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A JOINED MBL-ZONAL METHOD TECHNIQUE FOR EVALUATING RADIATIVE THERMAL LOADS INTO COMBUSTION CHAMBERS OF INDUSTRIAL GAS TURBINES

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ABSTRACT

Thermal radiation is typically one of the most important phenomenon to be taken into account in the evaluation of combustor walls thermal loads due to the high temperatures reached into them. A classical approach is based on the so called Zonal Method, originally developed by Hottel and Sarofim (1967), and actually widely employed in the industrial environment. Even if its accuracy has been largely demonstrated, its efficiency is affected by computational costly solution of $4^{th}-6^{th}$ *fold integrals constituting the DEAs. A direct integration is usually employed, subsequently smoothing the results in order to obey the conservation constraints. The last decades have seen a growing interest on developing new techniques able to simplify these time consuming direct numerical integrations. Among them one of the most promising approaches has been recently introduced by Yuen (2008) which is based on the classic Mean Beam Length concept. The emittance (absorptance) coefficients of the radiating (receiving) gas volume zones of cubic shape are treated as those of a grey gas filled hemisphere. According to Yuen (2008), a correlative expression has been employed for evaluating the MBL corresponding to each single volume zone. In the present work its application has been extended to more complex zone shapes by means of an Artificial Neural Network trained on a properly selected geometry database. In this way the DEA integral folds can be all reduced to the* 4^{th} *order, and, employing well known geometrical techniques (Walton, 2002), can be further decreased to lower order integrals. The proposed model has been compared with 3D benchmark test cases available in literature: the accuracy has been tested against the results of DTM, FVM and classical Zonal Method. Moreover an industrial application is shown. The geometry of the AE94.2 gas turbine combustion chamber has been considered; the gas mixture has been treated as a non-grey gas using the well known Weighted Sum of Gray Gases (WSGG) model. In comparison with the classical zonal method approach, the efficiency of the proposed one demonstrates the possibility of a zone refinement enabling a more accurate evaluation of radiative thermal loads at the same computational cost. Results are presented and discussed.*

NOMENCLATURE

A Area [m^2]
C zonal boundary contour [m]
D side length of a square or cubic zone [m]
E emittance [Wm^{-2}]
$\overline{gs}$, $\overline{gg}$ direct exchange areas [m^2]
H Lagrangian term [-]
H_j radiative energy leaving the $j-$th zone [kW/m^2]
I radiating energy intensity [kW/m^2]
k absorption coefficient [m^{-1}]
K number of volume zones [-]
L mean beam length [m]
M summation of N and K zones [-]
N number of surface zones [-]

$\vec{n}$ unit vector normal to the surface [-]
p,t number of zone subdivisions [-]
q radiative flux [Jm^2s^1]
r,S module of $\vec{S}$ vector [m]
$\vec{S}$ distance vector between any two zones [m]
s spatial distance [m]
$\overline{ss}$, $\overline{sg}$ direct exchange areas [m^2]
T absolute temperature [K]
w LSSM weights [-]
x, y, z absolute reference frame
Z number of grey gases modelling non-grey gas [-]

Greek
α absorptivity
δ Kronecker variable
ε emissivity
ϑ angle between $\vec{n}$ and $\vec{S}$
σ Stefan-Boltzmann constant
ξ, η, ζ transformed reference frame
ν vectorial differential elements walking on zonal contours

Subscripts
b black body
b body
f face
g volume zone
i,j dummy indexes
m,n dummy indexes
s surface zone
z index of a WSGG grey gas

Abbreviation
ANN Artificial Neural Network
DEA Direct Exchange Areas
DTM Discrete Transfer Method
FVM Finite Volume Method
LSSM Least Square Smoothing Method
MBL Mean Beam Length
RTE Radiative Transfer Equation
SNB Statistical Narrow Band Model
TEA Total Exchange Areas
WBM Wide Band Model
WSGG Weighted Sum of Gray Gases
ZM Zonal Method

INTRODUCTION

The analysis of thermal radiative loads on mechanical supports is a common problem to be faced of in the design of industrial components that have to work in a high-temperature environment. A classical example is represented by combustion chambers of turbomachines, where temperature raises up to approximately 1900 K: in such situations it is fundamental to predict the thermal radiative flux at the walls in order to properly design them, that is, checking the material properties suitability and the eventual need of a thermal barrier coat.

So far a lot of models and numerical approaches have been developed in order to perform this kind of analysis: at the present time the zonal method, firstly proposed by Hottel and Sarofim [1], is one of the most used due to its great accuracy compared to the reasonable computational time required. It is able to treat and analyse almost all kinds of media that could be involved in radiative exchange phenomena, that is, emitting/absorbing and scattering media with constant and non constant radiative properties. For the sake of simplicity in the present work media are always considered as only emitting/absorbing: this assumption enables to reduce the computational load of the method here presented, but at the same time does not cause any reduction of its applicability to scattering media too; moreover, it is a common industrial practice to neglect the soot formation in combustion chambers fueled by a methane mixture (CH_4), which implies the main cause of scattering phenomena to be avoided.

The resulting most radiating components of the medium are carbon dioxide (CO_2) and water vapour (H_2O); a model able to treat non-grey gas is then needed. According to [2], spectral methods such as the statistical narrow band model (SNB) and the wide band model (WBM) are among the most accurate, since they treat the radiation properties as varying across the spectrum bands; however, despite their accuracy, they are not widely used in industrial applications due to their greater computational demands. The Weighted-Sum-of-Gray-Gases (WSGG) model is preferred, due to its great accuracy compared to its simplicity of implementation.

Considering the classical zonal method, the evaluation of Direct Exchange Areas represents the most time consuming operations of the entire procedure. For the evaluation of radiative thermal exchanges between surface-surface, surface-volume and volume-volume zones the computation of 4^{th}, 5^{th} and 6^{th} fold integrals respectively is needed.

A lot of works have been published in literature which describes innovative techniques able to reduce the order of integrations. In the present paper an approach is introduced which relies on the idea originally proposed by Yuen [3]: he applied the MBL concept to the computation of the DEA integrals for a cubic mesh shape. The authors' proposal extends this idea to the treatment of a general mesh shape, exploiting an Artificial Neural Network trained by properly chosen deformed geometries. Obtained results show a good agreement with literature benchmarks, highlighting the possibility to perform a more efficient radiative analysis in comparison with the classical zonal method: finer zones could be used in front of the same time demand.

In the following sections an overview is provided of the techniques employed for developing a code able to compute heat radiative fluxes at the walls of a combustion chamber: in addition to describing the basics, that is, computation and usage of the Direct Exchange Areas (DEA), a brief resume is made of the Least-Square-Smoothing (LSSM) and WSGG methods used. Within the described procedure, the new proposal which this work is focused on is inserted in next paragraphs. Two benchmarks from literature are used to validate the approach. The last section is devoted to the description of an application to an industrial case.

THE ZONAL METHOD

The main assets of the zonal method are the easy implementation along with its accuracy and not demanding computations. It was originally proposed by Hottel [1], and then extended by a lot of authors (among the others, [4–9], etc.).

First of all, the enclosure involved in the radiation phenomena is subdivided into a finite number of isothermal volume (K) and surface area (N) zones with uniform radiative properties, the former regarding the medium and the latter the walls. Then an energy balance equation is applied between any two zones, which requires the preliminary computations of two different terms: the Direct Exchange Areas (DEA), representing the amount of energy which leaves a zone and arrives directly to another one, and the Total Exchange Areas (TEA), representing the amount of energy which leaves a zone and could eventually strike onto another one after being reflected by the medium which it passes through. They are both dependent on the geometry of the enclosure and on the radiative properties of the materials involved, that is, on the emittance coefficient of the surface area zones and on the absorptance coefficient of the medium. Focusing on the DEA terms, they have three different expressions depending on kinds of zones involved in the energy exchange, that is, surface-surface, surface-volume and volume-volume; they are shown in expressions (1), (2) and (3) respectively:

$$\overline{s_i s_j} = \int_{A_i}\int_{A_j} e^{-kS}\frac{\cos\vartheta_i \cos\vartheta_j}{\pi S^2} dA_j dA_i \tag{1}$$

$$\overline{g_i s_j} = \int_{V_i}\int_{A_j} e^{-kS}\frac{\cos\vartheta_j}{\pi S^2} k dA_j dV_i \tag{2}$$

$$\overline{g_i g_j} = \int_{V_i}\int_{V_j} e^{-kS}\frac{k^2}{\pi S^2} dV_j dV_i \tag{3}$$

where A_i and V_j are the area and volume values of the i-th and j-th surface and volume zones respectively, k is the absorption coefficient of the gas and e^{-ks} represents the so-called optical thickness, S is the distance between the centers of the zones involved in the energy exchange, and ϑ_i is the angle between the normal to the i-th surface zone and the direction of the line connecting the centers of the i-th surface zone to any other one.

The classical approach to the computation of these terms consists in the direct numerical integration, that is, every zones is subdivided into smaller elements and the summation of integrand terms evaluated over each of them is performed. Taking into account the surface to surface radiative exchange, the expression (1) is transformed into (4):

$$\overline{s_i s_j} = \frac{1}{\pi}\sum_{m=1}^{p}\sum_{n=1}^{t}\frac{\cos\vartheta_m \cos\vartheta_n}{r_{mn}^2} e^{-kr_{mn}} \Delta A_n \Delta A_m \tag{4}$$

where p and t are the numbers of elements discretizing the i-th and j-th zones respectively, r_{mn} is the distance between the m-th and n-th elements, ΔA_m and ΔA_n are the area values of the same m-th and n-th zones. Similar expressions can be derived for surface-volume and volume-volume DEAs too.

The computation of the above numerical discretized integrals represents the most time consuming operation of the entire zonal method. Supposing to uniformly subdivide the enclosure into cubical volume zones and square surface zones whose side length is D, it can be immediately demonstrated that the number of needed computations is O(D^4), O(D^5) and O(D^6) for the surface- surface, surface-volume and volume-volume DEAs, respectively. So far many efforts have been spent in order to reduce this time demanding discretization calculation; in the present work a new technique is proposed, which will be described in the following paragraphs.

The numerical approach to the computation of DEAs implies another problem to be faced of, that is, round- off and truncation errors prevent from obtaining the correct results. This could be extremely dangerous because, according to [2], DEAs must satisfy the conservation constraints represented by expressions (5) and (6), which are related to the surface and volume zones respectively:

$$A_i = \sum_{j=1}^{N}\overline{s_i s_j} + \sum_{k=1}^{K}\overline{s_i g_k}, \quad i = 1,2,...,N \tag{5}$$

$$4\alpha_i V_i = \sum_{j=1}^{N}\overline{g_i s_j} + \sum_{k=1}^{K}\overline{g_i g_k}, \quad i = 1,2,...,K \tag{6}$$

It is straightforward to note that an error in the resulting DEA terms means violating the conservation principle which the entire zonal method is based on. This is why a lot of effort have been spent in order to avoid this kind of problem, developing the so-called smoothing methods. The most employed ones are the Least Squares Smoothing Method (LSSM) [10] and the Generalized Lawson's Improved Smoothing Method [11]. According to [12], they are both accurate, but none of them could be said to be always better than the other; their accuracy is strictly dependent on the model used to treat the non-greyness of the medium.

In the present work the LSSM will be employed and a 4 gases WSGG model is implemented, as will be discussed in the next section.

LSSM was originally proposed in [10]: the concept which it is based on is that the conservative equations (5) and (6) have to be satisfied in such a way that the previously computed DEA terms are modified as little as possible. Defining the matrix X as composed by all the DEA terms, the lagrangian term (7) is introduced:

$$H = \sum_{i=1}^{M}\sum_{j=1}^{M}\frac{1}{2w_{ij}}\left(x'_{ij}-x_{ij}\right)^2 \tag{7}$$

where x_{ij} is the generic original DEA term previously computed, x'_{ij} is the new modified value that has to be computed, w_{ij} is one of the weights which can be arbitrarily defined depending on the desired goal, and $M = K+N$ is the dimension of the DEA term matrix, which is equal to the summation of the amounts of the surface and volume zones. Imposing the minimization of the H term along with the respect of the constraints (5) and (6) by means of the Lagrange multipliers, a linear system is obtained whose solution enables to define the new modified x'_{ij} values.

Once the DEA have been completely smoothed, the TEA terms can be directly computed, and the final linear system (8) is obtained:

$$\mathbf{T}\cdot\mathbf{h}_s = \mathbf{S}\cdot\mathbf{e}_{bs}+\mathbf{V}\cdot\mathbf{e}_{bg} \tag{8}$$

where the unknowns are the fluxes $h_s = \varepsilon_j A_j H_j$ of radiative energy leaving the surface A_j, and the other terms have the following expressions:

$$\begin{array}{rll} T_{ij} = & \frac{\delta_{ij}}{\varepsilon_{ij}} - \frac{\rho_j \overline{s_i s_j}}{\varepsilon_j A_j} & [\mathbf{T}] = N\times N \\ S_{ij} = & \overline{s_i s_j}\varepsilon_j & [\mathbf{S}] = N\times N \\ V_{ij} = & \overline{s_i g_j} & [\mathbf{V}] = N\times K \\ e_{bs_j} = & \sigma T_{s_j}^4 & [\mathbf{e}_{bs}] = N\times 1 \\ e_{bg_j} = & \sigma T_{g_j}^4 & \left[\mathbf{e}_{bg}\right] = K\times 1 \end{array} \tag{9}$$

with subscripts s and g referencing the corresponding term to surface and volume zones respectively, and with σ corresponding to Stefan-Boltzmann constant. Due to its large dimensions, the system (8) is finally solved by means of an indirect iterative solver.

WSGG model

As said above, in radiative phenomena developing inside combustion chambers of turbomachines fueled by methane a $CO_2 - H_2O$ mixture is involved. The zonal method as described in the previous section is not appropriate, since it deals with grey gases, that is, with uniformly absorbing/emitting media. This is why in industrial applications models are employed which are able to reproduce the properties of non-grey gases; among the others WSGG model [2] is widely used due to its accuracy compared to its simplicity. It assumes that the radiative properties of the non-grey gas can be approximated by a linear combination of the radiative properties relative to a certain amount of grey gases, that is, expression (10):

$$\varepsilon(T,s) = \alpha(T,s) \simeq \sum_{z=0}^{Z} a_z(T)\left(1-e^{-k_z s}\right) \tag{10}$$

where Z is the number of grey gases employed for modelling the real gas, and $a_z(T)$ are the coefficients determining the weight of the z-th grey gas. The model consists in solving a radiative problem for each of the Z grey gases independently one from the others, and then in summing together the resulting radiative fluxes.

This procedure is equivalent to solve the Radiative Transfer Equation (RTE) (11) for each grey gas, which balances the absorption and the emittance effects relative to each volume of medium:

$$\frac{dI_z}{ds} = k_z\left(I_{b_z} - I_z\right) \tag{11}$$

where $k_z I_{b_z}$ represents the radiative energy emitted from a medium whose emittance coefficient is k_z, and $k_z I_z$ is the radiative energy attenuation encountered by a beam passing through a medium whose absorptance coefficient is k_z. The computed I_z terms are then easily summed all together, in order to account for the radiating energy intensity passing through the non-grey gas.

The accuracy of the WSGG model strictly depends on the $a_z(T)$ weight coefficients; they are typically computed in an experimental way, and so they are specific for the non-grey gas they are computed for. In the past decades a lot of attention has been devoted to the computation of these coefficients for a $CO_2 - H_2O$ mixture; among the others, Truelove [13] and Smith [14]. In the present paper the more recent work of Bahador and Sunden [15] has been taken as reference, which takes account not only of the temperature dependency of the coefficients, but of their pressure dependency too.

APPLICATION OF MBL FOR DEA EVALUATION

The evaluation of Direct Exchange Areas represents a crucial task in view of computational costs for determining the radiative thermal exchanges. It involves integrals over surface and volume zones that are equal to those mentioned above in Eq. (1), (2) and (3), and that are of 4$^{\text{th}}$, 5$^{\text{th}}$ and 6$^{\text{th}}$ order, respectively.

Several efforts have been carried out in order to obtain simpler formulations for these terms and many authors have proposed different approaches to reduce the order of integration [16, 17]. For example, supposing that the exponential term e^{-kS} could be considered uniform over the zone sub-elements, by means of the introduction of the Stokes' theorem surface zone integrals $\overline{ss}$ can be converted into double line integrals strongly reducing the order of integration. Eq. (1) can be then recasted obtaining expression (12) [18]:

$$\overline{s_i s_j} = e^{-kS} \frac{1}{\pi} \oint_{C_{f,i}} \oint_{C_{f,j}} \ln\left(S_{f,i\to f,j}\right) d\vec{v}_{f,j} d\vec{v}_{f,i} \tag{12}$$

where $C_{f,i}$ and $C_{f,j}$ are the boundary contours of both surfaces involved and $S_{f,i\to f,j}$ is the distance between $d\vec{v}_{f,j}$ e $d\vec{v}_{f,i}$, which are vectorial differential elements walking on the contours. Unfortunately, about the gas volumes treatment, the $\overline{gg}$ DEA computation is more complex since the integration order grows up to the 6$^{\text{th}}$ and the attenuation term, relating to the gas absorptivity and scattering, has to be computed inside the integration process over the volume zones themselves. For all these reasons the most used technique is actually the direct numerical integration.

An interesting approach has been recently introduced in the framework of the Multiple Absorption Coefficient Zonal Method [3, 19]. It relies on the concept of Mean Beam Length in order to simplify the DEA evaluation for gas volume zones and provides a simple equation for its quick computation over simple cubic meshes.

The Mean Beam Length concept was introduced during the '50s in order to obtain a simple expression for the evaluation of the radiative emissive power of a gas volume. It is based on the formulation of the radiative flux leaving an hemisphere towards the center of its base. This approach enables a considerable simplification of the heat flux equation, Eq. (13).

$$q = E_{bg}\left(1 - e^{-kL_b}\right) \tag{13}$$

In particular, by means of this technique, the radiative flux emitted by a volume zone of a specific medium and passing through its external surfaces can be replaced by one emitted by an equivalent hemisphere, filled with the same medium, having an appropriate radius (actually the Mean Beam Length) such that the flux emitted by the hemisphere towards its base is equivalent to the real radiative thermal flux.

This method was often employed in the past for the computation of the radiative flux concerning an entirely isotherm gas volume having a simple shape; the idea is now to apply it to more general grid subvolumes of irregular shape. In this way the procedure complexity is shifted from the integral computation to the MBL evaluation.

Introducing MBL in Eq. (2, 3) leads to double area integrations over the bounding surfaces of the volume zones considered [17]. If the zones have an hexaedral shape, the following expressions (14) and (15) are obtained:

$$\overline{s_i g_j} = \sum_{f=1}^{6} \left[\left(1 - e^{-kL_{b,f}}\right) \int_{A_i} \int_{A_j} e^{-k_{ij}S} \frac{\cos\vartheta_i \cos\vartheta_j}{\pi S^2} dA_j dA_i \right] \tag{14}$$

$$\overline{g_i g_j} = \sum_{f_i=1}^{6} \sum_{f_j=1}^{6} \left[\left(1 - e^{-kL_{b,f_i}}\right) \left(1 - e^{-kL_{b,f_j}}\right) \right. \tag{15}$$
$$\left. \int_{A_i} \int_{A_j} e^{-k_{ij}S} \frac{\cos\vartheta_i \cos\vartheta_j}{\pi S^2} dA_j dA_i \right]$$

Moreover, if the transmission factor $e^{-k_{ij}S}$ is assumed to be uniform over integration intervals or sub-intervals, both Eq. (2) and (3) can be further simplified as Eq. (12). This approach enables to reduce the integral order and consequently the computational cost of DEA evaluation.

As mentioned above, a simple polynomial formulation has been proposed in literature [3] for the quick evaluation of the MBL for a cubical volume over one of its bounding surfaces, expressing its dependence from the optical thickness kD, Eq. (16). This equation has been provided for a variable optical thickness up to $kD = 8$.

$$\frac{MBL}{D} = 0.67 - 0.081kD + 0.0043(kD)^2 \tag{16}$$

Subsequently [19], it has been extended to a a wider range of optical thickness up to $kD = 25$. This extension has been carried out by means of a computer code that evaluates the MBL for a general tetragonal prism; the same relation has been upgraded to a 4$^{\text{th}}$ order polynomial obtaining:

$$\frac{MBL}{D} = a + b\,(kD) + c\,(kD)^2 + d\,(kD)^3 + e\,(kD)^4 \tag{17}$$
$$\text{with} \begin{cases} a = 6.70514 \cdot 10^{-1} \\ b = -\,8.58579 \cdot 10^{-2} \\ c = 6.60556 \cdot 10^{-3} \\ d = -\,2.51226 \cdot 10^{-4} \\ e = 3.65941 \cdot 10^{-6} \end{cases}$$

The resulting dependence of the MBL from the optical thickness according to Eq. (16) and (17) is plotted in Fig. (1). Both the results are compared with those provided by an exact and complete computation of the MBL for a cube.

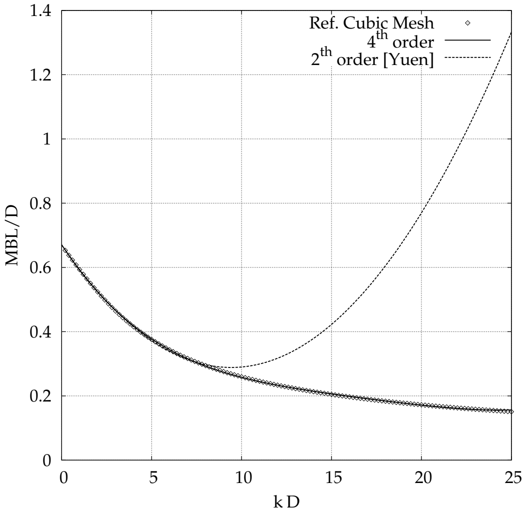

FIGURE 1. Comparison between 2^{nd} and 4^{th} order polynomial equations.

This diagram highlights the accuracy of the 4^{th} order polynomial equation in the whole range up to $kD = 25$ and also demonstrates the simplicity wherewith the value of MBL can be related as a function of optical thickness through a simple polynomial equation. Nevertheless, until now this kind of approach can be employed only for mesh made by cubic elements.

Of great interest is therefore the extension of the approximated model mentioned above to more general hexahedral geometry. The analytical approach is not feasible due to the large number of functional parameters. An Artificial Neural Network has been then applied in order to evaluate the MBL for hexaedral geometries; its application will be discussed in the following paragraphs.

ARTIFICIAL NEURAL NETWORK FOR MBL EVALUATION

Even if some simplifications can be made, the MBL evaluation is in general a time consuming process; then it could be of great interest to provide a reliable and fast approach in order to reduce the computational effort.

At the same time, the goal being to obtain the MBL of a generic shape volume, the problem which need to be faced of is the large amount of parameters that have to be handled, regarding both media properties and geometry peculiarities. To address this issue an Artificial Neural Network has been used.

For the computations described in the following, hexaedral multi-block structured meshes have been used as reference. In this way a specific subset of cell geometry shapes has been defined; in particular isothermal zones have been made coinciding with tetragonal prisms.

The first need has been to pick out the set of key geometrical parameters that are sufficient for uniquely defining the volume shape. Rather than using angle and/or edge ratios, a more general approach has been followed. A spatial transformation has been defined, that is able to modify a cubic shape expressed in the reference frame ξ, η, ζ into the actual volume zone shape expressed by coordinates x, y, z. Imposing as constraint that the surfaces of the volume zones must be planar, some degrees of freedom are removed, leading to a second order polynomial equation as shown in Eq.(18).

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \mathbf{A} \begin{pmatrix} \xi \\ \eta \\ \zeta \end{pmatrix} + \mathbf{B} \begin{pmatrix} \eta\zeta \\ \xi\zeta \\ \xi\eta \end{pmatrix} + \vec{D} \tag{18}$$

$$\text{with} \quad \mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \qquad \mathbf{B} = \begin{pmatrix} 0 & b_{12} & b_{13} \\ b_{21} & 0 & b_{23} \\ b_{31} & b_{32} & 0 \end{pmatrix}$$

Fifteen parameters result in Eq. (18) that have to be evaluated in order to clearly and univocally define the shape of the volume zone. These terms have been used as input parameter of the ANN together with the absorption coefficient k of the gas, resulting in a total amount of 16 input data.

ANN training The ANN needs to overcome a training phase through which it can be adapted to the studied problem. Since the number of considered parameters is large, in order to narrow the size of the training database a fractional factorial design has been employed for creating a set of cases which will be used for feeding the network. In Tab. (1) the two-level Fractional Factorial approach and the corresponding two-level Full Factorial one are compared in terms of number of computations. Considering the cube shape as starting reference geometry, the entire set of fifteen geometrical parameters presented in Eq. (18) have been set at the following two values: $\pm 50\%$ for the elements of the matrix A main diagonal, $\pm 10\%$ for the other terms.

In this way it has been possible to reduce the complete set of training geometrical shapes to just 128 cases; they have been employed as training inputs for the ANN along with 30 different values of k, which have been sampled in the range $0 \div 30$ not uniformly, but, according to Fig. (1), thickening sample points in the range $0 \div 8$ as the gradient is larger.

In order to check the reliability of the resulting trained ANN, the same curve shown in Fig. (1) has been reproduced. It has to be noticed that the training database does not comprehend the

TABLE 1. Comparison between two-level Full and Fractional Factorial design

Approach	Number of tests
Full factorial 2^{15}	32768
Fractional factorial 2^{15-8}	128

cubic shape which the curve above is referred to, and, for this reason, this can be considered a good test case to verify the convergence of the training process and also to ensure that the network has not been over-trained. The obtained results are shown in Fig. (2) referring to two differently structured ANNs: the first one, identified as ANN 1L, having a single hidden layer of 30 neurons, and the second one, called ANN 2L, having two different hidden layers of 8 and 9 neurons, respectively. It can be observed that the two-layer ANN has a better agreement to the reference values with respect to the single- layer one, especially for small values of optical thickness kD. In the following applications the two hidden layer ANN will be then employed.

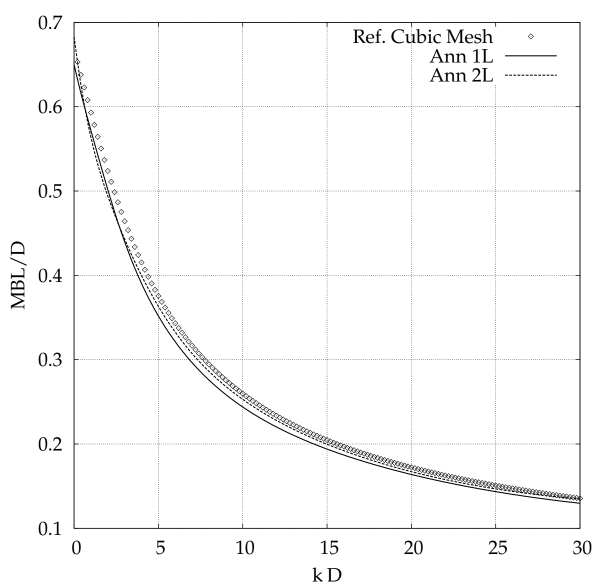

FIGURE 2. Comparison between reference values and ANN evaluations of MBL for cubic volume zones.

APPLICATIONS

In order to validate and evaluate robustness and reliability of the proposed model, the obtained results have been compared with those obtained by other authors on test cases available in the open literature. Two different analysis have been carried out which will be described in the following: the first one dealing with a grey gas having uniform temperature and the second one dealing with a non-grey gas at uniform temperature.

Grey gas test case

The method described in the previous paragraphs has been firstly applied to a classical test case for radiative heat exchange. It has been chosen among those presented in the framework of the project RADIARE [20] as benchmark of different computational methods for radiative heat transfer evaluation. It is characterized by a furnace having a simple geometry with dimensions 3m x 1m x 1m. The wall temperature is uniform $T_w = 1273K$; the bounding walls are grey-diffuse and analysis are performed for two different wall emissivities: $e_w = 0.5$ and $e_w = 0.8$. The gas is assumed as perfectly grey, having uniform temperature equal to $T_g = 1773K$ and the absorption coefficient equal to $k = 0.1m^{-1}$. A grid dependency analysis has been carried out highlighting the reliability and robustness of the proposed method for coarse computational grids too. In the performed application a grid 15x5x5 has been employed.

In Fig. (3) and (4) the obtained results are shown which correspond to the net radiative heat fluxes along the middle line of the upper surface. Thermal loads at the walls have been compared with those obtained by [20] by means of the Discrete Transfer Method (DTM) and the Finite Volume Method (FVM).

From these comparisons it appears that, for both wall emissivities, the radiative heat flux values evaluated by means of the joined MBL zonal method are in good agreement with those obtained by means of standard methods.

WSGG and uniform temperature test case

The second test case that has been investigated is related to a furnace with dimensions 2m x 2m x 4m. The media is steam at uniform temperature $T_g = 1000K$ and its non-greyness behaviour has been modelled by means of the WSGG approach described in [14]. The bounding walls have been considered as black, that is, having emissivity $e_w = 1$, and have temperature $T_w = 300K$. The joined MBL-zonal method results have been compared with reference data obtained by Liu [21] through a DTM model coupled with a Narrow Band Model for the characterization of the non-grey gas spectral properties behaviour.

In this case as well, a grid dependency analysis has been carried out in order to highlight the accuracy and reliability of the proposed model when dealing with coarse grids. It has to be noticed that the mesh used for this computation is not the uniform cubic one, but it is that shown in Fig. (5). The developed ANN

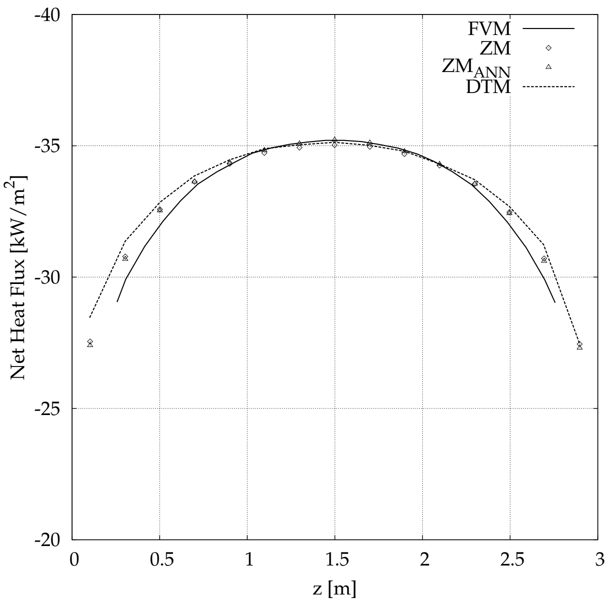

FIGURE 3. Net Radiative Heat Flux Comparisons between FVM, DTM, classical ZM and the proposed ZM_{ANN} for $\varepsilon = 0.5$.

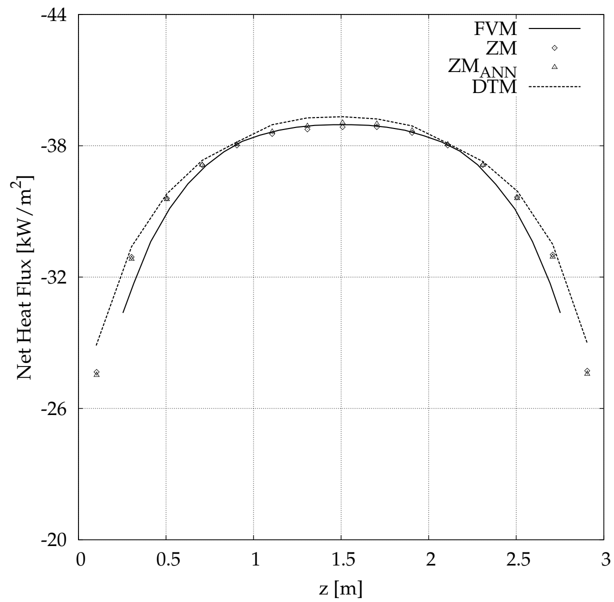

FIGURE 4. Net Radiative Heat Flux Comparisons between FVM, DTM, classical ZM and the proposed ZM_{ANN} for $\varepsilon = 0.8$.

is then applied for evaluating the MBL of each different volume zone.

The computed values have been compared with the results provided in literature by other authors [21–23] on the same test case and on a uniform cubic mesh. In Fig. (6), the radiative thermal loads related to the line with coordinates $x = 1, y = 2$ developing along the first half of z axis have been reported. The agreement with the other results is good. Some differences are however present, but most likely they are due to the lack of accuracy of the WSGG model [14] with respect to the Narrow Band Model: the latter is based on a spectral method which is implicitly more precise but computationally heavier and so it is not usually employed in industrial computations.

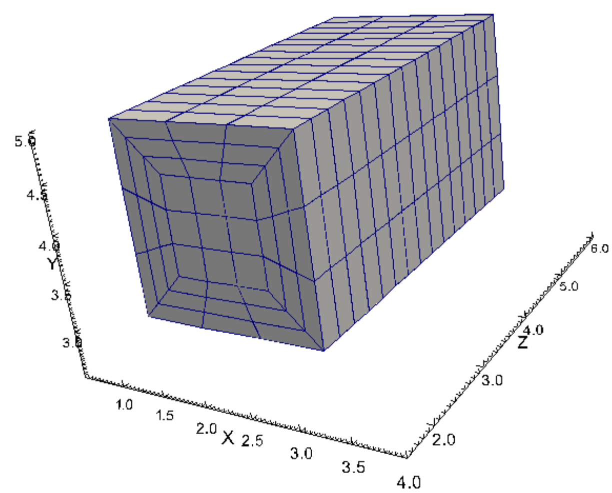

FIGURE 5. Non uniform hexaedral mesh used for the WSGG and uniform temperature test case.

INDUSTRIAL APPLICATION

As practical example a comparison between the classical zonal method and the technique presented in this work is made performing a thermal radiative analysis of the entire combustor complex of the AE94.2 gas turbine. The domain (Fig. 7) includes the flame tube, where the fuel enters and the flame is developed, and the entire mixing chamber, where the burned gases are mixed with the air before entering the first turbine stage. In the recently built combustion chambers, walls are ceramic coated: as temperature increases on the walls, radiative flux decreases between gas mixture and structure. This is why in the flame tube radiative flux is about a fifth of the convective one, but, in the mixing chamber, where there is not any protection layer, radiation effect raises to a half of the global thermal load. The volume zone subdivision is made in order to reproduce the 1D temperature profile obtained by a proper post-processing of a previous CFD-reactive computation. On the other side the surface area zone subdivision is made in order to reproduce the environment surrounding the combustor complex: at the outlet there is not a real wall, so an ideal surface is placed with unitary emittance and a supposed temperature is imposed in order to simulate the thermal load due

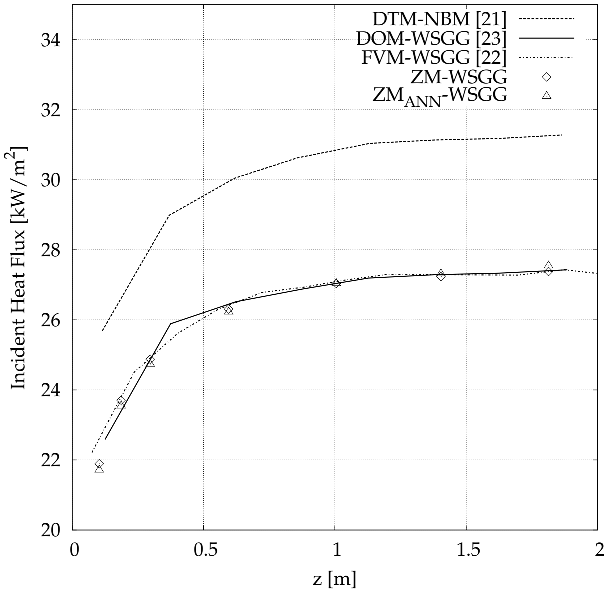

FIGURE 6. Net Radiative Heat Flux Comparison between literature results and those obtained by means of the joined MBL-zonal method on a non uniform hexaedral mesh.

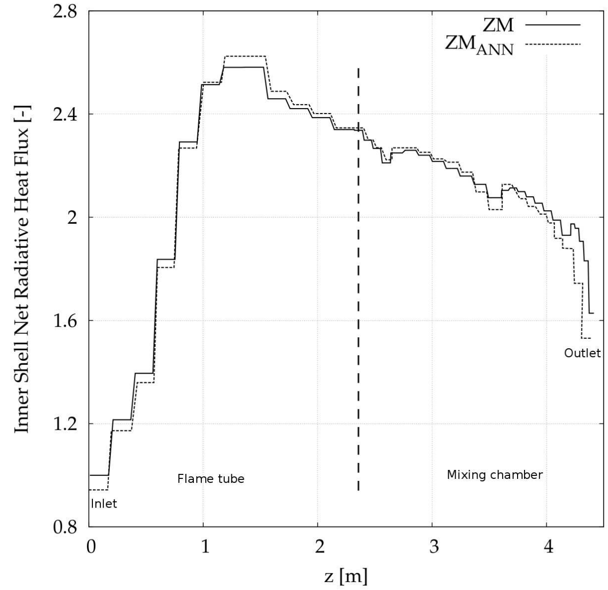

FIGURE 8. Net Radiative Heat Flux on the Inner Shell of the AE94.2 Combustor Symmetry Section: comparison between results obtained by means of the classical ZM and the ZM$_{ANN}$.

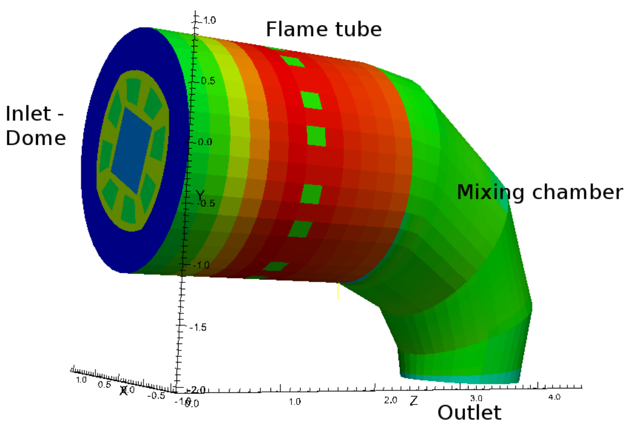

FIGURE 7. Domain Geometry and Dome Zones Subdivision of the Combustor Complex referred to AE94.2 Turbomachine.

to the outside gases; the shell walls are subdivided in such a way that the zone distribution takes into account the external cooling system: based on cooling system structure, fed by secondary air system, zones are thickened where temperature gradients are greater. The dome is subdivided in order to consider that the place which burners are inserted into are not cooled by any external system; this is why an annular subdivision is applied (Fig. 7). Since the gas mixture is made of carbon-dioxide CO_2 and water-vapour H_2O, the WSGG coefficients evaluated in [15] are employed; the average total pressure into the combustion chamber reaches $1.2MPa$, so the coefficients corresponding to $1.1MPa$ total pressure are chosen as they are the nearest available values.

Both the methods employ the same mesh, that is, a mesh made by $370 \times 40 \times 15$ elements along curvilinear abscissa, radial and tangential directions, respectively. The results are shown in Fig. (8) and (9): net radiative heat fluxes entering the inner and outer shells are plotted along the symmetry plane. Due to confidentiality reasons, the resulting fluxes are normalized with the flux entering the inner and outer shells at the initial abscissa which is computed by means of the classical zonal method.

As can be seen, in both cases values are in good agreement, with the fluxes computed by the proposed method being a little more conservative in the most heated section, that is, after the half of the flame tube, where the flame has its maximum extension. On the other side, a little less conservative results compared to those obtained by the classical zonal method are highlighted near the dome and near the outlet section.

Beyond the good agreement in term of fluxes value, what is of note is the great decrease of computational time demands. As a matter of fact, if it is supposed to have a perfectly cubic uniform mesh, in front of an amount of operations which is $O\left(D^6\right)$ for the classical approach, in the proposed method the amount decreases to be $O\left(D^2\right)$. In comparison with the 8 hours needed by the classical approach, the proposed one lasts about 2 hours when applied to the same mesh.

However the authors' approach shifts the load from the DEA

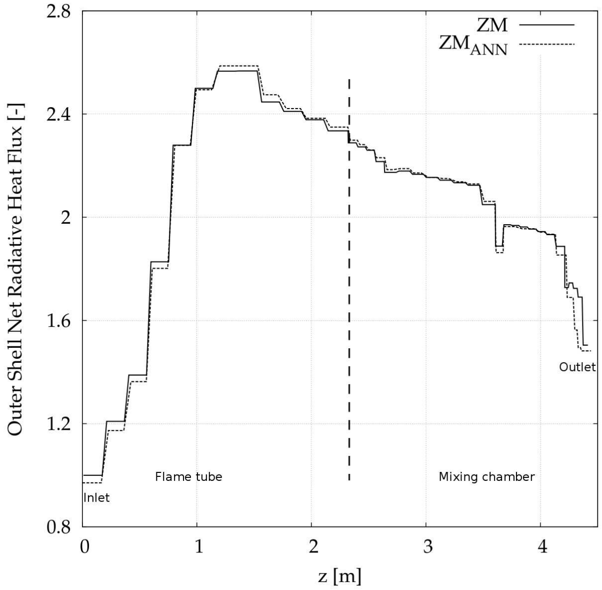

FIGURE 9. Net Radiative Heat Flux on the Outer Shell of the AE94.2 Combustor Symmetry Section: comparison between results obtained by means of the classical ZM and of the joined MBL-ZM.

evaluation to the ANN training. This is due to the fact that a precise mathematical integration has to be performed in order to organise the database for setting the ANN: lightened by means of the fractional factorial method, for the actual computation the time expense is about 1 hour when distributed on 20 CPUs.

In practice this enables to perform a more detailed radiative analysis: a refinement of the zones is enabled, maintaining the computational demand as large as that required by a classical zonal method approach running on a coarser zone subdivision.

CONCLUSIONS

Thermal radiation is typically one of the most important phenomenon to be taken into account in the evaluation of combustor walls thermal loads due to the high temperatures arising around them. An industrial classical approach to study radiative phenomena is based on the so called Zonal Method, originally developed by Hottel and Sarofim [1] and actually widely employed in the industrial environment. The computation of DEA terms is the most time consuming operation of the entire method, which consists in performing a direct integration of 4^{th}, 5^{th}, 6^{th} fold integrals on several volume and surface elements.

In order to reduce the demanded time a new technique is proposed which relies on the approach originally presented in Yuen [3]: based on the classic Mean Beam Length concept, he found a correlation which enables the evaluation of the MBL corresponding to uniform cubic mesh elements. In the present work this correlation has been extended to generic 3D hexaedral mesh elements by means of an artificial neural network. In this way, by means of well known geometrical techniques (Walton [18]), it is enabled the reduction of the integrals folds from 6 to 2 for the volume-volume DEAs, from 5 to 2 for surface-volume DEAs and from 4 to 2 for surface-surface DEAs.

The proposed model has been compared with 3D benchmark test cases available in literature: the accuracy has been tested against the results of DTM, FVM and classical Zonal Method approach, all of them with great agreement. Moreover a practical application is shown. The geometry of the AE94.2 gas turbine combustor complex has been considered; the gas mixture has been treated as a non-grey gas using the well known Weighted Sum of Gray Gases (WSGG) model. In comparison with the classical zonal method approach, by exploiting the greater efficiency of the described method, the possibility of a refinement of the zone subdivision has been highlighted, which would enable a more accurate evaluation of radiative thermal loads at the same computational cost.

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