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FAR AXISYMMETRIC WAKES IN TURBULENT STREAMS

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ABSTRACT

A time developing far axisymmetric wake at moderate Reynolds numbers ($Re = O(10^4)$) has been simulated without a generating body. When the wake achieved its self-similar state it was artificially combined with three different isotropic turbulent backgrounds with different intensities ($u'_{bg} = \sqrt{\overline{u^2}}$). It was found that the presence of the background enhances the wake's decay rate to one which depends on the initial ratio of the free-stream turbulence intensity to the wake centre-line deficit velocity, u'_{bg}/U_d. Moreover, the time required for enhancing the decay rate of the wake was also found to decrease with an increase in u'_{bg}/U_d. In no case (with $u'_{bg} > 0$) did the wake achieve a self-similar state. Finally, rms turbulence levels inside the wake were clearly influenced and eventually dominated by the free stream levels. A tendency to isotropy in the rms normal stresses (i.e. $u' \approx v' \approx w'$) was noticed within the wakes, whilst the turbulence shear stress seemed to be largely maintained.

INTRODUCTION

Research is in progress to investigate the influence of free-stream turbulence on the development of far axisymmetric wakes. Since the wake of any three-dimensional object becomes axisymmetric sufficiently far downstream and is often embedded in surrounding turbulence, such fundamental research will help to understand better many general problems.

Although far axisymmetric wakes in uniform streams have been thoroughly studied for more than half a century (e.g. Hwang & Baldwin, 1966, Chevray, 1968, Uberoi & Freymuth, 1970 and Gourlay *et al.*, 2001), the effect of free-stream turbulence on their development, on the other hand, has not. Raithby & Echert (1968) were the first to investigate the influence of turbulence on the axisymmetric wake of a heated sphere, using different mounting configurations, behind different grids. They reported that the average Nusselt number increases with the turbulence intensity and with the ratio of the sphere diameter to the scale of turbulence for values up to at least five. Later, Mujumdar & Douglas (1970) reviewed Raithby's & Echert's (1968) results in terms of the effect of the free-stream turbulence on the vortex shedding from the sphere. They noticed that when the free-stream was made turbulent by introducing turbulence generating grids typical velocity auto-correlations behind the sphere decayed without oscillation, thus indicating suppression of vortex shedding. In addition, they conducted similar experiments for circular and square cylinders which led to a different result – the vortex shedding is not suppressed and the Strouhal number stays the same even for highly turbulent free-streams.

Wu & Faeth (1994), more than two decades later, conducted measurements of a sphere's far wake in turbulent environments. Unlike Raithby & Echert (1968), who used grids to generate the turbulent surroundings, they used a turbulent pipe flow with the sphere mounted at the downstream end of the pipe. In comparison to non-turbulent environments, they noticed a faster self-similar decay rate of the wake and, while the wake was turbulent, the mean streamwise velocities scaled like a self-preserving laminar wake but with enhanced viscosity due to the turbulence. (Note that under uniform flow conditions the centerline velocity deficit in far laminar axisymmetric wakes is known to decrease as $(x - x_0)^{-1}$, Batchelor, 1967)

Mittal (2000) used Direct Numerical Simulation (DNS) to investigate the response of a sphere's wake to free-stream fluctuations. Since his research was directed towards particulate flows only low Reynolds numbers were simulated. He conducted several simulations with different amplitudes of the free-stream fluctuations and different sphere Reynolds numbers ($100 < Re < 350$). He found that in the presence of free-stream fluctuations the wake behaves like an oscillator and, at resonance, returns large amounts of kinetic energy to the surrounding fluid. With the same idea in mind, Bagchi & Balachander (2004) also used DNS to investigate the wake of a particle. Unlike Mittal (2000), who used a sinusoidally oscillating uni-

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form flow for the inlet condition, they used a frozen isotropic turbulent field superimposed on a uniform flow. In their simulations the particle Reynolds number varied between 50 to 600 and the particle diameter varied between 1.5 to 10 times the turbulent free-stream's Kolmogorov length scale. They found that the presence of the free-stream turbulence reduces the mean velocity deficit and that the wake profiles become flatter. In addition, they reported that with a turbulent free-stream the mean velocity profile in the particle's wake behaves like a self preserving laminar wake as was previously reported by Wu & Feath (1994). Finally, they noticed that by increasing the free-stream turbulence intensity the process of vortex shedding is largely suppressed, as was noticed by Mujumdar & Douglas (1970).

Tyagi *et al.* (2005) measured the effect of free-stream turbulence generated by a perforated plate on the wake of a sphere at different Reynolds numbers. They noticed that in the presence of the perforated plate the spectral peak could not be observed indicating again that the vortex shedding is attenuated by the free-stream turbulence. Later Tyagi *et al.* (2006) reported that the presence of the free-stream turbulence reduces the integral length scale in the near wake as well as the intensity of the velocity fluctuations, which they associated with the destruction of the horseshoe vortices in the wake by the free-stream turbulence.

Legendre *et al.* (2006) used Large-Eddy Simulation (LES) to simulate the wake of a spherical bubble and of a solid sphere in a turbulent pipe flow. They reported that the centreline velocity deficit decreased as $(x - x_0)^{-2}$ which is much faster than Wu & Faeth (1994) had reported for a different configuration and thus suggests a dependency on the turbulent surroundings. Later, Redford & Coleman (2007) reported on a general time-developing far axisymmetric wake in turbulent surroundings using DNS. In their simulation the generating body was not modelled and an initial wake was created using a series of ring vortices, which with time broke and developed into an axisymmetric wake and then combined with the free-stream turbulence. (It should be mentioned that their wake's self-similar behaviour in a uniform free-stream was found to be different to the theoretical prediction or any available experimental data on far axisymmetric wakes and that no explanation for that was given by them). They found, as expected, that the stronger the background turbulence, the shorter was the wake's decaying and merging time with the surrounding turbulence.

In the current study, a set of numerical experiments which involve the study of a general, time developing, far axisymmetric wake in uniform and several different turbulent surroundings is being analysed. The numerical experiments involve Direct Numerical Simulation (DNS) of far axisymmetric wakes, without a generating body, developing in time (using a similar approach to that of Gourlay *et al.*, 2001), as described below.

METHODOLOGY

The computational study in this research is being carried out using the University of Southampton's in-house triply-periodic pseudo-spectral Direct Numerical Simulation (DNS) code described by Redford & Coleman (2007). In the cases presented here, the computational domain's dimensions are $4\pi \times 16\pi \times 4\pi$ and the number of Fourier modes is $512 \times 2048 \times 512$, with each time step taking approximately 50 seconds using 512 processors on the UK's HPCx system. In all our simulations the grid size varied between $20\% - 110\%$ of the local Kolmogorov length scale and the wake's width was less than half the domain width. As an example of the satisfactory nature of both resolution and statistical convergence, Figure 1 shows an example of an energy spectrum in the wake (at $Re = 2U_d l_h/\nu \approx 4000$, where l_h is the wake's radial location (r) at which the velocity deficit is half its maximum value). The spectrum is compared with the universal result expected in the inertial subrange (eq.(1), e.g. Pope, 2000); its properties (i.e. E_{11} and ϵ) were directly calculated from the simulation data, and the agreement is very satisfactory. The classical result can be expressed as

$$E_{11} = \frac{27}{55}\epsilon^{2/3}\kappa^{-5/3}, \tag{1}$$

where, E_{11} is the one-dimensional spectrum of the axial velocity, ϵ is the local dissipation rate and κ is wave number.

The self-similar wake was generated using suitably massaged experimental data reported by Chevray (1968), as an initial condition. The reported mean velocity profiles at one disc diameter downstream, $\frac{x}{d} = 1$, were scaled such that the wake's initial Reynolds number (Re, based on the wake's half-width and the centre-line deficit velocity) was 10000. Those scaled profiles were then inserted into the computational flow field with turbulent statistics added using a modified version of Xie & Castro's (2008) digital filter. Since the initial flow field did not satisfy conservation, a so called correction stage (see Gourlay et al., 2001) takes place until the wake is fully developed and has achieved its self-similar state. Alternative methods, which might have reduced the computational time required for this correction stage, were not tried; recall that the Redford & Coleman (2007) technique did not yield the usual self-similar state.

Later, several isotropic background turbulent

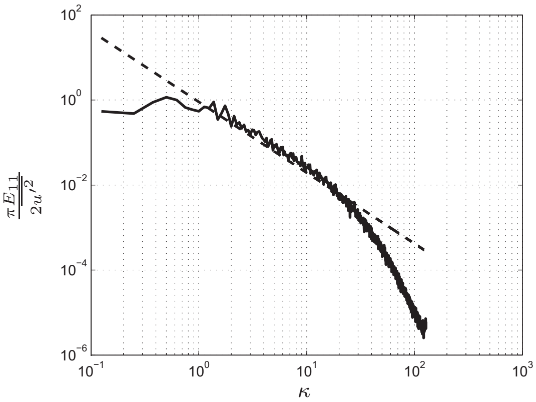

Figure 1: Example of comparison between a DNS one-dimensional energy spectrum and its universal form, at the wake's half width point; $Re \approx 4000$. Solid line: the DNS spectrum; dashed line: the universal spectrum, eq.(1).

fields were generated. One might anticipate that major parameters affecting the wake include u'_{bg}/U_d and L_x/l_h, where u'_{bg} and U_d are the rms velocities in the background turbulence and the wake's centre-line deficit velocity, respectively, and L_x and l_h are the integral scale in the background turbulence and the r-location of the wake's half width, respectively. In order to achieve a specific strength, u'_{bg}/U_d, of the background field, first a weaker one was generated and, using the forcing scheme of Sullivan *et al.* (1994), was prevented from decaying. Then the forcing constant (the turbulent kinetic energy) was increased until the required u'_{bg}/U_d was achieved. Notice that the integral length scale is only an output of the simulation and cannot be controlled *á priori*, unlike the rms velocity - using our technique only the ratio u'_{bg}/U_d can be imposed.

Finally, the self-similar wake and the turbulent background were then artificially combined, with two constraints: the background turbulence was inserted at all locations outside the mean wake (defined by the region in which the velocity is below 1% of the centre-line value) and at all places inside the mean wake where the turbulence kinetic energy (TKE) was smaller than 5% of the maximum TKE. When that initialising process was finished the whole field (wake plus background turbulence) was allowed to decay with time and its features analysed, with results illustrated in the following section.

RESULTS AND DISCUSSION

Wake in Uniform Background

In order to evaluate whether the current approach

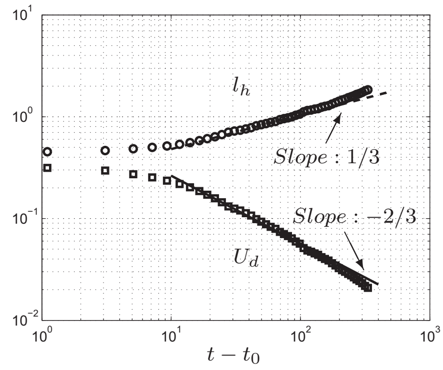

Figure 2: Wake development with time.

yielded the expected behaviour, we first examined the self-similar far-wake parameters in the absence of the background turbulence. After an initial adjustment time of about 20 non-dimensional time units, the deficit velocity and the half-width varied according to power laws very close to the expected ones: $U_d \propto (t - t_0)^{-2/3}$ and $l_h \propto (t - t_0)^{1/3}$ (Tennekes & Lumley, 1972), as can be seen in Figure 2, with t_0 determined by extrapolating a linear fit to the data plotted as $l_h^{-1/3}$ vs. t.

Reynolds stress profiles through the wake are shown in figure 3; these are qualitatively similar to existing data for spatially developing wakes (e.g. Uberoi & Freymuth, 1970), but quantitative agreement is neither found nor expected, as it is known that any particular self-similar state depends on the initial conditions (Johansson *et al.*, 2003). (Note that unprimed lower case u, v & w refer to the fluctuating components of velocity.) Finally, the wake's turbulent kinetic energy budget was calculated and is shown in Figure 4. Again, it is found to be in good qualitative agreement with previously reported data.

We conclude that a self-similar axisymmetric wake can be simulated using DNS without having to

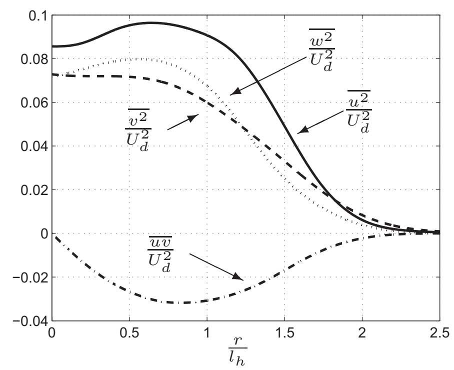

Figure 3: Averaged normalised Reynolds stress profiles.

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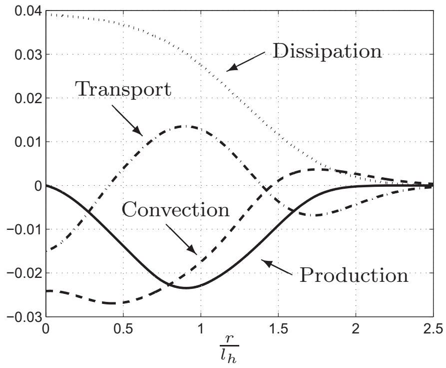

Figure 4: The non-dimensional turbulent kinetic energy budget of the self similar wake in uniform surroundings (the transport term was calculated from the balance).

u'_{bg}/U_d	L_x/l_h
0.1	0.75
0.18	0.5
0.37	0.6

Table 1: Defining parameters of the turbulent background at t_1.

include a generating body within the computational domain, which would not be feasible for such high Reynolds numbers.

The Turbulent Backgrounds

Table 1 presents the defining parameters of the different backgrounds at t_1, the moment when the wake and the background were combined. Notice that the variation of length scale ratio over the three cases is much lower than the (imposed) range of intensities. It would perhaps be possible to increase this range, while maintaining the same range of intensities, but this was not attempted; the current set of cases has the advantage of emphasising the effect of variations in u'_{bg}/U_d for an approximately constant imposed background length scale ratio of O(1), which would be expected to be the most critical in terms of the free-stream's influence on the wake.

Wake in Turbulent Background

The self-similar wake was artificially combined with all three turbulent backgrounds, as described earlier, and then allowed to decay with time. The first, not surprising, feature to be noted, which was also reported previously by other researchers (see Introduction), is that the presence of the turbulent background enhances the decay rate of the wake. Unlike Wu & Faeth (1994) and Legendre *et al.* (2006)

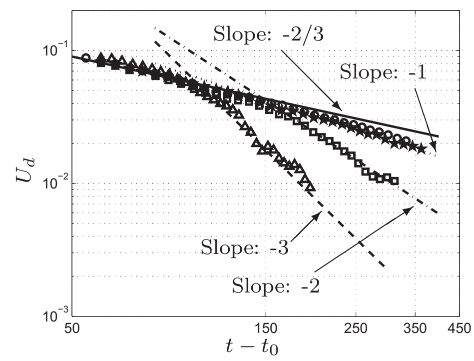

Figure 5: Wake's deficit velocity vs. time for the different wake-background combinations. Initial u'_{bg}/U_d: circle, 0; star, 0.1; square, 0.18; triangle, 0.37.

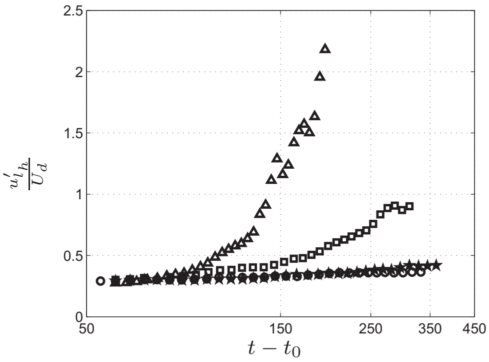

Figure 6: Comparison of u'_{l_h}/U_d vs. time for the different wake-background combinations. Initial u'_{bg}/U_d: circle, 0; star, 0.1; square, 0.18; triangle, 0.37.

who each noticed only one decay rate ($U_d \approx x^{-1}$ and $U_d \approx x^{-2}$, respectively) , we find that the wake decays quicker as the ratio u'_{bg}/U_d increases, as can be seen in Figure 5. Moreover, it is clear from our simulations that the effect of the turbulent background does not take place instantaneously but is directly related to its strength – the stronger the background the quicker it increases the decay rate of the wake, as also can be seen in Figure 5. By examining truncation errors and energy spectra it was found that the stronger the background turbulence, the longer the 'correction' stage was, but in all three cases it was shorter than 20 time units. (The correction stage was typified by energy spectra containing unphysical peaks, for example.) So the correction stage presumably has little influence on the time taken to affect the decay rate (which, as noted, decreases with increasing turbulence level).

Although the wake might appear to achieve a new self-similar state, since both U_d (Figure 5) and l_h (not shown) obey new power laws, which depend on

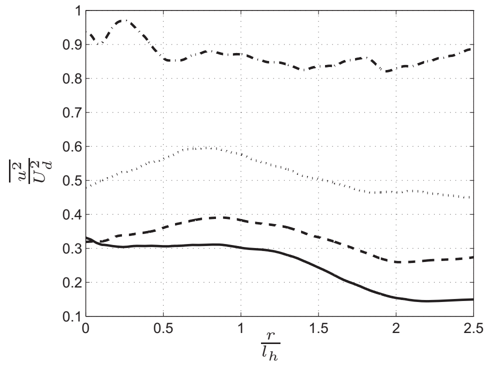

Figure 7: Profiles of $\frac{\overline{u2}}{U_d^2}$ at different times for the $u'/U_d = 0.18$ case. solid line - $t - t_0 = t_1$; dashed line - $t - t_1 = 114$; dotted line - $t - t_1 = 196$; dash-dot line - $t - t_0 = 298$.

the ratio u'_{bg}/U_d, neither velocity nor turbulence profiles can be collapsed in self-similar form, independent of time, contrary to what is found at much lower Reynolds numbers according to previously reported laminar flow data (see the introduction). For example, the time-variation of the ratio u'_{l_h}/U_d, where u'_{l_h} is the rms velocity at the wake's half-width location, is shown in Figure 6. Notice again that divergence from the background condition (for which the ratio is approximately constant) is, again, quicker and stronger as u'_{bg}/U_d increases.

The non-dimensional profiles of the three rms velocity components were also examined. They were noticed to be constantly changing with time, each from its original self-similar profile into one which was more closely uniform and clearly determined by the background's turbulence intensity - the stronger the background the stronger its effect on the wake's rms velocity profile; an example is given in Figure 7. Figure 8 shows why self-similarity is not achieved: the natural decay in the free-stream (i.e. background) turbulence intensity is significantly less rapid than the decay in the wake centre-line deficit velocity, so u'_{bg}/U_d (the major controlling parameter) continually rises with time. This is emphasised in Figure 9, which shows that the ratio of half-width intensity, u'_{l_h}, to free-stream intensity, u'_{bg}, continually falls (or, in the case of $u'_{bg}/U_d = 0.37$ initially, rises) towards what must be the eventual state in which $u'_{l_h} = u'_{bg}$. Since the wake's rms velocity profiles must eventually be determined by the background, which is decaying relatively slowly, no self-similar state can be achieved.

As mentioned earlier, the non-dimensional rms velocity profiles develop with time to ones which are eventually determined by their values in the (isotropic) background, so one might predict that at least for the two stronger backgrounds the turbulence

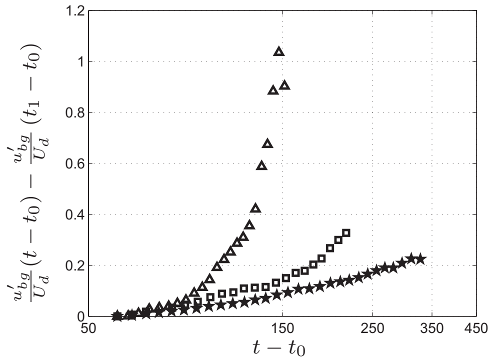

Figure 8: The change in u'_{bg}/U_d vs. time for the different wake-background combinations. Initial u'_{bg}/U_d: star, 0.1; square, 0.18; triangle, 0.37.

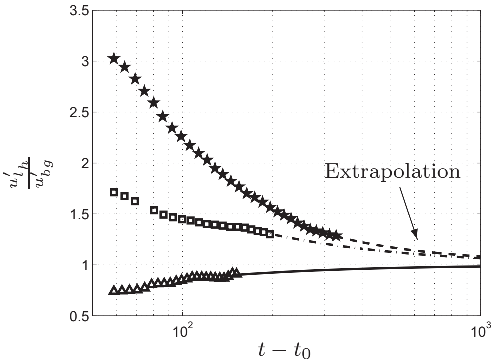

Figure 9: Variation with time of the ratio of axial intensity at $r = l_h$ to free-stream intensity. Initial u'_{bg}/U_d: star, 0.1; square, 0.18; triangle, 0.37.

normal stress profiles inside the wake will eventually become both isotropic and uniform. They do indeed become more isotropic and, as illustrated (for $\overline{u^2}$) in Figure 7, uniform, Careful examination of the turbulent shear stress profiles inside the wake reveals that $\overline{uv}$ does not, however, decay to zero as rapidly as the rms velocity profiles change towards uniformity; an example for the mean shear stress profile's behaviour with time is given in Figure 10. That behaviour is presumably associated with the continuing presence of shear in the mean flow and it shows that the wake turbulence does not become strictly isotropic.

CONCLUSIONS

A time-developing far axisymmetric wake was artificially combined with three different isotropic turbulent backgrounds. The presence of freestream turbulence has been shown to enhance the wake's decay rate, which increases with the initial u'_{bg}/U_d. Moreover, our simulations clearly show that no new

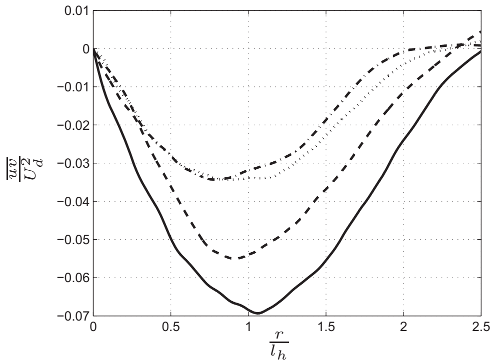

Figure 10: Non-dimensional mean turbulent shear stress profiles at different times at the wake's half width for the $u'_{bg}/U_d = 0.18$ case, dash-dot line - $t - t_0 = t_1$, dotted line - $t - t_1 = 80$, dashed line - $t - t_1 = 114$ and solid line - $t - t_0 = 153$.

self-similar state is achieved even after some time, because the free-stream turbulence intensity and the wake's centre-line velocity deficit decay at different rates. Finally, it was found that whilst the turbulent shear stress profile remains non-uniform, driven by the mean shear, the background turbulence eventually causes normal stress wake profiles to become uniform, with stress values equal to those in the free stream.

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