Modal instability analysis of light jets

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1. Objective

The linear instability dynamics of low-density subsonic jets are characterized by an investigation of their eigenmode spectra and their response to forcing input. Temporal eigenmodes (commonly called global modes) are computed as two-dimensional wavepacket structures in the \((r, x)\) plane, associated with a complex frequency that indicates temporal growth or decay. The flow response to external forcing is represented by the perturbation energy gain (response energy versus forcing energy), which defines the pseudospectrum\textsuperscript{1}. The relevance of linear eigenmode analysis for the interpretation of real jet dynamics, and indeed its capacity for quantitative predictions, is demonstrated by a comparison with experimental results reported by Hallberg & Strykowski\textsuperscript{2}. The principal goal of this study is to provide a detailed discussion of typical features of the eigenmode spectra that are obtained for jets. Many of these observations apply to other types of shear flows as well.

The main questions to be addressed are:

- What physical interpretation (if any) can be given for the various classes of eigenmodes that are encountered?
- Is global instability in low-density jets connected to an isolated unstable mode, or is it a continuous branch of modes that becomes unstable at strong density contrasts?
- Is the response to forcing in subcritical jets dominated by the resonance of the least stable mode, or do non-normal effects play a decisive role?
- Does modal analysis accurately predict the transition from amplifier- to oscillator-behaviour?

2. Methodology

The linear perturbation equations are obtained from a low Mach number formulation\textsuperscript{3} of the Navier–Stokes equations. They are discretized in cylindrical coordinates \((r, x)\) using finite elements\textsuperscript{4}. Eigenvalues and singular values are then computed with ARPACK routines. Laminar baseflows are obtained as exact steady solutions of the Navier–Stokes equations by Newton iteration. The inflow pipe and nozzle are explicitly included in the numerical domain, both for baseflow and for perturbation computations.

3. Flow configurations

Three physical parameters strongly affect the dynamics of light jets: the Reynolds number \(Re\), the density ratio \(S = \rho_{\text{jet}}/\rho_{\infty}\) and the shear layer momentum thickness at the nozzle exit \(\theta\). We vary \(Re\) and \(S\) in our investigation of spectra and pseudospectra, for a constant value of \(\theta/D = 1/40\), where \(D\) is the jet diameter. \(\theta\) is varied for comparison with experimental results at fixed values of \(S\).

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The Reynolds number controls the streamwise development of the baseflow. At $Re = 400$, the shear layer spreads rather quickly, and the potential core is roughly 5 diameters long. This length is typical for turbulent mean flows at high Reynolds number. A laminar baseflow at $Re = 1000$ spreads significantly slower. We focus on values $400 \leq Re \leq 1000$, and we find qualitative changes in the eigenmode spectrum over this interval. Increasing the Reynolds number far beyond 1000 is not expected to yield physically meaningful results, as the baseflow becomes nearly parallel, and markedly different from turbulent mean flows. The density ratio controls the destabilization of the jet by baroclinic effects. Very light jets (small value of $S$) display absolute instability, and as a result are globally unstable.

4. Results

4.1. Eigenmodes

Few global mode spectra for jets have so far been published. Nichols & Lele obtained results mainly for the supersonic regime, Qadri shows one spectrum for a $Re = 470$, $S = 0.14$ jet, using a low Mach number formulation very similar to the one employed here. Garnaud et al. conducted global mode analysis of subsonic jets both in incompressible and in fully compressible settings. The present results are consistent with all these earlier studies, but we aim to go further in the analysis of numerous unexplained observations.

By varying the Reynolds number between 400 and 1000, we obtain eigenmode spectra that show a gradual transition from a dominant isolated mode to a continuous branch behaviour. The black symbols in figure 1 are eigenvalues of axisymmetric perturbations in a flow with $Re = 400$ and a slightly stable density ratio $S = 0.3$. The isolated marginally unstable mode with $St = 0.35$ is well converged with respect to numerical parameters, the corresponding eigenfunction well contained inside the computational box. The parabola-like continuous branch that is found at lower growth rates arises from the shear instability of the jet. Consistent with our previous studies, it is found to converge poorly with respect to domain size, owing to the truncation of the eigenfunctions (not shown here). Many details about this branch remain to be investigated; among them are the roles of jet-column and shear-layer scalings in the eigenfunctions, the implications of the strong non-normality between individual modes on this branch, and the very extended downstream support of the associated eigenfunction wavepackets.

At a Reynolds number of 1000 and slightly stable density ratio $S = 0.5$, no isolated mode is identified in the eigenmode spectrum (see figure 2). Instead, it is the continuous branch that rises up near the unstable half plane, and that at lower values of $S$ is observed to cross over. The “zip-fastener” behaviour of the branch (visible in figure 2 at high Strouhal numbers) has been observed before, but has not yet been well understood.

4.2. Pseudospectra

The pseudospectrum characterizes the flow response to forcing input at a given (complex) frequency. Its values (formally the resolvent norm) are represented by logarithmic colour contours in figures 1 and 2. Lower values (blue) denote a more energetic system response. It is observed that in both cases the forcing response at real frequencies is dominated by a resonance of the stable eigenmodes. There is no clear trace of important non-normal effects at real frequencies. Interestingly, the continuous mode branch aligns neatly with a contour line of the pseudospectrum. We surmise that it is the primary “function” of the continuous branch to produce an accurate pseudospectrum in the upper part of the complex frequency plane. This will be the starting point for further analysis. We clearly observe that the pseudospectrum values at real frequencies converge rather easily with respect to domain size and numerical resolution, contrary to the eigenmodes.

4.3. Comparison with experiments

Figure 3 compares the onset of linear global instability to experimentally measured critical values for the onset of self-excited synchronized oscillations in pure Helium jets. The agreement is very encouraging. We will carry out a similar comparison for a density ratio of $S = 0.5$. The present results indicate that our linear instability analysis captures well the physical origin of oscillator behaviour in light jets.

References

Fig. 1. Spectrum and pseudospectrum of a jet at $Re = 400$ and density ratio $S = 0.3$. Growth rate $\omega_i$ is plotted versus Strouhal number. Black dots mark eigenvalues; colour contours denote the $\log_{10}$ of the resolvent norm.

Fig. 2. Spectrum and pseudospectrum of a jet at $Re = 1000$ and density ratio $S = 0.5$. Growth rate $\omega_i$ is plotted versus Strouhal number. Black dots mark eigenvalues; colour contours denote the $\log_{10}$ of the resolvent norm.

Fig. 3. Neutral curve of global instability of a pure Helium jet ($S = 0.14$). Reference experiments$^2$ and present computations.