radius) of the sources is 0.007$\delta$. Four test cases are considered in our DNS study. Cases 1-4 correspond to four different source separation distances, $d = 0.049\delta, 0.115\delta, 0.279\delta$ and 0.541$\delta$. These values are chosen to study the effect of turbulent eddy sizes on the interaction of two plumes. The Schmidt number
\( Sc \equiv \frac{\nu}{\alpha} \) of the scalar is equal to 1.0.

![Diagram](image)

Figure 1: Schematic of the computational domain, two point sources (A and B) are separated horizontally by distance \( d \).

For the simulations, we solved numerically the momentum transport equations for an incompressible flow and the advection-diffusion equation for a passive scalar given by:

\[
\frac{\partial u_i}{\partial x_i} = 0, \quad (1)
\]

\[
\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j \partial x_j}, \quad (2)
\]

and

\[
\frac{\partial c}{\partial t} + u_j \frac{\partial c}{\partial x_j} = \alpha \frac{\partial^2 c}{\partial x_j \partial x_j}. \quad (3)
\]

Here, \( u_i \) is the instantaneous velocity in the \( x_i \)-direction with \( i = 1, 2 \) or 3 representing the along-wind \( x \), vertical \( y \), and spanwise \( z \) directions, respectively, \( c \) is the instantaneous concentration, \( p \) is the pressure, \( \rho \) is the density of the fluid, and \( \alpha \) is the molecular diffusivity of the scalar.

The DNS were conducted using an in-house code developed using the FORTRAN 90/95 programming language and fully-parallelized using the message passing interface (MPI) library. The code is based on a fully conservative and fully implicit finite difference discretization scheme and utilizes a staggered grid arrangement. Numerical simulations were conducted on the Western Canada Research Grid (Westgrid), which is one of the seven consortia that provides high-performance computational resources across Canada.

For the velocity field, we applied periodic conditions to the streamwise (\( x \)) and spanwise (\( z \)) directions, no-slip condition to the bottom wall (\( y = 0 \)), and free-slip condition to the top boundary (\( y = \delta \)). For the concentration field, we imposed a zero concentration condition at the inlet boundary, and zero Neumann conditions to other boundaries. The computational domain was discretized using \( 512 \times 128 \times 384 \) grid nodes in the \( x \)-, \( y \)-, and \( z \)-directions, respectively.

3 Result Analysis

In this section, the results of our numerical simulations are discussed, which include the velocity statistics, mean concentration, the root mean square (RMS) concentration fluctuations, correlation functions, and concentration PDF.

**Velocity Statistics**

Figure 2a shows the vertical profile of the normalized mean streamwise velocity \( \bar{u}/\bar{u}_\tau \). The obtained DNS result on the mean velocity profile is compared against the classical log law of the wall based on von Karman’s two-layer turbulent boundary-layer model. Figures 2b and 2c exhibit the vertical profiles of the three components of RMS velocities (or, velocity standard deviations) and the Reynolds shear stress. All quantities have been non-dimensionalized using the friction velocity \( u_\tau \). In figure 2b, \( u_{rms} \), \( v_{rms} \), and \( w_{rms} \) denote the RMS velocity fluctuations in the along-wind, vertical and spanwise directions, respectively.

Due to a lack of numerical and experimental data for the open channel at \( Re_\tau = 395 \), we used a plane channel data to validate the flow field. Figure 2 shows DNS data of half plane channel of Kim et al. (1987) at \( Re_\tau = 395 \). The upper boundary of the open channel coincides with half of the plane channel. The agreement between two DNS results is good, except for
$v_{rms}$ close to the upper boundary of the open channel. The value of $v_{rms}$ of the open channel approaches zero at the upper boundary, while that of the plane channel reaches a non-zero value. This discrepancy is attributed to the non-penetrative condition used at the upper boundary of the open channel.

**Mean Concentration**

The scalar transport equation represented by equation 3 is linear with respect to the concentration field. Therefore, the total mean concentration produced by the two point sources is superposable, which is equal to the sum of the mean concentration fields produced by each of the sources. The streamwise evolution of the lateral profiles of the total (resultant) mean concentration, $C_T$, are shown in figure 3. The total mean concentration is non-dimensionalised using the maximum of the total mean, $(C_T)_m$, at any downstream position. In the streamwise direction close to the sources, the profiles show a dual-peak for the cases with larger source separation distance (i.e. cases 3 and 4). The lateral position of the dual-peak coincides with the lateral position of the sources. The dual-peak is replaced by a single peak at downstream fetches where the two plumes spread significantly, except for case 4, in figure 3d. Due to the equal strength of the sources, the lateral position of the single peak is at the middle of the two plumes.

In the following section, the results of RMS of concentration fluctuations are discussed.

**RMS Concentration Fluctuations**

Two major sources of concentration fluctuations are the meandering of the instantaneous plume, and the entrainment of uncontaminated fluid into the plume. The latter causes internal concentration fluctuations inside the plume. Figure 4 shows the streamwise variation of the intensity of concentration fluctuation $i = c_{rms,A}/C_A$ along the source line for the plume released from source A, where $c_{rms,A}$ and $C_A$ are RMS of concentration fluctuations and the mean concentration, respectively. The values of $i$ predicted using the well-known fluctuating plume model (FPM) of Gifford (1959) are also demonstrated in this figure. FPM only takes into account the effect of meandering of the instantaneous plume. As evident in figure 4, FPM is only capable of predicting the intensity at the locations very close to the source, where the meandering effect is strong. As the meandering effect decreases, the agreement between DNS and FPM also decreases. It can be seen in this figure that the intensity calculated using FPM is almost zero for $x/\delta \geq 0.5$. This reveals that the meandering is negligible at these downstream locations.

The profiles of RMS of concentration fluctuations of the total plume $c_{rms,T}$ in the spanwise direction at the source height are shown in figure 5. In the figure, the value of $c_{rms,T}$ has been non-dimensionalized using its maximum value $(c_{rms,T})_m$ at any downstream position. At all downstream locations shown in figure 5, the profiles of $c_{rms,T}$ of cases 1 and 2 exhibit a dual-peak pattern. In contrast, profiles of $c_{rms,T}$ for cases 3 and 4 show a quadruple-peak pattern at $x/\delta = 1$ and 2. In the far downstream regions (for $x/\delta = 4$ and 6), the quadruple-peak is replaced by a dual-peak in case 3, but it is preserved in case 4. The behavior of the spanwise profile of $c_{rms,T}$ can be explained as follows.

The production term in the transport equation for the variance of the concentration fluctuations is proportional to the mean concentration gradient. Therefore, locations where the mean concentration gradient is zero (local extrema of the mean concentration profile) can be also potentially the locations for local minima of the variance. Similarly, locations where the absolute mean concentration gradient is maximum can be also potentially the locations for local maxima of the variance. However, when the instantaneous plume undergoes the meandering stage, the fluctuations can be transported to locations of maximum absolute mean concentration gradient by turbulent eddies from the regions with zero or small mean concentration gradient.
or vice versa. This mechanism smoothens the local maxima and minima of the variance, leading to a single peak for the variance profile.

As shown in figure 4, for the simulated ground plume, the meandering effect is small for \( x/\delta \geq 0.5 \). Therefore, the local minima of \( c_{\text{rms},T} \) should be located at the extrema locations of the total mean concentration profiles, and the locations of local maxima of \( c_{\text{rms},T} \) should correspond to the locations of maximum absolute total mean concentration gradient. As an example, in figure 6, the total mean concentration \( C_T \) and RMS \( c_{\text{rms},T} \) for case 1 and 3 are plotted at four downstream locations. At all downstream locations, the extrema locations in the \( C_T \) profile occur at the locations with local minimum \( c_{\text{rms},T} \), and the locations of maximum absolute gradient of \( C_T \) coincide with local maximum \( c_{\text{rms},T} \). Similar behavior is also observed in cases 2 and 4.

In this section, it is observed that the shape of RMS concentration fluctuations of the total plume can be predicted using the knowledge of the strength of meandering and the mean concentration profile of the total plume. The latter can be easily obtained as sum of the mean concentration of each plume. This observation implies that the knowledge of each single plume (without considering the interference effect) is enough to predict the approximate shape of the total RMS concentration fluctuations. In the following section, the interference effect on the second- and third-order concentration moments are further studied using
the second- and third-order correlation functions.

**Correlation Functions**

The non-dimensionalized form of the scalar covariance, $c_A c_B$ is the second-order correlation function which is defined as

$$
\rho^{[1][1]} = \frac{c_A c_B - c_A^2 - c_B^2}{2(c_A^2 + c_B^2)^{0.5}}
$$

where $c_A^2$, $c_B^2$ and $c_T^2$ are the variance of concentration A, B and total plume, respectively. The second-order correlation function shows the quality of mixing process whilst the scalar covariance shows the total amount of mixing (Costa-Patrzy and Mydlarski, 2008).

Following Yee *et al.* (2003), the third-order correlation function is defined as

$$
\rho^{[1][2]} = \frac{c_A c_B c_T}{(c_A^2 + c_B^2)^{0.5}} = \frac{c_T c_A - c_A^2}{3(c_A^2 + c_B^2)^{0.5}}
$$

where $c_T^2 = c_A^2 + c_B^2 - c_A c_B$.

The streamwise variation of the second- and third-order correlation functions at the midpoint in the spanwise direction between the plumes at the source height are shown in figure 7. Based on Yee *et al.* (2003), it is observed that the shape of $\rho^{[1][2]}$ profile is sensitive to either the meandering or the internal fluctuations, whilst the shape of $\rho^{[1][1]}$ profile is not. Also, when the meandering effect is strong compared to the internal fluctuations, $\rho^{[1][2]}$ shows a similar behavior to $\rho^{[1][1]}$, but when the internal fluctuations are dominant, $\rho^{[1][2]}$ shows a more complicated behavior than $\rho^{[1][1]}$. This means that $\rho^{[1][1]}$ contains the effect of internal concentration fluctuations that are not exhibited in $\rho^{[1][2]}$. As shown in the previous subsection, the meandering is negligible for $x/\delta \geq 0.5$ for all cases considered in this study. Therefore, it is not expected that the general trend of $\rho^{[1][2]}$ be similar to that of $\rho^{[1][1]}$ for $x/\delta \geq 0.5$. In general, a constructive or destructive interference of the two plumes in the second-order concentration moment (positively or negatively valued $\rho^{[1][1]}$, receptively) does not necessarily imply a constructive or destructive interference in the third-order concentration moment (positively or negatively valued $\rho^{[1][2]}$, receptively). For example, at $x/\delta = 1$ in case 1, the interference is constructive in the second-order concentration moment ($\rho^{[1][1]} > 0$), however, it is negative in the third-order concentration moment ($\rho^{[1][2]} < 0$).

The equation for $\rho^{[1][1]}$ can be also equivalently expressed using the instantaneous and mean concentrations as

$$
\rho^{[1][1]} = \frac{c_A c_B - C_A C_B}{(c_A^2 + c_B^2)^{0.5}},
$$

where $c_A c_B$ is the correlation between the two instantaneous plumes and $C_A$ and $C_B$ are mean concentrations of plumes A and B, respectively. Figure 7a shows four stages in the streamwise evolution of $\rho^{[1][1]}$ at the midpoint in the spanwise direction between the two plumes.

1. **First Stage**

   At this stage, there is no interaction between the two plumes. In other words, the mean plume width $\sigma_a$ and instantaneous plume width $\sigma_r$ are smaller than the source separation distance $d$ (i.e., $\sigma_a < d$ and $\sigma_r < d$) and the location at the midpoint between the two plumes is never exposed to both plumes simultaneously such that $c_A c_B = 0$ and $C_A C_B = 0$. As a result, $\rho^{[1][1]}$ maintains zero identically.

2. **Second Stage**

   At this stage, the mean plume width $\sigma_a$ exceeds the sources separation distance $d$ but the instantaneous plume width $\sigma_r$ is still smaller than the source separation distance (i.e., $\sigma_r \leq d < \sigma_a$). This means that the two mean plumes overlap (i.e., $C_A C_B \neq 0$) but not yet the instantaneous ones (i.e., $c_A c_B = 0$). In this stage, at the midpoint location, a relatively large eddy brings a patch of concentration from one of the plumes to the midpoint, while simultaneously taking a patch of concentration from the other, off this location (causing the concentration contributed by one plume to increase and that contributed by the other plume to decrease). As a result, $\rho^{[1][1]}$ reduces to a negative value at this stage.

3. **Third Stage**

   At this stage, the mean plume width $\sigma_a$ and instantaneous plume width $\sigma_r$ exceed the source separation distance $d$ (i.e., $d \leq \sigma_a$ and $d \leq \sigma_r$) and instantaneous plumes overlap in addition to the mean (i.e., $c_A c_B \neq 0$ and $C_A C_B \neq 0$). At
this stage, eddies at the midpoint location transport a patch of concentration from both plumes together, causing the two concentrations to either increase or decrease simultaneously. This internal mixing causes $\rho^{[1][1]}$ to increase towards a positive value after reaching its minima.

4. Fourth Stage
At this stage, the mean plume width $\sigma_\alpha$ and instantaneous plume width $\sigma_\chi$ are much larger than the source separation distance $d$ (i.e., $d \ll \sigma_\alpha$ and $d \ll \sigma_\chi$). In this range, the two plumes are completely mixed (i.e., $\rho^{[1][1]} = 1$) and the total plume behaves as a single source plume. None of the cases in this study could reach this theoretical stage. In order to capture this stage, a very large channel length is required.

Figure 7a also shows that $\rho^{[1][1]}$ evolves slower as the source separation distance $d$ increases. This is because for a large value of source separation distance, the two plumes require a greater downstream distance to interact sufficiently. We will further discuss this matter in the following subsection.

Concentration PDF
The concentration PDF at point $x(x, y, z)$ is defined as

$$p(\chi; x)dx \equiv \Pr\{\chi \leq c(x) < \chi + dx\}, \quad (7)$$

where $\Pr\{\cdot\}$ denotes the probability and $\chi$ is the value of the instantaneous concentration. Figure 8 shows the PDF of non-dimensionalized concentration ($p(\chi/C)$) at the midpoint in the spanwise direction between the plumes at source height for both single and total plumes of case 1. The skewness and kurtosis of these PDFs are shown in table 1. A distribution with a skewness of zero is symmetric about its peak. If the skewness is negative, then more elements of the distribution have values lower than those expected for a distribution with a zero skewness. This results in a longer tail to the left of the PDF peak. In contrast, distributions with positive skewness have a longer tail to the right of the PDF peak, indicating that more elements of the distribution have values higher than those expected for a distribution with a zero skewness. As the kurtosis increases, the data becomes more clustered around the mean, indicating a lower variability in the data set. However, if the tails get “fatter”, the likelihood of occurrence of very rare events increases.

The shape of the PDF in figure 8 and the values of skewness and kurtosis in table 1 show that the skewness and kurtosis of the single plume increase from $x/\delta = 0.5$ to $x/\delta = 3$, and after that, they both decrease. As mentioned above, when the kurtosis increases, more of the variance in a distribution is due to the occurrence of very rare events. On the other hand, the entrainment of patches of uncontaminated fluid into the plume is the main source of concentration variance in this study. Therefore, these rare events happen when a diluted fluid patch reaches the measurement location. This causes the concentration level at the measurement location to decrease for a very short period of time and then it returns back to its mean value. This explains the increment in the skewness and kurtosis of concentration from $x/\delta = 0.5$ to $x/\delta = 3$. However, when the instantaneous plume size is large, a diluted patch has more time to mix with its surrounding fluid before reaching the measurement location. This causes the concentration level of the patch to reach almost that of the surrounding fluid. Therefore, the measurement point located at the center of the instantaneous plume does not experience a sudden decrease in the concentration level. This justifies the reduction in the skewness and kurtosis of the concentration towards that of a Gaussian profile (Sk=0 and Ku=3) from $x/\delta = 3$ to $x/\delta = 6$.

The streamwise variation of the skewness and kurtosis values of the total plume is very similar to that of the single plume, except from $x/\delta = 0.5$ to $x/\delta = 1$, where the kurtosis decreases. At downstream location $x/\delta = 6$, PDFs of the single and total plumes al-

![Figure 8: PDF for case 1 at the midpoint in the spanwise direction between the plumes at source height at streamwise locations $x/\delta = 0.5$ (a), 1 (b), 3 (c), and 6 (d), for the total plume (○) and for the single plume (●).](image)

<table>
<thead>
<tr>
<th>Case</th>
<th>Sk</th>
<th>Ku</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single Plume</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x/\delta = 0.5$</td>
<td>0.09</td>
<td>2.44</td>
</tr>
<tr>
<td>$x/\delta = 1$</td>
<td>0.19</td>
<td>2.73</td>
</tr>
<tr>
<td>$x/\delta = 3$</td>
<td>0.77</td>
<td>4.39</td>
</tr>
<tr>
<td>$x/\delta = 6$</td>
<td>0.32</td>
<td>3.30</td>
</tr>
<tr>
<td>Total Plume</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case 4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x/\delta = 0.5$</td>
<td>-0.74</td>
<td>3.26</td>
</tr>
<tr>
<td>$x/\delta = 1$</td>
<td>-0.14</td>
<td>2.74</td>
</tr>
<tr>
<td>$x/\delta = 3$</td>
<td>0.54</td>
<td>3.91</td>
</tr>
<tr>
<td>$x/\delta = 6$</td>
<td>0.17</td>
<td>3.10</td>
</tr>
</tbody>
</table>

Table 1: The values of skewness (Sk) and kurtosis (Ku) for cases 1 and 4.
most collapse on each other, which shows that the total plume acts like a single plume (because the source separation distance is very small compared to the downstream distance at $x/\delta = 6$). Also at this downstream location, the skewness and kurtosis values are very close to that of a Gaussian profile, which reveals that the concentration PDFs exhibit almost a Gaussian distribution. In order to show a perfect Gaussian distribution, further downstream development use of a much larger computational domain would be necessary.

Figure 9 shows the PDF of case 4 at the midpoint in the spanwise direction between the plumes at source height. The shape of the profiles for this case is totally different from that of case 1 at downstream locations close to the sources ($x/\delta = 0.5, 1$ and 3). At these locations, the measurement point is not within instantaneous plumes, therefore it is not expected that the entrainment of uncontaminated fluid patches would cause the skewness and kurtosis to increase. The PDF profiles at $x/\delta = 0.5$ and 1 (figure 9a,b) show that the probability of zero concentration state is prominent, because as it is observed in figure 7a, the correlation between the two plumes is zero at these downstream locations (indicating that the midpoint location is mostly exposed to the uncontaminated fluid). This causes the instantaneous concentration to be very intermittent which then results in a positive skewness and a large kurtosis as shown in table 1. Further downstream of these sources ($x/\delta = 3$), the size of the instantaneous plumes increases, therefore the measurement location is exposed to plumes for a longer period of time, and to clean fluid for a shorter period of time. As such, the probability of zero concentration reduces significantly at the midpoint location and the instantaneous concentration becomes less intermittent. This also reduces values of skewness and kurtosis at this downstream location. In a far downstream region for $x/\delta = 6$, the probability of finding zero concentration for both single and total plumes is zero. This reveals that the midpoint location is always within the instantaneous plumes, or in other words, the two single instantaneous plumes are overlapped at this downstream location\(^1\).

It is observed that the total plume PDF at the midpoint location approaches that of a single plume as downstream distance increases and as the source separation distance decreases. For a large source separation distance, the single and total PDF profiles at the midpoint have an exponential form, however, for a small source separation distance, they approach a Gaussian distribution at far downstream locations.

\(^1\)A sufficient condition to recognize that the two plumes overlap instantaneously is that the probability of the occurrence of zero instantaneous concentration is zero identically at the midpoint between the two plumes. However, this is not a necessary condition, because it is possible that the two plumes are overlapped but the probability of zero concentration is still non-zero due to the presence of patches of uncontaminated fluid at the measurement location.

4 Conclusions

In this study, we performed DNS of continuous releases of two passive plumes from two concentrated point sources into a turbulent open channel flow. The sources are located in the vicinity of the wall to represent the ground source releases. The mean concentration (first-order concentration moment) of the total plume can be obtained as a sum of the mean concentration of each plume. However, other concentration moments (e.g. second-order and higher) are not superposable and the interference of the two plumes should be taken into account for calculations of higher-order concentration moments of the total plume.

Using the fluctuating plume model of Gifford (1959), it is shown that for the ground plumes of this study, the effect of meandering is negligible for $x/\delta \geq 0.5$. It is also observed that the spanwise profile of RMS concentration fluctuations can be predicted based on the knowledge of a single plume without a need for information on interference of two plumes. The effect of the interference of the two plumes on the second- and third-order concentration moments is studied using correlation functions. The streamwise variation of the second-order correlation function at the midpoint location between the two plumes reveals that there are four stages in the mixing of two plumes. The analyses on the shape of the PDF of the single and total plumes show that for a source separation distance of large value, the PDF profiles exhibit exponential behaviors at downstream locations close to the sources. As downstream distance increases and the source separation distance decreases, the PDF approaches a Gaussian distribution.

References


