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An analytical model for transition in Plane Poiseuille Flow and its experimental validation

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Abstract

Transition to turbulence in Plane Poiseuille Flow initiated by Transient Growth (TG) associated with four streamwise elongated vortices is studied. Although its growth is less than that associated with two vortices, this combination is chosen because of its ability to generate inflection points in the velocity profile. The final paper will include comparison between the analytical predictions, results obtained by Direct Numerical Simulation (DNS) and experimental measurements.

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Keywords: Transition; Transient Growth; Linear Stability; Plane Poiseuille Flow

1. Extended Abstract

Linear Stability Theory (LST) based on normal modes fails to predict instability in wall-bounded flows, in particular for Plane Poiseuille Flow (PPF) which has the critical $Re$ (based on centerline velocity and half channel height, which are also used to normalize all length scales in the present paper) value of 5772, whereas experimental investigations have shown that the transitional Reynolds number, i.e. the lowest Reynolds number for which turbulence can be sustained, is $Re \sim 1000$.

An alternative instability mechanism was first proposed by, who showed that for an inviscid fluid, the streamwise velocity component of a 3d disturbance with no streamwise variation grows linearly with time (unlike the exponential growth of normal modes). The inclusion of viscosity leads to the TG scenario in which the disturbance increases transiently and may reach a significant amplitude that can trigger nonlinear mechanisms before its eventual long-time exponential decay owing to viscous effects.

It has been found by for PPF that the initial structure for which the kinetic energy amplifies the most (the so called optimal disturbance) is independent of the streamwise coordinate. Furthermore, two types of disturbances were found: symmetric and anti-symmetric ones; the latter achieving higher energy growth. The anti-symmetric disturbance consists of a counter-rotating vortex pair (CVP) whereas the symmetric one consists of two CVP’s, one in the top half of the channel and the other in the bottom half (i.e. four vortices, see also).

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Transition scenarios involving the combination between PPF, CVP’s and a secondary disturbance, in the form of noise, have been investigated by $^9$. The CVP’s modify the baseflow by forming streaks and generating inflection points. The spanwise inflection points play a key role in the destabilization of the flow during the self-sustaining process proposed by $^10$. Stability analysis of the perturbed base-flow (PPF with the addition of a CVP) has been performed for the anti-symmetric disturbance $^{11}$. They have found that sinuous type instabilities, associated with spanwise inflection points, are more dominant, except at small vortex amplitudes.

The purpose of the present work is to investigate the transition scenario initiated by TG of four streamwise elongated vortices. This scenario has been much less studied in comparison with the TG initiated by a pair of vortices due to their greater TG potential. However, as demonstrated by $^{12,13}$ the maximal energy growth is not the essential parameter, rather it is the ability to generate inflection point in the base velocity profile. The final paper will include DNS, secondary stability analysis based on minimal modes (similar to the analysis presented by $^{12,13}$) and comparison with experimental results. In the following we present preliminary theoretical and numerical results.

Fig. 1 presents various TG scenarios for $Re = 5000$ and $\beta = 3$. The red curve corresponds to the optimal disturbance obtained numerically for many modes. Its initial structure consists of a CVP. The black curve corresponds to the symmetric disturbance, consisting of four vortices obtained numerically for many modes. The blue curve corresponds also to the symmetric disturbance, obtained analytically using only five modes. Although its maximal growth is almost half of the optimal disturbance, we shall use it as TG disturbance.

Fig. 2 presents the energy growth of an initial disturbance obtained by the DNS ‘Channelflow’ $^{14}$. The broken blue curve corresponds to the initial disturbance associated with the blue curve in Fig. 1. The initial disturbance amplitude is $A = 0.25\%$ relative to the centerline velocity. When small amplitude noise (0.4% $A$) is added, rapid transition occurs following a relatively long TG stage (the red curve).

The temporal development of the vortical structures during the transition scenario shown in Fig. 2 is presented in Fig. 3. To identify the vortical structure, $Q$, the second invariant of the velocity gradient tensor, is used. Initially, the disturbance consists of four streamwise vortices (Fig. 3A). By $t = 204$ the structures experience a sinuous instability (Fig. 3B), followed by the breakdown of the left and right pairs into segments of two streamwise vortices on each side ($t \sim 220$, Fig. 3C). The segments on the left and right are alternating in the streamwise direction. Next ($t \sim 260$), each of the top and its corresponding bottom segments on the right hand side of the channel (as well as on the left hand side) are connected by a vortex sheet (Fig. 3D). Shortly before transition, the structures become more localized, forming complex structures resembling vertical $\Lambda$ vortices $t \sim 300$ (Fig. 3E).

References

Fig. 2. Energy growth obtained by DNS based on 5 even modes and a secondary disturbance in form of white noise for $Re = 5000$ and $\beta = 3$. The unperturbed TG is given for reference (dashed line).

Fig. 3. Vortex dynamics during transition as obtained from DNS. The structures are shown by iso-surfaces of the $Q$ definition. (A) $t = 0$, $Q/Q_{\text{max}} = 0.7$; (B) $t = 204$, $Q/Q_{\text{max}} = 0.3$; (C) $t = 228$, $Q/Q_{\text{max}} = 0.3$; (D) $t = 260$, $Q/Q_{\text{max}} = 0.3$; (E) $t = 304$, $Q/Q_{\text{max}} = 0.12$.