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CRITERIA FOR BOUNDARY LAYER TRANSITION

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ABSTRACT

New results are deduced to assess the validity of proposed transition indicators when applied to situations other than boundary layers on smooth surfaces. The geometry employed utilizes a two-dimensional square rib to disrupt the boundary layer flow. The *objective* is to determine whether some available criteria are consistent with the present measurements of laminar recovery and transition for the flow downstream of this rib. For the present data -- the proposed values of thresholds for transition in existing literature that are based on the freestream turbulence level at the leading edge are not reached in the recovering laminar run but they are not exceeded in the transitioning run either. Of the *pointwise* proposals examined, values of the suggested quantity were consistent for three of the criteria; that is, they were less than the threshold in laminar recovery and greater than it in the transitioning case.

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INTRODUCTION

In order to predict heat, mass and momentum transfer and entropy generation adequately for flows undergoing transition from laminar to turbulent states, it is necessary to predict in some sense where transition phenomena begin to dominate laminar effects and then where transition is essentially complete. A *question* is whether one can identify a useful transition indicator for situations other than boundary layers on smooth surfaces. Accordingly, an *objective* of the present study is to determine whether available criteria are consistent with the occurrence of transition (or laminar recovery) in a different situation, the flow downstream of a rib roughness. In general, this test asks whether -- for a given criterion -- when its measured value is below its suggested threshold the flow remains laminar and when the threshold is exceeded the flow will start transition towards a turbulent state. This unique assessment is not sufficient to demonstrate that a proposed criterion is valid for all cases but it can prove that one is not universal. Alternatively, one could say an aim is to find which criteria can be removed from consideration as not consistent with the data here.

As noted by Sharma et al. [1], about 75-80 per cent of total profile loss can be attributed to the suction surface boundary layers for typical turbine airfoils. Since the skin friction coefficient C_f can differ by a factor of over five to more than an order-of-magnitude between laminar and turbulent boundary layers (Figure 5 by Mayle [2]), it is important to be able to predict the occurrence of transition reasonably. Low-pressure turbines operating at altitude can have chord Reynolds numbers of the order of 50,000 or less so large regions of the blades can be expected to be non-turbulent [1,2]. While assuming the flow to be completely turbulent may be considered to be conservative, such an approach would lead to poor predictions of losses and efficiency and would prohibit optimization in these cases.

For boundary layers with freestream turbulence and negligible streamwise pressure gradients, many investigators have examined stability and transition by analyses, computations and experiments [3]. A reasonably clear picture is evolving for some aspects [4-6]. Criteria and correlations for the start of transition or its precursors have been proposed. Typical recommended indicators for onset of transition are both pointwise and integral. Details of some approaches are explained later in the relevant sections.

In the present study we treat a different flow situation with some comparable phenomena but effectively different initial conditions, in order to obtain evidence of which proposed transition criteria *may* be common and, possibly, universal – or are not consistent and should be rejected for this purpose. (Whether a universal criterion exists is not known.) The geometry employed utilizes a two-dimensional square rib to disrupt the boundary layer flow (see Figure 2 of Becker et al. [7]) -- somewhat like the perpendicular transport of streamwise momentum induced by film cooling in gas turbine blades/vanes.

Placing a two-dimensional rib in a developing laminar boundary layer can be expected to induce a number of flow phenomena. Ahead of the rib a recirculating spanwise vortex will form below a stagnation line on the front face, somewhat as visualized by Werle (Figure 69 of [8]) and by Wilhelm, Härtel and Kleiser (Figure 1 of [9]). The curvature of the flow above this vortex may lead to streamwise vortices [10]; such vortices are likely to persist downstream. A separation bubble forms above the rib with reattachment there or on the main surface downstream of the rib, depending on the rib aspect ratio [10]. Downstream another recirculating region is formed below the main flow before it reattaches as in the flow behind a backward-facing step [11,12]. If streamwise vortices are not already part of the flow, they may be formed in the shear layer here. Consequently, the effective initial conditions for the flow beyond reattachment resemble the laminar or turbulent jet from an two-dimensional slot, impinging on the flat plate at a slight angle, or from a two-dimensional separation bubble.

The experiments of Liepmann [13] on flow behind a two-dimensional rectangular roughness element "showed that there exist three primary and readily distinguishable cases.

> 1) The inflection-type profiles prevailing in the wake of the obstacle and immediately downstream from it are gradually reduced to the normal Blasius-type profile
>
> 2) The inflection-type profile still changes to a Blasius profile but the disturbances -- that is, the induced laminar waves -- are so strong when the Blasius profiles are attained that transition occurs almost immediately."
>
> 3) Effects of an external disturbance such as sound waves.

Klebanoff and Tidstrom[14] later reported comparable data for flow behind a circular wire but their main emphasis was on the "recovery zone" (case 1). Figures 4a and 4b of Becker et al. [7] demonstrate these observations for Liepmann's cases (1) and (2), respectively. An inflectional mean velocity profile is known to give a "frictionless instability" and is a sufficient condition for amplification of disturbances (p. 445 of [15]). A rib inherently induces an inflectional profile immediately downstream but, as shown by their Figure 4a and suggested by Liepmann [13], it does not necessarily lead to transition immediately in this case.

EXPERIMENT AND PROCEDURES

With transition induced by a square rib, Becker et al. [7] measured the evolution of the Reynolds stresses, $(v')^2$ and $\overline{uv}$, in addition to the usual mean streamwise velocity component and its fluctuation, at $y^+ > \sim 0.1$ in transitional boundary layers on a flat plate. To measure the wall-normal component close to the surface, two-component laser Doppler anemometry (LDA) was used with the INL (Idaho National Laboratory) Matched-Index-of-Refraction (MIR) flow system. Due to its large size, this system allows significantly improved spatial resolution compared to most previous MIR experiments. Further details of the system and the estimated uncertainties of the experiment are provided by Becker et al. and by Stoots et al. [16]. The flat plate model was about 600 mm wide, 12 mm thick and 2.4 m long with a NACA 0009 shape for the leading edge and the forward face of the roughness element was located at x = 1150 mm. Element heights of 2, 4 and 6 mm were used. Experiments with a smooth flat plate (k = 0) were also conducted to obtain measurements for comparison to accepted boundary layer theory for laminar flows.

Velocity and turbulence measurements were primarily obtained with a TSI two-component laser Doppler velocimeter operated in the forward scattering mode with custom Uni. Erlangen receiving optics. The LDA data acquisition was normally operated in the "coincidence mode" requiring near simultaneous detection of samples in both coordinate directions, i.e., from the same scattering particle. Results are calculated via Reynolds averaging of the entire time series as for fully-turbulent flows. Thus, the streamwise velocities are represented as time-mean values and their fluctuations about these means, U + u (not a base flow and perturbation from it as in stability analyses) with the mean fluctuations being provided as root-mean-square fluctuations, u'. Comparable averaging is employed for the other statistics. It is recognized that such treatment may not describe the significant difference between the unsteady laminar fluctuations and unsteady turbulent spots occurring during the transition process and others have used conditional sampling to examine those regions independently [17-19] --- but it gives useful statistics which are consistent with many computational approaches. It is also consistent with definitions employed in the bypass transition computations of Brandt, Schlatter and Henningson [5].

For convenience, the differing experimental runs are identified by letters: capitals A, B and C for 6, 4 and 2 mm ribs, respectively, and lower case a, b and c for approximate freestream velocities of 1.25, 0.75 and 1.75 m/s. The smooth plate is labeled Sm. These identifiers are given with the data in Table 1.

The database provides a series of profiles downstream (and upstream) from the rib at each run combination of rib height and freestream velocity. Tabulated mean results are available for eight sets of experimental conditions. Data available include U, V, u', v', $\overline{uv}$ and their other statistical means plus distributions of integral boundary layer parameters. The measurements with a square rib gave the following ranges of nominal parameters:

$$k^+ \approx 5.5 \text{ to } 21, \quad 0.3 < k/\delta^* < 1, \quad 180 < Re_k < 740,$$
$$6 \times 10^4 < Re_{x,k} < 1.5 \times 10^5,$$
$$Re_{\theta,max} \approx 660, \quad -125 < (x-x_k)/k < 580.$$

(Here x_k is the location of the downstream face of the rib.) Results covered boundary layers which retained their laminar characteristics through those where a turbulent boundary layer was established shortly after reattachment beyond the forcing rib. Thus, this range of data is suitable to determine which criteria predict laminar recovery or transition to turbulence when they occur and which do not.

For small ribs (e.g., $k^+ \approx 11$ and less), disturbances evolved from the inflectional region above the separated region downstream of the rib but laminar mean velocity profiles were recovered as by Klebanoff and Tidstrom [14]. For larger ribs ($k^+ > 14$) significant fluctuation levels u' and v' –- either from low-frequency laminar disturbances or turbulence or both -- appeared in the reattaching shear layer as well and the viscous layer of a turbulent boundary layer began evolving soon after reattachment. These levels are higher than would be induced on a smooth plate by the freestream turbulence of about one per cent at the same Re_x.

Various investigators determine or define the start of transition in various ways. Liepmann [13] used a surface tube and later observations of a hot wire signal on an oscilloscope: in the latter case, the first appearance of large sudden velocity fluctuations close to the plate -- the so-called turbulent bursts -- was taken as the indication of transition. Suder, O'Brien and Reshotko [20] examined five definitions: (1) distortion of mean velocity profiles, (2) change of shape factor, (3) divergence of skin friction coefficient, (4) amplitude of rms velocity fluctuation u' and (5) intermittency γ_w observed with a wall sensor. Hernon, Walsh and McEligot [21] define "transition onset" as the observation of one "turbulent spot" per ten seconds by a wall sensor; they note that this value corresponds to an intermittency of about 0.001 or 0.1 per cent. Fransson, Matsubara and Alfredsson [22] define a "transitional Reynolds number" to be that where $\gamma_y = 0.5$ at $(y/\delta^*) = 1.4$ and choose the values of $\gamma_y = 0.1$ and 0.9 to define where transition starts and ends, respectively. Unfortunately, not all investigators specify their definitions of transition or its precursors.

Since the LDV time series from the present measurements were random in time, one cannot deduce an intermittency γ from them directly. Emmons [23], Nolan et al. [19] and others have suggested that in transitional flows one may predict the various flow properties from the relation

$$f = (1 - \gamma)\, f_{lam} + \gamma\, f_{turb}$$

where f is the property of interest. For the present study, we arbitrarily identified a "start" of transition and "end" of transition by comparison to the predicted local skin friction coefficients for developed laminar and turbulent boundary layers,

$$C_{f,lam} \approx (0.441/Re_\theta) \quad \text{and} \quad C_{f,turb} \approx 0.02193/Re_\theta^{0.2324}$$

The laminar correlation is from the Blasius solution and the turbulent one is based on fitting the direct numerical simulation (DNS) results of Spalart [24] for the low-Reynolds-number range $300 < Re_\theta < 670$. An "apparent" intermittency may then be calculated as $\gamma = (C_f - C_{f,lam}) / (C_{f,turb} - C_{f,lam})$ with C_f being the measured skin friction coefficient. The start of transition is then defined as the position where γ is first observed to be greater than zero and the end is taken as the location where it becomes unity. (This intermittency is not γ_y or γ_w measured with hot wire or hot film sensors.) Comparable results are obtained by using the measured shape factor H as the indicator. In practice, we identify the last station where γ is approximately zero within about five per cent and the first station where it is greater -- and likewise for the end; thus, $Re_{\theta,tr}$ and $Re_{\theta,turb}$ are bracketed rather than being determined exactly. Table 1 provides the deduced locations for reattachment and transition plus these transitional Reynolds numbers for the eight sets of experimental conditions. This non-zero value of γ does not necessarily equate to the value of transition onset commonly taken as the location of minimum C_f. (For example, the DNS of bypass transition by Brandt, Schlatter and Henningson [5] for Tu_{in} = 4.7 per cent and turbulence length scale of five give this $\gamma \approx 0.12$ at $C_{f,min}$.) However, for the present purposes a precise determination of a transition onset is not critical; the occurrence of transition – or not – is alone enough to disqualify the proposed criteria which are not consistent with the measurements.

EXPERIMENTS EXAMINED

Data presented by Becker et al. [7] include $U_\infty\{x\}$, $H\{x\}$, $U\{x.y\}$, $u'\{x.y\}$, $U^+\{x,y\}$, $(u'\{x,y\})^+$, $(v'\{x.y\})^+$ and $(\overline{uv}\,\{x,y\})^+$. (Tabulations of the data are available from the first author.) The downstream developments for Runs Ba and Bc were examined in detail by Becker et al. [7], in terms of physical coordinates (y/k) and wall coordinates (y^+) and Runs Aa, Ab and Ac were treated to a lesser extent. The smooth plate case apparently remained laminar with fluctuations to (and presumably beyond) the last station at $Re_x \approx 2 \times 10^5$, consistent with the level of freestream turbulence, and four others did after reattachment beyond the rib ($Re_k \approx$ 179, 251, 318 and 358). For $Re_k \approx 502$ and greater, the boundary layer became turbulent ($\gamma \approx 1$) before the end of the test section. Runs Ba and Bc appeared laminar ($\gamma \approx 0$) for one station after reattachment as suggested by Liepmann [13]. In the case of Run Ac ($Re_k \approx$ 741) no evidence of laminar conditions was seen after reattachment --- this situation may correspond to suppression of turbulent flow rather than being a question of receptivity [25]. Since Run Bc had the most gradual laminar-to-turbulent transition, it is the transitioning run which will be analyzed in the greatest detail. In all runs the Reynolds numbers Re_θ were in the range where Bejan (Table 6.2 of [26]) suggested that transition may be induced by inviscid buckling.

Preliminary measurements were obtained by Becker et al. [7] for a *flat plate without a rib* installed and with a negligible pressure gradient in order to qualify the facility and to serve as a reference condition (k = 0) for the effects of square roughness elements. The Reynolds numbers Re_x and Re_θ were about 2 x 10^5 and 290, respectively, at the last station and freestream turbulence (u'_∞ / U_∞) was about 0.7 to 0.8 percent along the plate. Based on the observations of Abu-Ghannam and Shaw [27] and Mayle [2] and typical transition criteria, these conditions are expected to produce a laminar boundary layer with growing disturbance levels but no transition before the end of the plate. Becker et al. showed that at all stations the streamwise mean velocity profile agreed well with the Blasius profile and, accordingly, the shape factor was close to 2.6, the expected laminar value. From $Re_x \approx 3.5 \times 10^4$ to 1.87×10^5, the rms fluctuation in the streamwise component, u', varied monotonically from near zero at the wall to its freestream value as the Blasius variable η increased. The last two profiles at $Re_x \approx 1.95 \times 10^5$ and 2.03×10^5 show slight maxima in the boundary layer of $u'/U_\infty \approx 0.01$ or $u'/u'_\infty \approx 1.3$ at $\eta \approx 1.3 - 2.2$ and a slight increase in shape factor (about 2.59 to 2.67). Examination of the profile of streamwise fluctuations at $Re_x \approx 1.95 \times 10^5$, the penultimate measuring station, shows the maximum fluctuation is slightly larger than would be predicted for a Stokes layer and this fluctuation grows while the freestream turbulence is near constant in the streamwise direction. The u'{y} profile observed, with its peak near (y/δ^*) $\approx$ 1.5, is representative of so-called Klebanoff "modes" [3,28] or Klebanoff distortions [6] of bypass transition. Based on these considerations we conclude that *without a rib* the present flow is typical of bypass transition forced by a low level of freestream turbulence.

Run Ba corresponds to Liepmann's case (1) = recovery to a laminar boundary layer, while Run Bc represents his Case (2). One may consider the various profiles *before reattachment and immediately after* as *effective initial conditions* for the ensuing boundary layers. Some of these profiles are presented as Figure 1. Freestream turbulence for these runs is slightly higher than for the smooth plate, being about $0.9 < (u'_\infty / U_\infty) < 1.3$ mostly; this increase may be a consequence of pressure waves from the unsteady reattachment region. At $x^* = ((x-x_k)/k) \approx 23.8$ both mean velocity profiles ($U\{y\}/U_\infty$) cross the recirculating region or "separation bubble" induced by the rib so there are reverse flows near the wall and inflection regions further away. (Scaling on the rib height is convenient for this configuration and, as seen in the figures, some variables have maxima in the shear layer from the top of the rib.) Both flows show some turbulent transport ($-\rho\overline{uv}$ identified via inverted triangles labeled "tau_{turb}" on the subfigures) but for Run Ba it is small; for Run Bc it might already be considered to be substantial in the wake of the rib. Unfortunately, the LDV and its processing did not provide the spanwise component, turbulence length scale or useful frequency spectra for further details of the initial disturbance field.

For *recovering Run* *__Ba__*, Re_x at $x^* \approx 23.8$ (before reattachment) is approximately the same as for the smooth plate at the same location since U_∞ is about the same. The momentum thickness Reynolds number Re_θ is likewise approximately the same numerically, despite presence of the recirculating region, but the shape factor at $H \approx 3.5$ is higher than for a Blasius profile. The fluctuation ($u'\{y\}/U_\infty$) has a slight peak of about 0.7 per cent at $(y/k) \approx 0.35$ near the dividing streamline where U = 0 but, in the boundary layer outside the internal shear layer from the rib, it increases monotonically to its freestream magnitude of about 1.2 per cent. Likewise, in this latter region v'{y} increases continuously to its freestream value. By the next set of profiles after reattachment ($x^* \approx 38.3$), the streamwise mean velocity profile is closer to that of a normal laminar boundary layer but still has a slight inflection region so H is a bit higher at about 2.9 rather than 2.6 and the skin friction coefficient C_f is about fifteen per cent lower than on the smooth plate at the same location. The maximum value of (u'/U_∞) is in the freestream but there are minor peaks near the wall and in the wake of the rib at $(y/\delta) \approx 0.12$ and 0.38, respectively. Except in the immediate vicinity of the wall, v'{y} increases monotonically to the freestream. So for the purpose of CFD (computational fluid dynamics) predictions, one could prescribe initial conditions as a pre-transitional laminar boundary layer with internal disturbances (u'/U_∞) to about 0.007 and 0.0095 at $(y/\delta) \approx 0.12$ and 0.38, respectively, and forcing freestream turbulence levels of one per cent or so. By $x^* \approx 61.3$, H is 2.63 and C_f is within two per cent of the Blasius prediction there. After this station (u'_{max}/U_∞) begins to grow until it is about 6.8 per cent at the end of the plate where $Re_\theta \approx 316$ but H is 2.60 and C_f is still slightly below the Blasius prediction.

For *transitioning Run* *__Bc__* again mean attachment is in the range $23.8 < x^* < 38.3$ but due to the different freestream velocity, Re_x is about 1.5×10^5 there. This value of Re_x occurs further downstream on the smooth plate and – for it at this downstream station -- $Re_\theta \approx 244$, $H \approx 2.5$, C_f is about the same as laminar theory and (u'_{max}/U_∞) ≈ 0.0075 occurs in the free stream. Accordingly, as shown in Figure 1c, the upstream fluctuation profiles are significantly different than for Run Ba near reattachment. In Run Bc at the station before reattachment, $Re_\theta \approx 228$ -- which is less than the Blasius prediction for this Re_x -- and $H \approx 4.3$, typical of flow immediately downstream of a rib. The u'{y} and $\overline{uv}$ {y} profiles demonstrate considerable modification of the laminar boundary layer by the rib but v'{y} increases monotonically with y to its freestream value. From $(y/k) \approx 0.2$ to the dividing streamline (u'/U_∞) has a broad maximum of about four per cent in the flow from the impinging shear layer back towards the rib;

outward from this point it decreases monotonically to the freestream.

After reattachment at $x^* \approx 38.3$ the boundary layer of Run Bc appears like a Blasius profile but a slight inflection region persists near $(y/k) \approx 0.4$; at this station $Re_\theta \approx 278$, $H \approx 2.7$ and C_f is lower than the Blasius prediction. The streamwise fluctuation (u'/U_∞) has peaks over seven per cent at $(y/k) \approx 0.3$ and 1.3 while at the latter point (v'/U_∞) shows a maximum of about three per cent. (At this Re_θ on the smooth plate (u'_{max}/U_∞) is about one per cent.) Thus, for this run the initial conditions for CFD calculations could be described as a laminar boundary layer with high disturbance levels within the boundary layer but forcing freestream turbulence levels still only about one per cent or so. By the next set of profiles at $x^* \approx 50.8$ our intermittency γ is 0.26 so transition is well underway. By $x^* \approx 138$ ($Re_x \approx 2.1 \times 10^5$) transitioning Run Bc has $H \approx 1.61$, $C_f > C_{f,turb}$ and $(u'_{max})^+ \approx 2.4$ at $y^+ \approx 15.3$, comparable to a developed turbulent boundary layer.

INTEGRAL CRITERIA

Many investigators have suggested correlations of integral criteria for the start of transition or its precursors. Several of these are presented as functions of the freestream turbulence at the leading edge; for a Blasius boundary layer these can be converted to other convenient parameters. For example, Andersson, Berggren and Henningson [29] suggest $(Re_{x,tr})^{1/2} Tu_{LE}(\%) > 1200$ which can be phrased as $Re_{\delta,tr} Tu_{LE}\ (\%) > 6000$ for a Blasius boundary layer. An effect of the rib is to introduce distortions in the form of increased fluctuation levels into the boundary layer. So, at our effective initial conditions, the level of fluctuations is higher within the boundary layer than a smooth plate at the same momentum Reynolds number.

With the present data in this section, when needed, the quantity (u'_∞/U_∞) is evaluated *locally* rather than at the leading edge. That is, Tu_{LE} is replaced by $Tu = (u'_\infty\{x\}/U_\infty)$. The recommended values used in the comparisons were taken or derived from the original literature cited; these values are specified in the paragraphs on individual criteria below.

For the smooth plate all proposed *integral transition criteria* examined grew gradually but *were well below their recommended threshold values* except the momentum Reynolds number Re_θ [26] which varied from about 130 to 290. The streamwise variations of several proposed "integral" transition indicators are displayed in Figure 2 for Runs Ba and Bc. Reattachment occurs in the range $23 < x^* < 38$ so the first several values plotted represent the recirculating flow behind the rib. Included also are the shape factor H indicated by the solid circles and the local momentum Reynolds number Re_θ shown by open circles at each station. In Run Ba (Figure 2a) transition does not occur before $x^* \approx 288$, its last station ($Re_x \approx 2.1 \times 10^5$). In Run Bc (Figure 2b) the first indication of transition from C_f data is at x^* about fifty and the first approximately turbulent value is at $x^* \approx 138$, its last measurement station (also $Re_x \approx 2.1 \times 10^5$).

Bejan [26] proposes that transition occurs when a boundary layer can undergo inviscid buckling as indicated by his buckling number N_B exceeding unity; he evaluates this proposal as $94 < Re_{\theta,tr} < 660$. This range may be considered to be too broad to provide useful information for transition onset. For example, in the present experiment both the recovering laminar data and the transitioning data satisfy this criterion. In Run Bc transition occurs with Re_θ between 278 and 287. But in Run Ba, $Re_\theta > 290$ for the last five stations (Re_x about 1.9×10^5 and higher) and C_f remains within about two per cent of the Blasius prediction.

In 1963 van Driest and Blumer and then in 1997 Mayle and Schulz suggested pointwise criteria for transition onset (i.e., applying at a point x,y in the boundary layer) but, since they have single values at each station, their behavior will be discussed here with the proposed integral criteria. The transition threshold proposed by **Mayle and Schulz** [30] may be phrased as $((u'_{max})^+/3)$ greater than unity. For Run Ba this quantity (diamonds labeled MS) increases gradually from less than 0.2 as u' grows in the pre-transitional boundary layer but does not reach unity by the last station; it grows approximately semi-logarithmically with x^*. One can see that for Run Bc it is about 0.4 at the last laminar station and has increased to about unity by $x^* \approx 51$, the first transitional one. (It then decreases some as the flow becomes fully turbulent since the maximum value of $((u')^+/3)$ is about 0.9 in the viscous layer of a low-Reynolds-number turbulent wall flow [31].)

In 1963 **van Driest and Blumer** [32] defined a "vorticity Reynolds number" based on the ratio of the inertial stress to the viscous stress at a point in a laminar boundary layer and hypothesized that it had a value Tr at which transition occurs. The idea appears somewhat like Bejan's [33] treatment. Conceptually, one could calculate pointwise values of this parameter from the measurements of Becker et al. [7] but we use the approximate integral result of van Driest and Blumer instead. Using the Pohlhausen approach to accommodate effects of a pressure gradient and/or freestream turbulence on this Reynolds number, they developed a proposed criterion (their eqn. 6) that transition occurs when

$$C\, Re_\delta\, (u'_\infty/U_\infty)^2 + B\,\Lambda + A = (Tr/Re_\delta)$$

Constants were evaluated by fitting the measurements of Dryden [34], Hall and Hislop [35] and Schubauer and Skranstad [36] to give the threshold value

$$((Re_\delta + 3.36\, Re_\delta^2\, (u'_\infty/U_\infty)^2\,)\,/\,9860\,) = 1$$

for transition in a flow with zero pressure gradient. This grouping is presented in Figure 2 as triangles with the label vD. For the recovering laminar run it is about 0.3 to 0.4 from reattachment to near the end of the model, increasing to about 0.55 at the last station ($x^* \approx 288$). In the transitioning flow, the grouping in parentheses is about 0.4 for most of the transition region and then finally exceeds 0.8 when our γ reaches 0.95 at the penultimate station ($x^* \approx 111$). Thus, requiring a value of unity for this grouping would not serve as an indicator or precursor of *transition onset* for the conditions of this run. (While the present data cannot yield γ_y, the DNS of Schlatter and colleagues [5] show its magnitude to be comparable to our γ as a turbulent boundary layer is approached.)

Andersson, Berggren and Henningson [29] suggest that transition occurs when the disturbance energy in the boundary layer, their E, reaches a specific value (which has not been evaluated). For the present data, we *approximate* this quantity non-dimensionally as

$$E_\delta = (3\,Re_\delta / (2\,\delta\,U_\infty^2)) \int_0^\delta [(u')^2 + (v')^2]\,dy$$

and evaluate it at pertinent stations. Our integral disturbance energy values E_δ are presented as crossed squares and labeled ABH-E in the figures. For the laminar data of Run Ba it grows approximately semi-logarithmically with x*, reaching a value of about eight (extrapolated) at the last station ($x^* \approx 288$). In Run Bc, at the last laminar station ($x^* \approx 38$) E_δ is about 6.5 and at the first transitional one ($x^* \approx 51$) it is over sixteen; as the end of transition is approached it settles to about fifty ($x^* \approx 111$). One might infer that an appropriate threshold value of our E_δ would fall between ten and fifteen for transition onset. Evaluation with further pre-transition and transitional data is needed.

In addition to analyzing the growth of the disturbance energy in a boundary layer, **Andersson, Berggren and Henningson** [29] developed a transition correlation. Its equivalent group $Re_\delta Tu(\%)$ is plotted in Figure 2 as open squares labeled ABH. Its threshold would be 6000 but a lower value could be reasonable when using local ($u'_\infty\{x\}/U_\infty$). This local group is about 2000 to 3000 in the recovering laminar run and it also is for most of the transition region of Run Bc so it does not discriminate between laminar recovery and the start of transition in the present case.

As reported by Hernon [37], the criterion for *transition onset* suggested by **Fransson, Matsubara and Alfredsson** [22] can be transformed to

$$Re_{\theta,tr}\, Tu_{LE} \approx 745$$

using Tu in per cent. In the figure its grouping is labeled as FMA and is represented by squares with a diagonal slash. In laminar Run Ba it is mostly about 230 to 320, with one value about 380, and appears to be gradually increasing at the end of the model. For transitioning Run Bc the grouping is about 330 at the last laminar and first transitional stations, $x^* \approx 38$ and 51, respectively. In the transition region it then increases slowly approaching 500 when the flow is deemed turbulent ($x^* \approx 138$).

Mayle [2] proposed that *transition onset* for zero pressure gradient (zpg) could be forecast by the correlation

$$Re_{\theta,tr}\, Tu_{LE}^{5/8} \approx 400$$

with the freestream turbulence level Tu_{LE} in per cent. In his Figure 10 he showed reasonable agreement with data for Tu between 0.3 and eight per cent (but on his page 518 he cautions against using this correlation for pressure gradients if Tu is less than three per cent). In Figure 2 the group $Re_\theta Tu^{5/8}$ is denoted by inverted triangles and is labeled Mayle. Since (u'_∞/U_∞) is *about* one per cent downstream from the rib it does not differ much from Re_θ and the grouping of Fransson, Matsubara and Alfredsson [22] which tend to obscure it. Mayle's group gradually increases from about 200 to 330 in Run Ba. In Run Bc after reattachment it gradually increases from about 310 to 390 through transition and then is about 450 at the turbulent station ($x^* \approx 138$), i.e., it is below his proposed threshold for transition onset until our γ is over unity. It might be appropriate to include dependence on u'_{max} or such in the boundary layer.

POINTWISE CRITERIA

In examining their data for bypass transition, **Suder, O'Brien and Reshotko** (p. 80 of [20]) concluded that "a critical value for the peak rms of the velocity fluctuations within the boundary layer of 3 to 3.5 per cent" U_∞ gave turbulent bursting regardless of the transition mechanism. After reattachment, the *recovering laminar Run Ba* ($Re_k \approx 358$, $k^+ \approx 11.1$) essentially retains laminar shape factors and friction coefficients to the last station ($x^* \approx 288$). However, there is evidence of a pre-transitional flow evolving [3]. At the first station after reattachment at $x^* \approx 38.3$ the profile ($u'\{y\}/U_\infty$) increases monotonically to the freestream value (Figure 1). Then from $x^* \approx 138$ to 263 a maximum appears within the boundary layer, growing to about six per cent of U_∞ at $x^* \approx 263$. It is interesting, perhaps, that this value corresponds to $(u')^+$ greater than two in the range about $15 < y^+ < 20$ in wall coordinates, approaching the value for a fully-turbulent wall layer even though the skin friction coefficient still agrees with the laminar value. The ($u'\{y\}/U_\infty$) profiles correspond to the so-called Klebanoff modes which have peak values being at a near constant value of y/δ^* and growing linearly with $(Re_x)^{1/2}$ [3]. In Run Ba the peak values do grow linearly with Re_θ. Various investigators show the peak values for the smooth flat plate at various wall distances but generally in the range $1.1 < (y/\delta^*) < 1.5$ [3,5,38] while downstream of our rib we find it at

$1.0 < (y/\delta^*) < 1.1$, slightly closer to the wall. Kendall (Figure 5 of [28]) shows a slide from Klebanoff [39] that demonstrates a dependence of (y/δ^*) for the peak fluctuation varying with turbulence length scale; therefore, one explanation of our different position could be a different length scale induced by the rib. The peak Reynolds shear stress $(\overline{uv})^+$ is found near $y^+ \approx 15$ and increases from 0.009 to 0.28 in the range $38.3 < x^* < 263$. While it is apparent that transition will occur further downstream, it does not happen immediately after exceeding 3.5 per cent of U_∞. On the other hand, Run Bc ($Re_k \approx 502$, $k^+ \approx 14.2$) already has (u'/U_∞) above 3.5 per cent at the first station after reattachment ($x^* \approx 38$) where the flow still appears laminar (Figure 1) -- and then transition starts before the next station ($x^* \approx 51$).

Walsh has suggested that the occurrence of transition might be forecast in terms of entropy generation [40]. By application of boundary layer and other approximations, Rotta has suggested that dissipation in an unheated turbulent boundary layer may be evaluated as

$$[\tau_{visc} + \tau_{turb}]\,(\partial U/\partial y) \approx [\mu(\partial U/\partial y) - \rho\,\overline{uv}\,]\,(\partial U/\partial y) = TS_{ap}'''$$

(reported as eqn. 23.8d in the Schlichting text [15]) so that the volumetric entropy generation rate can be calculated ***approximately*** as

$$(S''')^+ \approx (\partial U^+/\partial y^+)^2 - (\overline{uv})^+\,(\partial U^+/\partial y^+)$$

where $(S''')^+$ is defined as $T\nu S_{ap}'''/(\rho u_\tau^4)$. Here the first term represents the direct (or mean) entropy generation rate S_{dir}''' or S_{mean}''' and the second is called the indirect (or turbulent) entropy generation rate S_{indir}''' or S_{turb}'''. For a laminar boundary layer on a flat plate without freestream turbulence, S''' and its integrals can be calculated from the Blasius or Pohlhausen solutions [15]. One sees his turbulent dissipation term to be equivalent to the main contributor to production of turbulent kinetic energy. Several investigators have adopted this idea to measure or predict entropy generation (e.g., Moore and Moore [41], O'Donnell and Davies [42], Stieger and Hodson [43], Hyhlik and Marsik [44]).

Walsh hypothesized that a laminar boundary layer with disturbances will become significantly unstable if $\partial S'''/\partial y$ is not negative, i.e., transition onset occurs when $\partial S'''\{y\}/\partial y$ exceeds zero. Later Walsh [45] suggested that, if the pointwise value of indirect entropy generation rate exceeds that for the direct entropy generation rate someplace in the boundary layer, transition will follow; with the Rotta suggestion for dissipation above, this idea can be written as $|-\rho\,\overline{uv}/(\mu\partial U/\partial y| > 1$. This value is also suggested by Bejan's constructal theory [33,46]. It is interesting that Liepmann (p. 16 of [47]) essentially used this occurrence as a *definition* of *transition* (his "practical" critical Reynolds number R_2).

We first examine the idea that the gradient of S''' gives a useful transition indicator for the recirculating region immediately behind the rib (Figure 3). Here the pointwise entropy generation rate is non-dimensionalized with boundary layer quantities as $(S''')^* = (TS_{ap}'''\delta/(\rho U_\infty^3)) = (S''')^+\delta^+(C_f/2)^{3/2}$. Before reattachment both cases have rib wakes with inflectional velocity profiles, leading to positive gradients of $\partial S_{ap}'''/\partial y$ [40]; apparently this criterion alone is not sufficient to forecast the occurrence of transition. For both runs, the volumetric entropy generation rate $(S''')^*$ peaks at the edge of the shear layer induced by the rib, with the largest values at the first station after the rib (Ba being slightly larger than Bc), and decreases downstream; meanwhile the values near the wall increase in the streamwise direction. The high values are almost all from *direct* entropy generation. One sees that $\partial S_{ap}'''/\partial y$ is negative at the wall with its absolute value decreasing to zero at the center of the recirculating region where the $U\{y\}$ is a minimum. Thus, there is an inherent change to a positive slope at this condition; it then increases through the inflection region where it reaches a maximum. So, in these cases. the suggested transition indicator of $\partial S'''/\partial y$ is related to the inflectional velocity profile which is recognized to cause a "frictionless instability" [15]. However, while both runs satisfy this criterion, Run Ba does not show rapid transition afterwards.

The continued development of $(S''')^*$ is presented in Figure 4 for the transition occurring in the range $38.3 < x^* < 111$ in Run Bc. In the region near (y/k) of unity all profiles show a slight, gradual maximum to persist from the mixing layer created above the rib. For the first two stations, corresponding to the last laminar station and the first transitional one, the values near the wall are smaller, about one-half of their maxima. Then as the flow proceeds downstream and undergoes transition towards a fully-turbulent boundary layer, $(S''')^*$ grows in the wall region as more "indirect" entropy generation occurs there. As shown by Rotta [48] and McEligot et al. [49], for turbulent boundary layers most entropy generation occurs in the viscous layer adjacent to the wall.

The comparison of indirect versus direct entropy generation rates has also been hypothesized to be a potential transition criterion by Walsh [45]. In the recirculating region upstream (not shown), $(S_{turb}'''\{y\}/S_{mean}'''\{y\})$ for recovering Run Ba remains less than five per cent at the data points available. In contrast, for transitioning Run Bc, by $x^* \approx 24$ -- the last station before reattachment, $(S_{turb}'''/S_{mean}''')$ has pointwise values exceeding two (at the minimum of $U\{y\}$ this quantity will become infinite, unless there is no perturbation shear stress, so it becomes a question of the limiting behavior in the vicinity). The evolution of the indirect (turbulent) contribution relative to the direct (mean) entropy generation after reattachment is demonstrated in Figure 5 for Run Bc. One sees peak values of $(S_{turb}'''/S_{mean}''')$ greater than unity, increasing as the boundary layer undergoes transition. The first profile after transition does not differ much from the prior one evaluated as pre-transitional, then from $x^* \approx 73.8$ the turbulent transport and accompanying indirect entropy generation grow and spread outward (however, the values at the outer positions are exaggerated by the normalization as the mean velocity gradient approaches zero). On the other hand, Run Ba has only slight growth of about 0.006 to 0.03 from $x^* \approx 38.3$ to 146 ($210 < Re_\theta < 250$) then finally to about 0.28 at $x^* \approx 263$ ($Re_\theta \approx 302$) near the end of the model. So these observations agree with the idea of a transition indicator.

Liepmann {pgs. 14,16 of [47]) defined his R2 ("where amplification of disturbances has already taken place and to such an extent that complete breakdown of the laminar motion occurs") as "the Reynolds number at which the apparent shear $\tau_A = -\rho\,\overline{uv}$ due to amplified boundary layer oscillations at any point in the boundary layer becomes equal to the laminar shear $\tau_L = \mu \partial U/\partial y$ in the boundary layer." Commenting that the "so-called 'critical layer' is usually near the wall," he then approximated τ_L by τ_W (his p. 17), giving the approximate criterion that $(\overline{uv})^+$ be equal to one. Sharma et al. [1] and then Mayle [2] interpreted this approximation as identifying the onset of transition; since it is often been impractical to measure $\overline{uv}\{y\}$, Sharma et al. also developed a criterion in terms of the streamwise fluctuation, $(u')^+ > 3$ instead. However, with their measurements of Reynolds shear stress Becker et al. [7] are able to present development of $(\overline{uv}\{y^+\})^+$ directly in their Figures 5b and 6b.

In addition to profiles of mean streamwise and rms fluctuating velocities, Figure 1 near reattachment evaluates this transition onset model (identified as "tau_{turb}," inverted diamonds); a value of unity or more would indicate that transition should begin. In both runs, peaks of this quantity appear near $(y/k) \approx 1.8$, towards the edge of the inflection region caused by the wake of the rib. However, for recovering Run Ba the magnitude does not exceed 0.02 near reattachment while in Run Bc it peaks at about 0.3 near the dividing streamline at $(y/k) \approx 0.6$ within the recirculating region and grows to more than two by the first station after reattachment. Further downstream for Run Ba the peak Reynolds shear stress $(\overline{uv})^+$ is found near $y^+ \approx 15$ and increases from 0.009 to 0.28 in the range $38.3 < x^* < 263$. For Run Bc, after its last laminar profile ($x^* \approx 38$) it rapidly decreases to values less than unity during transition as it should to approach behavior of a typical viscous layer in a turbulent wall flow developing in a negligible pressure gradient.

Mayle, Schulz and Bauer [50] developed and solved approximate governing equations for the normal stress $(u')^2$ and Reynolds shear stress of the unsteady fluctuations induced within the laminar boundary layer by freestream turbulence. They define transition as the point where their predictions diverge from the data of Roach and Brierley [51] and conclude – from comparison to six experiments with and without varying $U_\infty\{x\}$ -- that it occurred when $(\overline{uv})^+$ is about one-third. They also indicate that this position was either upstream or at the location of the minimum C_f. As one can tell from Figure 1 (as described in the paragraph above), their suggestion is also consistent with our observations for Runs Ba and Bc.

The criterion suggested by **Fasihfar and Johnson** [52] is that when $(u'\{y\}/U\{y\})$ exceeds 0.23 transition will occur. In the recirculating region for both runs, near the dividing streamline and near the wall this threshold is exceeded (not shown). For recovering Run Ba, from $x^* \approx 38$ to 263 the value is above the threshold near the wall and gradually decreases outwards as u' increases downstream in the pre-transitional flow but U{y} remains near a Blasius profile. That is, the criterion predicts transition although apparently it does not happen in this case. In transitioning Run Bc at the last laminar station it is above 0.23 from $(y/k) \approx 0.8$ to the wall, with values near and over two close to the wall. Then during the transition process, near the wall it gradually decreases in the downstream direction as it begins to approach a value of about 0.4 at the wall in accordance with the observations of Alfredsson et al. [53] for a fully-developed turbulent flow. Above the level of the rib this quantity remains below its threshold recommendation, gradually decreasing with y at all downstream stations..

Liepmann (p. 42 of [13] and p. 16 of [47]) considered the *correlation coefficient* $C = -\overline{uv}/(u'v')$ in the context of pre-transitional growth of Tollmien-Schlichting instabilities and used it in estimating the location where laminar flow would break down. He commented (p. 42 of [13]) that C reaches a maximum value of 0.2 for the Tollmien-Schlichting waves while for turbulent boundary layers it is 0.3 approximately. **Sharma et al.** [1] then employed the data of Liepmann for "natural" transition ($Tu_{LE} \approx 0.11$ per cent) and their own data for bypass transition ($Tu_{LE} \approx 2.4$ per cent) to infer a correlation coefficient of 0.53 for the onset of transition. They claim that value is close to the 0.45 typical for turbulent flows. Apparently neither study actually measured the Reynolds shear stress in the boundary layer. The different values cited are cause to examine its variation in our Runs Ba and Bc where it has been measured.

In the recirculating regions for recovering Run Ba, C reaches 0.35 but is mostly less (the results for C are not plotted in the current paper). In contrast, for transitioning Run Bc, by the last station before reattachment ($x^* \approx 23.8$) the correlation coefficient grows to over 0.6 between the dividing streamline and $(y/k) \approx 2$. After reattachment in *recovering Run Ba*, for $x^* \approx 138$ and less, C has a maximum value of about 0.1 between $(y/k) \approx 2$ and the wall. Further downstream at $x^* \approx 262$, C increases to levels of about 0.6 (with considerable scatter since to the magnitudes of u', v' and $\overline{uv}$ are small). With *transitioning Run Bc*, at the last laminar station at $x^* \approx 38.3$, C has a peak over 0.6 near $(y/k) \approx 1.8$ and low values below the level of the rib. But the magnitude and location of this peak change considerably by the first transitional station ($x^* \approx 50.8$, $\gamma \approx 0.2$, $Re_\theta \approx 289$) and the profile evolves to have a minimum in this region during the transitional process. Meanwhile a peak of about 0.4 gradually appears at about half the rib height

($y^+ \approx 13$) in the later stages of transition. The profile of the correlation coefficient at $x^* \approx 111$ ($Re_\theta \approx 374$, $\gamma \approx 0.95$) has a maximum of about 0.6 very near the wall at $(y/k) < 0.1$, corresponding to $y^+ \approx 2.5$, and then decreases to a value of 0.2 - 0.3 in the outer boundary layer beyond the level of the rib. At the last measuring station ($x^* \approx 138$, $\gamma > 1$, $Re_\theta \approx 389$), $C \approx 0.3$ across much of the turbulent boundary layer and increases slightly near the edge (where magnitudes are small again). In Run Bc transition apparently begins when the peak value of C is between 0.45 and 0.6; in contrast, for Run Ba the peak value of C increases from about 0.1 to 0.6 as $(u'\{y\}/U_\infty)$ grows from about two to six per cent but γ is still only about 0.01 at this last position ($x^* \approx 263$). So some value of C may serve as a precursor to transition. Thus, it would be desirable to obtain further data with closer Δx^* and more heights k^+ in the range between these two runs.

CONCLUDING REMARKS

Successful comparison of a proposed transition indicator to a specific set of measurements cannot serve as a sufficient proof that the indicator is universal. But it is necessary that the criterion agree with any specific data available to be continued to be considered as possibly universal. The data of Becker et al. [7] provide such a simple test. Whether or not the behavior of a quantity is consistent with the idea of a transition indicator requires comparison to recovering laminar flows and transitional ones, e.g., our Runs Ba and Bc, respectively. In the present study, many proposed criteria failed the test but a few showed promise.

Seven proposed transition criteria were examined that employed integral parameters, such as local Reynolds numbers, disturbance energy, etc. This examination of proposed integral criteria for transition reinforces the need for useful pointwise criteria. For the present data -- with its higher fluctuation levels induced within the boundary layer – the proposed values of the thresholds for transition which are based on the freestream turbulence level at the leading edge are not reached in the recovering laminar run and they are not exceeded in the transitioning process either (until after the flow is turbulent in some cases) when evaluated on a local basis. It is not surprising that they fail to provide useful criteria since the rib temporarily decouples the boundary layer fluctuations from the freestream, providing higher values in the boundary layer than the freestream forcing would cause. However, our results are consistent with some criteria which include quantities internal to the boundary layer, such as $(u'_{max})^+$ by Mayle and Schulz [30] and, possibly, our E_δ approximating Andersson, Berggren and Henningson [29].

For seven criteria which apply at a point in the flow, three had values that were less than the threshold for recovering Run Ba and greater than it in transitioning Run Bc: (1) $(S_{indir}'''\{y\}/S_{dir}'''\{y\}) > 1$ [45], (2) $(\overline{uv}\{y\})^+$ or $(\overline{uv}_{max})^+ > 1$ [47] and (3) $(\overline{uv}\{y\})^+ > (1/3)$ [50] at any location y. And there may be a value of the correlation coefficient C that could serve as a criterion. The idea of Walsh [45] is that transition will ensue when the indirect entropy generation rate exceeds the direct entropy generation rate. Since the approximate estimate of S_{indir}''' was used in the comparison, we can see that this criterion is equivalent to saying that when production of turbulence kinetic energy is greater than direct (mean) dissipation, transition will occur. (Andersson, Berggren and Henningson [29] made a comparable suggestion in discussing reviews by Savill.)

Effectively the criteria of Liepmann [47] and of Mayle, Schulz and Bauer [50] both employ $(\overline{uv}\{y\})^+$ as the variable; the difference is the recommended threshold value: unity or one-third, respectively. To discriminate between them and to resolve whether C can serve, further examinations of experimental measurements are necessary. For example, with a geometry like the present experiment, more detailed data in the range $11 < k^+ < 14$ are desirable.

It is recommended that these surviving transition indicators (and any others we have not considered) be compared to further transition measurements from experiments with pressure gradients, heat transfer, separation bubbles, other roughnesses, injection or suction, acoustics, compressibility and other non-canonical recovering and transitioning boundary layers. It would be interesting to see whether any survive this broader inspection to remain considered as *possibly* universal. However, such a study is beyond the limited scope of the present work.

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NOMENCLATURE

{ }	function of
C	correlation coefficient, $\overline{uv}/(u'v')$
f	variable
k	roughness height
p	pressure
S	entropy generation rate
T	temperature

U, V	mean velocity components in streamwise and wall-normal directions, respectively
u, v, w	velocity fluctuations about means in streamwise, wall-normal and spanwise directions, respectively
u', v'	root mean square of velocity fluctuations
u_τ	friction velocity, $(\tau_W / \rho)^{1/2}$
$\overline{uv}$	mean fluctuation product in Reynolds shear stress ($-\rho \, \overline{uv}$)
x, y, z	coordinates in streamwise, wall-normal and spanwise directions, respectively

Non-dimensional quantities

C_f	skin friction coefficient, $2 \tau_W /(\rho U_\infty^2)$
E_δ	integral of disturbance energy
H	shape factor, δ^* / θ
k^+	roughness height, $k u_\tau / \nu$
Re	Reynolds number; Re_δ, based on boundary layer thickness, $U_\infty \delta / \nu$; Re_{δ^*}, based on displacement thickness, $U_\infty \delta^* / \nu$; Re_θ, based on momentum thickness, $U_\infty \theta / \nu$; Re_k, based on roughness height, $U_\infty k / \nu$; Re_x, based on streamwise position, $U_\infty x / \nu$
$(S''')^+$	pointwise volumetric entropy generation rate, $T \nu S_{ap}''' / (\rho u_\tau^4)$
$(S''')^*$	pointwise volumetric entropy generation rate, $T S_{ap}''' \delta / (\rho U_\infty^3)$
Tu	turbulence intensity, $[(\overline{u^2} + \overline{v^2} + \overline{w^2})/3]^{1/2} / U_\infty$ or u'_∞ / U_∞
U^+	mean velocity, U / u_τ
x*	distance downstream from roughness element, $(x - x_k) / k$
y^+	wall-normal coordinate, $y u_\tau / \nu$

Greek symbols

γ	intermittency
δ	boundary layer thickness; δ^*, displacement thickness
η	Blasius parameter, $y (U_\infty / (\nu x))^{1/2}$
Λ	Pohlhausen pressure gradient parameter [van Driest and Blumer, 1963], $(- \delta^2 / \mu U_\infty) \, dp/dx$
θ	momentum thickness
μ	absolute viscosity
ν	kinematic viscosity, μ / ρ
ρ	density
τ	shear stress; τ_W, wall shear stress

Superscripts

$(\_)^+$	normalization by wall units, ν and u_τ
$(\_)'$	root mean square
$(\_)''$	per unit surface area
$(\_)'''$	per unit volume
$\overline{(\ )}$	time mean value

Subscripts

A	apparent [Liepmann, 1945]
dir	direct
indir	indirect
k	based on roughness height
L	laminar
lam	laminar
LE	leading edge
max	maximum
mean	based on mean value
min	minimum
tr	transition
turb	turbulent
visc	viscous
w	wall
x	based on streamwise position
y	based on wall-normal distance
δ	boundary layer edge
∞	freestream value

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Table 1. EXPERIMENTAL RUN IDENTIFICATION, POSITIONS OF REATTACHMENT AND MOMENTUM REYNOLDS NUMBERS FOR TRANSITION AND LAMINAR RECOVERY PROCESSES.

Re_k	0	179	251	318	358	502	529	741
k^+	0	5.5	7.1	11.2	11.1	14.2	16.4	21.1
k/δ^*	0	0.297	0.351	0.683	0.593	0.702	0.882	1.043
Run	Sm	Ca	Cc	Ab	Ba	Bc	Aa	Ac
x_{reatt}^*	---	10.0	10.0	16.7	23.8	23.8	25.0	16.7
	---	-24.5	-24.5	-25.0	-38.3	-38.3	-33.3	-25.0
x* for								
last $\gamma \approx 0$	>end	>576	>576	>175	>288	38.3	33.3	???
$\gamma > 0$	---	---	---	---	---	50.8	41.7	25.0
$\gamma \approx 1$	---	---	---	---	---	138	58.3	33.3
$Re_{\theta,lam}$	>289	>293	>343	>221	>315	>279	>277	???
$Re_{\theta,tr}$	---	---	---	---	---	<289	<298	<331
$Re_{\theta,tu}$	---	---	---	---	---	<389	<320	<372
Re_θ for								
$H \approx 2.6$	>289	>293	>343	>221	>315	>278	>277	???
$H < 2.6$	---	---	---	---	---	<287	<298	<331
$H \approx 1.6$	---	---	---	---	---	<389	<331	<387

Some notes on the physical meanings of the quantities in the three subtables at the bottom of this Table may be useful. There is a heading "x* for" above three rows. The row $\gamma > 0$ indicates the first measuring station at which the skin friction coefficient showed transition to have started. For example, taking the row of $\gamma > 0$ and the column of $Re_k = 502$, x* about 50.8 was that station. The second set gives the last values of Re_θ for which the flow was recognized as still laminar (corresponding to last $\gamma \approx 0$ in the set above), the first where the flow was considered to be undergoing transition and the value where γ indicated a turbulent boundary layer. As an example, taking the row of $Re_{\theta,lam}$ and the column of Re_k =179, the flow was apparently laminar to the end of the plate where Re_θ is about 293 so $Re_{\theta,lam}$ would have been greater than this value.

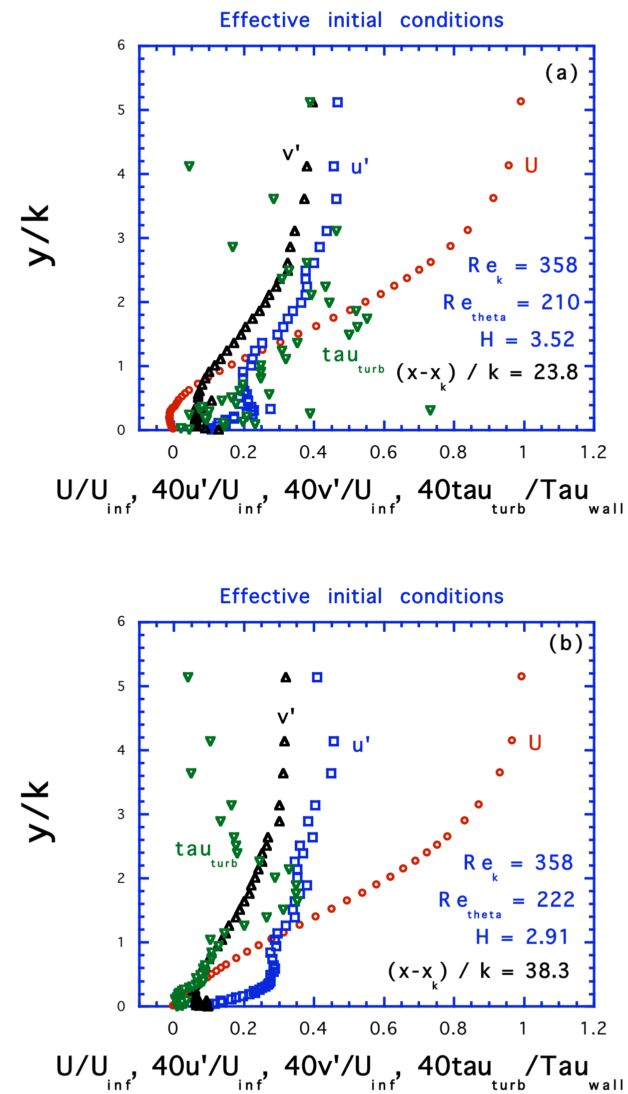

Figure 1. EFFECTIVE INITIAL CONDITIONS DOWNSTREAM OF A RIB. RECOVERING LAMINAR BOUNDARY LAYER, RUN Ba, $Re_k \approx 358$, $k^+ \approx 11.1$, BEFORE (a) AND AFTER (b) REATTACHMENT.

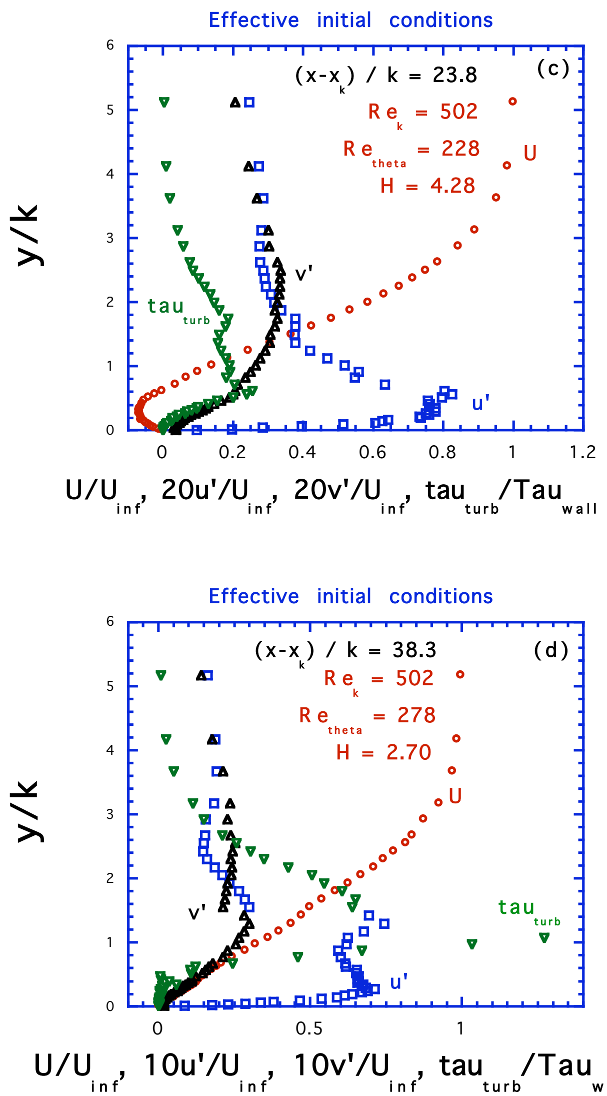

Figure 1. EFFECTIVE INITIAL CONDITIONS DOWNSTREAM OF A RIB. TRANSITIONING RUN Bc, $Re_k \approx 502$, $k^+ \approx 14.2$, BEFORE (c) AND AFTER (d) REATTACHMENT. IT SHOULD BE NOTED THAT THE SCALINGS OF THE VARIABLES DIFFER.

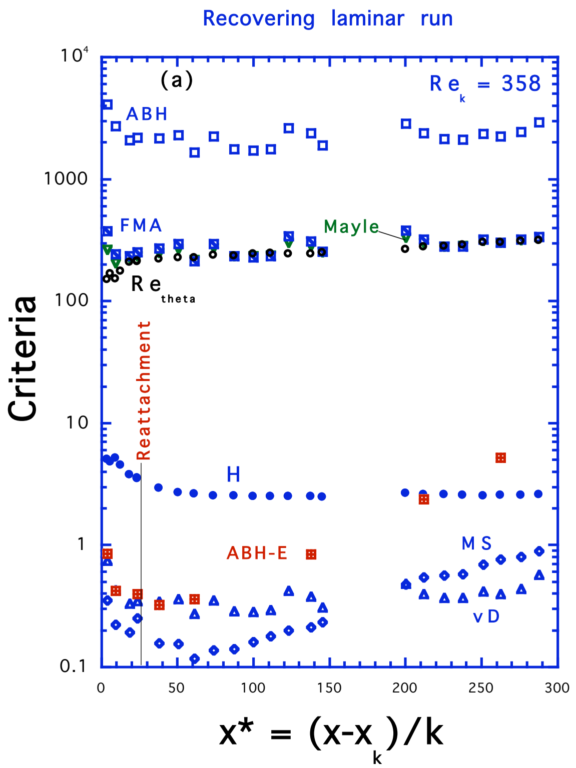

Figure 2a. STREAMWISE EVOLUTION OF PROPOSED INTEGRAL TRANSITION CRITERIA FOR RECOVERING RUN Ba ($Re_k \approx 358$, $k^+ \approx 11.1$).

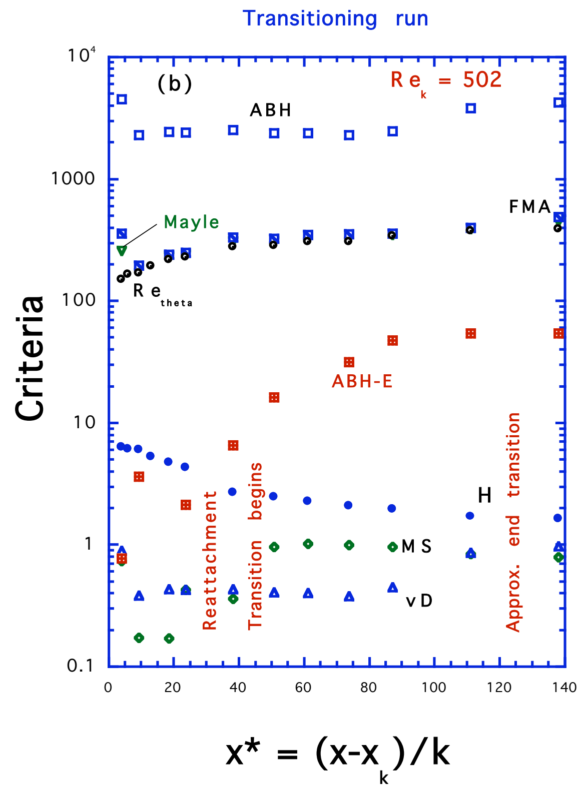

Figure 2b. STREAMWISE EVOLUTION OF PROPOSED INTEGRAL TRANSITION CRITERIA FOR TRANSITIONING RUN Bc ($Re_k \approx 502$, $k^+ \approx 14.2$).

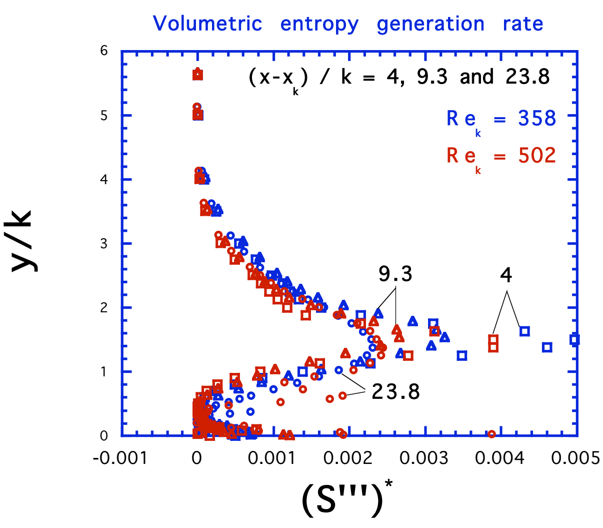

Figure 3. EVOLUTION OF POINTWISE ENTROPY GENERATION RATE FOR RUNS Ba AND Bc WITH $k^+ \approx 11.1$ AND 14.2, RESPECTIVELY.

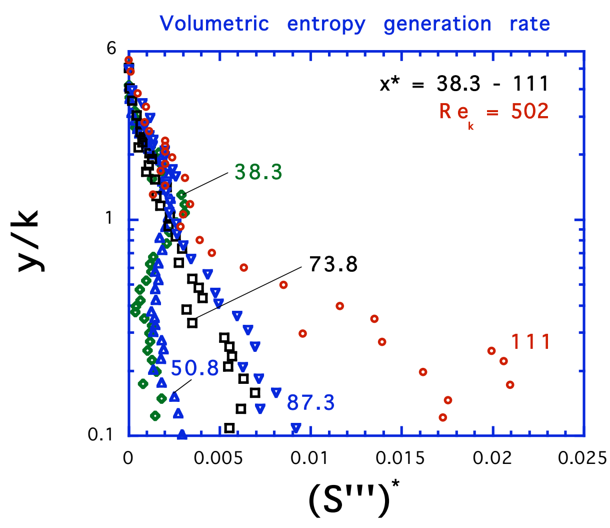

Figure 4. DEVELOPMENT OF POINTWISE ENTROPY GENERATION RATE IN A TRANSITIONING BOUNDARY LAYER, RUN Bc, $Re_k \approx 502$, $k^+ \approx 14.2$.

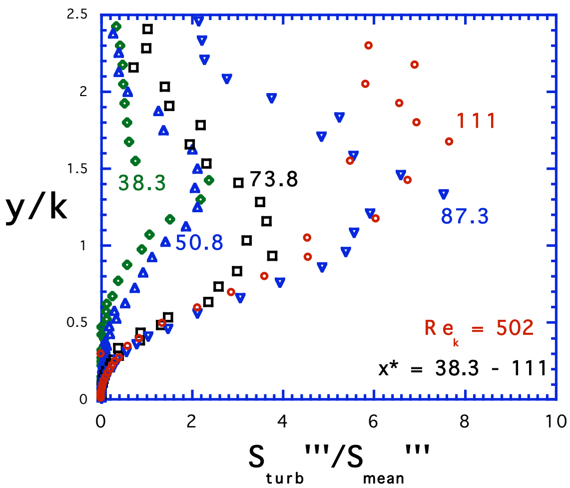

Figure 5. EVOLUTION OF INDIRECT (TURBULENT) ENTROPY GENERATION RATE IN A TRANSITIONING BOUNDARY LAYER, RUN Bc, $Re_k \approx 502$, $k^+ \approx 14.2$.

APPENDIX

At the recommendation of an interested reviewer the following additional figures are included for those readers who prefer to use coordinate scaling other than rib height. To place the present data in perspective, trends of some of their integral parameters are compared in Figure A1. The Blasius prediction (solid lines) is for the classical canonical flow without freesteam turbulence imposed. The DNS predictions (centerline curves) are for a case of bypass transition with freestream turbulence $Tu_{in} = [(\overline{u^2}+\overline{v^2}+\overline{w^2})/3]^{½}/U_\infty$ of 4.7 per cent and a non-dimensional length scale of five by Brandt, Schlatter and Henningson [5]. Present measurements for a plate without a rib are indicated by circles. Also included as an example of our most extreme situation are measurements from Run Ac ($Re_k \approx 741$) which becomes predominantly turbulent immediately after reattachment. For Run Ba with height $Re_k \approx 358$, the rib is located at $Re_{x,k} \approx 1 \times 10^5$. For Runs Ac and Bc, the rib location is at $Re_{x,k} \approx 1.4 \times 10^5$ and their heights are $Re_k \approx 741$ and $Re_k \approx 502$, respectively.

The smooth plate case apparently remained laminar to (and presumably beyond) the last station at $Re_x \approx 2 \times 10^5$, consistent with the level of freestream turbulence, and four others did after reattachment beyond the rib ($Re_k \approx 179$, 251, 318 and 358). For $Re_k \approx 502$ and greater, the boundary layer became turbulent ($\gamma \approx 1$) before the end of the test section. Runs Ba and Bc regained γ about zero after reattachment as suggested by Liepmann [1943]. By $Re_x \approx 2.1 \times 10^5$ *transitioning Run Bc* has $H \approx 1.61$, $C_f > C_{f,turb}$ and $(u'_{max})^+ \approx 2.4$ at $y^+ \approx 15.3$, comparable to a developed turbulent boundary layer. In the case of Run Ac ($Re_k \approx 741$) no evidence of laminar conditions was seen after reattachment --- this situation may correspond to suppression of turbulent flow rather than being a question of receptivity [25]. Since Run Bc had the most gradual laminar-to-turbulent transition, it is the transitioning run which is analyzed in the greatest detail.

Immediately after the rib where a recirculation region occurs, the values of Re_θ become very low (upstream agrees with the laminar smooth plate results). This reduction is a consequence of the definition with the recirculating region giving negative values of the integrand and significantly lower values of U{y} over most of the boundary layer compared to the smooth reference. They then increase as the dividing mean streamline approaches the plate. Since the first mean velocity profile after reattachment appears laminar near the wall for runs Ba and Bc, their Re_θ values approach agreement with the Blasius prediction. Run Ba then continues to follow the Blasius result until near the end of the plate where turbulent fluctuations begin to increase in the apparently laminar boundary layer (possibly so-called Klebanoff "modes" [3]). Without a rib, the present data follow the Blasius solution closely. For the DNS of bypass transition, as the freestream turbulence affects the laminar boundary layer Re_θ grows faster than for pure laminar flow; near $Re_x \approx 1.3 \times 10^5$ turbulent spots appear in the boundary layer [Walsh et al., in preparation] and the growth rate of Re_θ increases. It is interesting, but probably fortuitous, that once Run Bc begins its transition process, its values of $Re_\theta\{Re_x\}$ are approximately the same as the DNS results: transition of Run Bc appears to begin between $Re_x \approx 1.56 \times 10^5$ and $Re_x \approx 1.66 \times 10^5$. Since Run Ac is apparently turbulent before $Re_x \approx 1.60 \times 10^5$, it rapidly grows to a thicker boundary layer and higher Re_θ.

The shape factor $H_{12} = (\delta^*/\theta)$ generally mirrors the behavior of $Re_\theta\{Re_x\}$. The value 2.59 is given by the Blasius solution [15]. The present Run Ba after reattachment and the smooth data agree with this prediction. Immediately after the rib, values of H_{12} are high due to the low magnitude of θ and increase in δ^* (for the latter the recirculating region gives a positive contribution to the integrand and in general the quantity $(1 - (U/U_\infty))$ is increased). Runs Ac and Bc decrease to H_{12} about 1.5, a typical value for developed turbulent boundary layers, as do the DNS bypass transition predictions by the end of their calculations.

The behavior of the skin friction coefficient C_f is as one would expect from the trends of Re_θ and H_{12}. (Values are not shown for the recirculating region behind the rib where they are negative due to the reversed flow at the wall.) The data from Run Ac demonstrate the approximate magnitude to be expected for turbulent boundary layers. The DNS predictions and the measurements from Run Bc both reach these levels by their last stations giving intermittency γ about unity there. Both Runs Ac and Bc have the same rib location but, with $Re_k \approx 741$ for Run Ac and $Re_k \approx 502$ for run Bc, the former reaches this γ sooner. Run Ba and our smooth experiment both still agree with the Blasius solution towards the end of the plate (except for a couple low points at their last stations, possibly too close to the end of the test section).

Again these results for integral parameters confirm the impression that Run Ba undergoes *laminar recovery* while run Bc is a *transitioning* case.

Figures 1b, 1d, 4 and 5 have been re-scaled with y/δ as the ordinate and are presented as Figures A2a, A2b, A3 and A4, respectively. The labels adjacent to the data symbols in Figures A3 and A4 provide the locations Re_x for each profile.

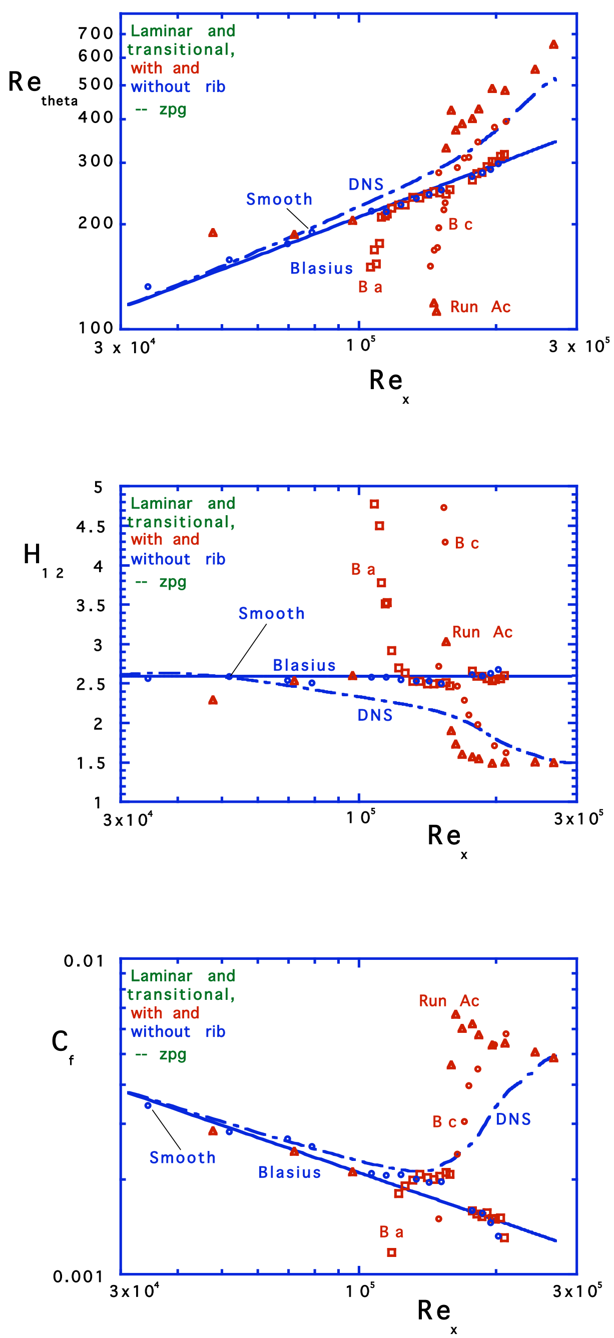

Figure A1. EVOLUTION OF INTEGRAL PARAMETERS FOR EXPERIMENTAL RUNS EMPHASIZED PLUS COMPARISON TO BLASIUS PREDICTIONS FOR PURE LAMINAR BOUNDARY LAYERS AND DNS FOR BYPASS TRANSITION BY BRANDT, SCHLATTER AND HENNINGSON [5]: (a) MOMENTUM THICKNESS REYNOLDS NUMBER, (b) SHAPE FACTOR AND (c) SKIN FRICTION COEFFICIENT.

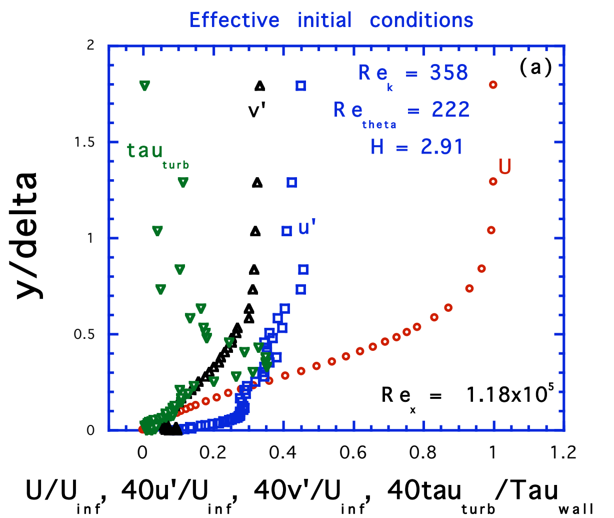

Figure A2a. EFFECTIVE INITIAL CONDITIONS DOWNSTREAM OF A RIB. RECOVERING LAMINAR BOUNDARY LAYER, RUN Ba, (k/δ) ≈ 0.204, AFTER REATTACHMENT.

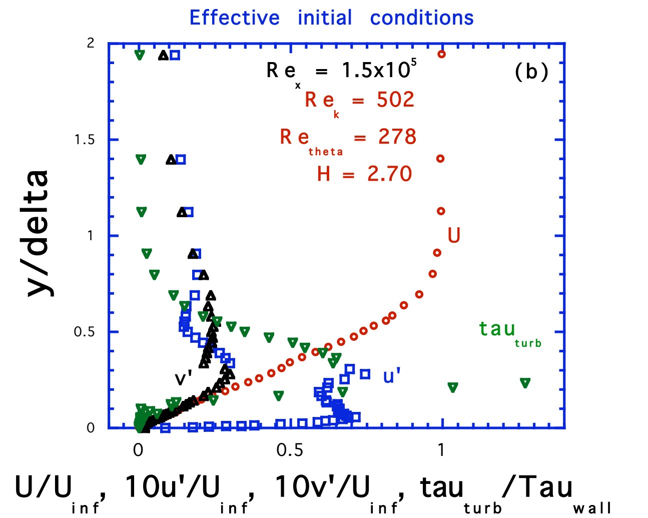

Figure A2b. EFFECTIVE INITIAL CONDITIONS DOWNSTREAM OF A RIB. TRANSITIONING RUN Bc, (k/δ) ≈ 0.241, AFTER REATTACHMENT. IT SHOULD BE NOTED THAT THE SCALINGS OF THE VARIABLES DIFFER.

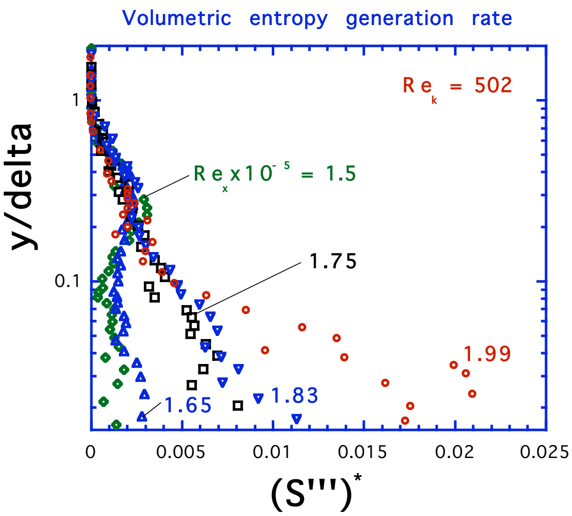

Figure A3. DEVELOPMENT OF POINTWISE ENTROPY GENERATION RATE IN A TRANSITIONING BOUNDARY LAYER, RUN Bc, $Re_k \approx 502$, $k^+ \approx 14.2$.

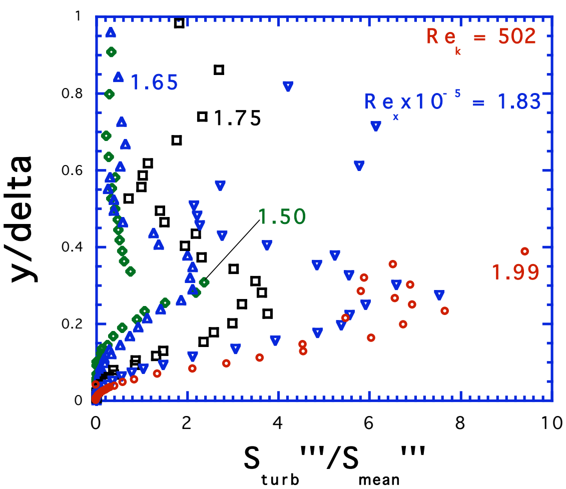

Figure A4. EVOLUTION OF INDIRECT (TURBULENT) ENTROPY GENERATION RATE IN A TRANSITIONING BOUNDARY LAYER, RUN Bc, $Re_k \approx 502$, $k^+ \approx 14.2$.