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OPTIMAL OPERATION OF A GAS TURBINE COGENERATION PLANT IN CONSIDERATION OF EQUIPMENT MINIMUM UP AND DOWN TIMES

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ABSTRACT

It has become important for operators to determine operational strategies of energy supply plants appropriately corresponding to energy demands varying with season and time from the viewpoints of economics, energy saving, and recently reduction in CO_2 emission. Especially, cogeneration plants produce heat and power simultaneously, which increases alternatives for operational strategies. This makes it more important for operators to determine operational strategies of cogeneration plants appropriately. In this paper, for the purpose of assisting operators or operating plants automatically, an optimal operational planning method based on the mixed-integer linear programming is developed to determine the operational strategy of equipment so as to minimize the operational cost, in consideration of equipment minimum up and down times for each piece of equipment to be operated with appropriate numbers of startups and shutdowns. In the numerical study, the proposed method is applied to the daily operational planning of a gas turbine cogeneration plant for district energy supply. It is clarified how the constraints for minimum up and down times affect the operational strategy and cost. Through the study, the validity and effectiveness of the proposed method is ascertained.

NOMENCLATURE

a : slope for input-output relationship of equipment [MWh/m^3]
b : intercept for input-output relationship of equipment [MWh/h]
E : electric power [MWh/h]
F : natural gas flow rate [m^3/h]
f : hourly operational cost [yen/h]
$\boldsymbol{h}$: constraint function vector
J : objective function, or daily operational cost [yen/d]
K : number of sampling time intervals
k : index for sampling time intervals
N : number of binary variables
P : degree with which minimum up or down time is not satisfied
Q : heat flow rate [MWh/h]
p, q, r, s : continuous variables for replacement [h]
T : operation period [h]
t : continuous up or down time [h]
Δt : sampling time interval [h]
$\boldsymbol{x}$: continuous variable vector for energy flow rates [MWh/h, t/h, m^3/h]
$\boldsymbol{Y}$: initial values for continuous variable vector $\boldsymbol{y}$ [h]
$\boldsymbol{y}$: continuous variable vector for continuous up and down times [h]
$\boldsymbol{Z}$: initial values for binary variable vector $\boldsymbol{z}$
$\boldsymbol{z}$: binary variable vector for on/off status of equipment
δ : on/off status of equipment
φ : unit cost for energy charge of input energy [yen/kWh, yen/m^3]
ψ : unit cost for penalty [yen/h]
$\underline{(\ )}$: lower limit
$\overline{(\ )}$: upper limit
$\breve{(\ )}$: upper bound

Subscripts

down : continuous down time
elec : electricity
gas : natural gas

k : index for sampling time intervals
m : index for pieces of equipment
up : continuous up time
water : feedwater

Superscripts
a : auxiliary machinery
e : exhaust heat

Abbreviations for Equipment (Subscripts)
AR : steam absorption refrigerator
CT : cooling tower
GB : gas-fired auxiliary boiler
GT : gas turbine generator
P : pump for supplying cold water
TR : electric compression refrigerator
WB : waste heat recovery boiler

INTRODUCTION
As one of the technologies for efficient energy utilization, cogeneration plants have the potential of high economic and energy saving characteristics, and have been extensively installed into districts and buildings. In order to utilize this potential, not only installation but also design and operation are considered to be important issues. It has become important for operators to determine operational strategies of energy supply plants appropriately corresponding to energy demands varying with season and time from the viewpoints of economics, energy saving, and recently reduction in CO_2 emission. Especially, cogeneration plants produce heat and power simultaneously, which increases alternatives for operational strategies. This makes it more important for operators to determine operational strategies of cogeneration plants appropriately.

For the operational planning of cogeneration plants, a method based on the mixed-integer linear programming (MILP) has been proposed to operate the plants rationally so that they attain the minimum operational cost for heat and power supply, and its effectiveness has been ascertained by comparing this cost-minimizing strategy with conventional electric-/thermal-following ones [1]. In this method, however, dynamic characteristics of equipment, by which states such as mass flow rates, pressures, and temperatures of equipment change transiently, are neglected, and the operational strategy of equipment is determined statically and independently at each sampling time interval which is set to take account of variations in energy demands. Namely, it is not considered that energy inputs of equipment at a sampling time interval affect not only energy outputs at the same one but also energy outputs at subsequent ones, and that there is a coupling of operational strategy at different sampling time intervals. Therefore, the strategy obtained by this method does not necessarily mean the optimal one for a longer period if dynamic characteristics of equipment are taken into account. Moreover, the method tends to make transition of on/off status of equipment sensitive even to small variations in energy demands, which is one of the problems to be solved for implementing the method into a real-time operational advisory system. This is also an important problem from the viewpoints of deterioration and life as well as their relevant costs of equipment [2]. However, it is very difficult to take account of dynamic characteristics of equipment, because it makes the optimal operational planning problem excessively complex and large-scale.

To cope with the aforementioned drawbacks of the conventional optimal operational planning method, an alternative one has been proposed by incorporating equipment startup/shutdown cost as part of the operational cost to be minimized. When a piece of equipment is started or shut down, extra energy is consumed until it attains a steady state. The equipment startup/shutdown cost is due to this extra energy consumption. Although this method also neglects dynamic characteristics of equipment, it considers a coupling of operational strategy at all the sampling time intervals over a period considered, and makes transition of on/off status of equipment less sensitive. Therefore, the method is considered one of the simple and effective approaches for a real-time operational advisory system. However, the corresponding optimal operational planning problem is formulated as a large-scale MILP one, and it generally takes excessive computation time to solve it. To solve it efficiently, a solution method has been proposed, where the number of candidates for on/off status of equipment is reduced using information on upper and lower bounds for the optimal value of operational cost [3]. However, the information is derived only from local sampling time intervals, and this may limit the effectiveness of the method. Another solution method based on the dynamic programming and MILP has been proposed, and has been applied to the determination of the operational strategy of a gas turbine cogeneration plant [4]. In this method, to solve the problem more efficiently, the number of candidates for on/off status of equipment is reduced using global information on upper and lower bounds for the optimal value of operational cost over a period obtained by the dynamic programming.

To cope with the aforementioned drawbacks of the conventional optimal operational planning method, another one can also be considered by incorporating equipment minimum up and down times in place of equipment startup/shutdown cost. This method makes each piece of equipment be operational and stopping continuously during the times longer or equal to the specified minimum up and down times, respectively. The method is widely employed for unit commitment of power generation units [5]. Although constraints for minimum up and down times can be formulated explicitly as quadratic equations [6], they are often treated implicitly. For example, although a solution method based on the Lagrange relaxation and dynamic programming has been proposed, it needs a relatively complex solution procedure and leads only to a feasible solution due to a duality gap, which means that there exists a difference between upper and lower bounds for the objective function [7].

In this paper, an optimal operational planning method is developed in consideration of equipment minimum up and down times for each piece of equipment to be operated with appropriate numbers of startups and shutdowns. The constraints for minimum up and down times are treated as quadratic equations explicitly, and are transformed into linear equations by adding new variables and constraints, which leads to the optimal solution by the MILP. In addition, a penalty method is also introduced so that the constraints for minimum up and down times are relaxed in case that they are not perfectly satisfied even by any combinations of on/off status of equipment.

First, a summary of the optimal operational planning problem is described. Then, a formulation of the equipment minimum up and down times is presented, which is followed by a solution method and an extension. Finally, the method is applied to the daily operational planning of a gas turbine cogeneration plant for district energy supply, and it is investigated how the consideration of the equipment minimum up and down times affects the operational strategy and cost.

OPTIMAL OPERATIONAL PLANNING

Basic Formulation

The optimal operational planning problem for an energy supply plant considered in this paper is such that the operational strategy of constituent equipment is determined so as to minimize the operational cost and to satisfy energy demands estimated over the period T. The current time is considered as the initial time, at which the operational status of equipment is assumed to be known. The time after T from the initial time is considered as the terminal time, at which the operational status is assumed to be without any constraints. In applying this method to a real-time operational advisory system, the energy demand prediction and operational planning should be repeated as time passes.

To formulate the optimal operational planning problem, the period T is discretized into the K identical sampling time intervals of Δt, i.e.,

$$\Delta t = T / K \tag{1}$$

In the following formulation, a quantity at the kth sampling time interval is designated by the argument k ($k = 0, 1, \cdots, K$). A quantity at the initial time is designated by $k = 0$.

As the operational strategy, continuous and binary variable vectors, $\boldsymbol{x}(k)$ and $\boldsymbol{z}(k)$, are used to express energy flow rates and on/off status of equipment, respectively. By using these variables, performance characteristics of equipment and energy balance relationships are expressed as follows:

$$\boldsymbol{h}_k(\boldsymbol{x}(k), \boldsymbol{z}(k)) = \boldsymbol{0} \quad (k = 1, 2, \cdots, K) \tag{2}$$

where $\boldsymbol{h}_k$ is the constraint function vector. Performance characteristics of equipment are expressed approximately by linear equations with respect to $\boldsymbol{x}(k)$ and $\boldsymbol{z}(k)$, and $\boldsymbol{h}_k$ becomes a linear function with respect to $\boldsymbol{x}(k)$ and $\boldsymbol{z}(k)$. The hourly operational cost can be expressed by a linear equation with respect only to $\boldsymbol{x}(k)$ as $f_k(\boldsymbol{x}(k))$. For concrete forms of f_k and $\boldsymbol{h}_k$, refer to the previous paper [1] and the appendix.

Formulation for Minimum Up and Down Times

For a piece of equipment, the continuous up time $t_{\mathrm{up}}(k)$ at the sampling time interval k is calculated as

$$t_{\mathrm{up}}(k) = \{t_{\mathrm{up}}(k-1) + \Delta t\}\delta(k) \quad (k = 1, 2, \cdots, K) \tag{3}$$

where $\delta(k)$ is the binary variable for on/off status of equipment. This equation means that if $\delta(k) = 1$, then the continuous up time $t_{\mathrm{up}}(k)$ at the sampling time interval k is calculated by adding the sampling time interval Δt to the continuous up time $t_{\mathrm{up}}(k-1)$ at the previous sampling time interval $k-1$, and that else if $\delta(k) = 0$, then the continuous up time $t_{\mathrm{up}}(k)$ at the sampling time interval k is reset at zero. By using this continuous up time, the constraint for the minimum up time is expressed as

$$\{t_{\mathrm{up}}(k-1) - \underline{t}_{\mathrm{up}}\}\{\delta(k-1) - \delta(k)\} \geq 0 \quad (k = 1, 2, \cdots, K) \tag{4}$$

where $\underline{t}_{\mathrm{up}}$ is the minimum up time. This equation means that the piece of equipment can be shut down, or $\delta(k-1) = 1$ and $\delta(k) = 0$ only if $t_{\mathrm{up}}(k-1) \geq \underline{t}_{\mathrm{up}}$.

Similarly, for a piece of equipment, the continuous down time $t_{\mathrm{down}}(k)$ at the sampling time interval k is calculated as

$$t_{\mathrm{down}}(k) = \{t_{\mathrm{down}}(k-1) + \Delta t\}\{1 - \delta(k)\} \quad (k = 1, 2, \cdots, K) \tag{5}$$

This equation means that if $\delta(k) = 0$, then the continuous down time $t_{\mathrm{down}}(k)$ at the sampling time interval k is calculated by adding the sampling time interval Δt to the continuous down time $t_{\mathrm{down}}(k-1)$ at the previous sampling time interval $k-1$, and that else if $\delta(k) = 1$, then the continuous down time $t_{\mathrm{down}}(k)$ at the sampling time interval k is reset at zero. By using this continuous down time, the constraint for the minimum down time is expressed as

$$\{t_{\mathrm{down}}(k-1) - \underline{t}_{\mathrm{down}}\}\{\delta(k-1) - \delta(k)\} \leq 0 \quad (k = 1, 2, \cdots, K) \tag{6}$$

where $\underline{t}_{\mathrm{down}}$ is the minimum down time. This equation means that the piece of equipment can be started up, or $\delta(k-1) = 0$ and $\delta(k) = 1$ only if $t_{\mathrm{down}}(k-1) \geq \underline{t}_{\mathrm{down}}$.

These constraints for minimum up and down times are used with the initial values of $\delta(k)$, $t_{\mathrm{up}}(k)$, and $t_{\mathrm{down}}(k)$, or $\delta(0)$, $t_{\mathrm{up}}(0)$, and $t_{\mathrm{down}}(0)$.

Equations (3) to (6) are applied to the pieces of equipment whose minimum up and down times are considered, and are added to the basic constraints of Eq. (2). As a result, all the constraints are expressed as follows:

$$\boldsymbol{h}_k'(\boldsymbol{x}(k), \boldsymbol{y}(k-1), \boldsymbol{y}(k), z(k-1), z(k)) = \boldsymbol{0} \quad (k = 1, 2, \cdots, K) \tag{7}$$

where $\boldsymbol{y}(k)$ is the continuous variable vector composed of $t_{\mathrm{up}}(k)$ and $t_{\mathrm{down}}(k)$ of the pieces of equipment whose minimum up and down times are considered, and $\boldsymbol{h}_k'$ is the renewed constraint function vector, and is a nonlinear or quadratic function with respect to $\boldsymbol{y}(k-1)$, $z(k-1)$, and $z(k)$.

Solution Method

The above formulation of the optimal operational planning problem results in the following mixed-integer nonlinear programming (MINLP) one:

$$\left.\begin{array}{ll}
\text{min.} & J = \sum_{k=1}^{K} f_k(\boldsymbol{x}(k))\Delta t \\
\text{sub. to} & \boldsymbol{h}_k'(\boldsymbol{x}(k), \boldsymbol{y}(k-1), \boldsymbol{y}(k), z(k-1), z(k)) = \boldsymbol{0} \\
 & \qquad (k = 1, 2, \cdots, K) \\
 & \boldsymbol{y}(0) = \boldsymbol{Y} \\
 & z(0) = \boldsymbol{Z} \\
 & \boldsymbol{x}(k) \geq \boldsymbol{0} \quad (k = 1, 2, \cdots, K) \\
 & \boldsymbol{y}(k) \geq \boldsymbol{0} \quad (k = 0, 1, \cdots, K) \\
 & z(k) \in \{0, 1\}^N \quad (k = 0, 1, \cdots, K)
\end{array}\right\} \tag{8}$$

where $\boldsymbol{Y}$ and $\boldsymbol{Z}$ are the vectors for the initial values of $\boldsymbol{y}(k)$ and $z(k)$, respectively, at the sampling time interval $k = 0$. The seventh equation in Eq. (8) means that $z(k)$ is a binary variable vector with a dimension of N. This problem includes nonlinear equations, and can become large-scale as the numbers of N and K increase. Therefore, it is difficult to solve the problem directly.

In this paper, a solution method is proposed by transforming the MINLP problem into a MILP one. For this purpose, the nonlinear terms due to the products of $t_{\mathrm{up}}(k-1)$ and $\delta(k-1)$, $t_{\mathrm{up}}(k-1)$ and $\delta(k)$, $t_{\mathrm{down}}(k-1)$ and $\delta(k-1)$, and $t_{\mathrm{down}}(k-1)$ and $\delta(k)$ in Eqs. (3) to (6) are replaced by the nonnegative continuous variables $p(k)$, $q(k)$, $r(k)$, and $s(k)$, respectively, as follows:

$$\left.\begin{array}{l}
p(k) = t_{\mathrm{up}}(k-1)\delta(k-1) \\
q(k) = t_{\mathrm{up}}(k-1)\delta(k) \\
r(k) = t_{\mathrm{down}}(k-1)\delta(k-1) \\
s(k) = t_{\mathrm{down}}(k-1)\delta(k)
\end{array}\right\} (k = 1, 2, \cdots, K) \tag{9}$$

This replacement makes Eqs. (3) to (6) the following linear equations:

$$\left.\begin{array}{l}
t_{\mathrm{up}}(k) - q(k) - \Delta t\delta(k) = 0 \\
p(k) - q(k) - \underline{t}_{\mathrm{up}}\{\delta(k-1) - \delta(k)\} \geq 0 \\
t_{\mathrm{down}}(k) - t_{\mathrm{down}}(k-1) + s(k) - \Delta t\{1 - \delta(k)\} = 0 \\
r(k) - s(k) - \underline{t}_{\mathrm{down}}\{\delta(k-1) - \delta(k)\} \leq 0
\end{array}\right\} \quad (k = 1, 2, \cdots, K) \tag{10}$$

In addition, since Eq. (9) is still nonlinear, the following equations are employed in place of it:

$$\left.\begin{array}{l}
\left.\begin{array}{l}
p(k) \leq \tilde{t}_{\mathrm{up}}\delta(k-1) \\
t_{\mathrm{up}}(k-1) + \tilde{t}_{\mathrm{up}}\{\delta(k-1) - 1\} \leq p(k) \leq t_{\mathrm{up}}(k-1)
\end{array}\right\} \\
\left.\begin{array}{l}
q(k) \leq \tilde{t}_{\mathrm{up}}\delta(k) \\
t_{\mathrm{up}}(k-1) + \tilde{t}_{\mathrm{up}}\{\delta(k) - 1\} \leq q(k) \leq t_{\mathrm{up}}(k-1)
\end{array}\right\} \\
\left.\begin{array}{l}
r(k) \leq \tilde{t}_{\mathrm{down}}\delta(k-1) \\
t_{\mathrm{down}}(k-1) + \tilde{t}_{\mathrm{down}}\{\delta(k-1) - 1\} \leq r(k) \leq t_{\mathrm{down}}(k-1)
\end{array}\right\} \\
\left.\begin{array}{l}
s(k) \leq \tilde{t}_{\mathrm{down}}\delta(k) \\
t_{\mathrm{down}}(k-1) + \tilde{t}_{\mathrm{down}}\{\delta(k) - 1\} \leq s(k) \leq t_{\mathrm{down}}(k-1)
\end{array}\right\}
\end{array}\right\} \quad (k = 1, 2, \cdots, K) \tag{11}$$

where $\tilde{t}_{\mathrm{up}}$ and $\tilde{t}_{\mathrm{down}}$ are upper bounds for $t_{\mathrm{up}}(k-1)$ and $t_{\mathrm{down}}(k-1)$, respectively. The constraints of Eq. (11) are because, for example, the first and second equations in Eq. (11) means that if $\delta(k-1) = 0$, then $p(k) = 0$, and that else if $\delta(k-1) = 1$, then $p(k) = t_{\mathrm{up}}(k-1)$, which makes the first equation in Eq. (9) valid indirectly. This procedure can linearize the nonlinear terms without any approximations and transform the optimal operational planning problem into the following MILP one:

$$\left.\begin{array}{ll}
\text{min.} & J = \sum_{k=1}^{K} f_k(\boldsymbol{x}(k))\Delta t \\
\text{sub. to} & \boldsymbol{h}_k''(\boldsymbol{x}(k), \boldsymbol{x}'(k), \boldsymbol{y}(k-1), \boldsymbol{y}(k), z(k-1), z(k)) = \boldsymbol{0} \\
 & \qquad (k = 1, 2, \cdots, K) \\
 & \boldsymbol{y}(0) = \boldsymbol{Y} \\
 & z(0) = \boldsymbol{Z} \\
 & \boldsymbol{x}(k) \geq \boldsymbol{0} \quad (k = 1, 2, \cdots, K) \\
 & \boldsymbol{x}'(k) \geq \boldsymbol{0} \quad (k = 1, 2, \cdots, K) \\
 & \boldsymbol{y}(k) \geq \boldsymbol{0} \quad (k = 0, 1, \cdots, K) \\
 & z(k) \in \{0, 1\}^N \quad (k = 0, 1, \cdots, K)
\end{array}\right\} \tag{12}$$

where $\boldsymbol{x}'(k)$ is the continuous variable vector composed of $p(k)$, $q(k)$, $r(k)$, and $s(k)$ for the pieces of equipment whose minimum up and down times are considered, and $\boldsymbol{h}_k''$ is the renewed constraint function vector, and is a linear function with respect to all the variables.

The resultant MILP problem can be solved using a commercial solver, for example, GAMS/CPLEX [8].

Extension

With increases in $\underline{t}_{\mathrm{up}}$ and $\underline{t}_{\mathrm{down}}$, the constraints for minimum up and down times become severe and difficult to be satisfied. In such a case, the constraints should be relaxed to obtain a feasible solution. Here, the constraints for minimum up and down times are relaxed by introducing new variables

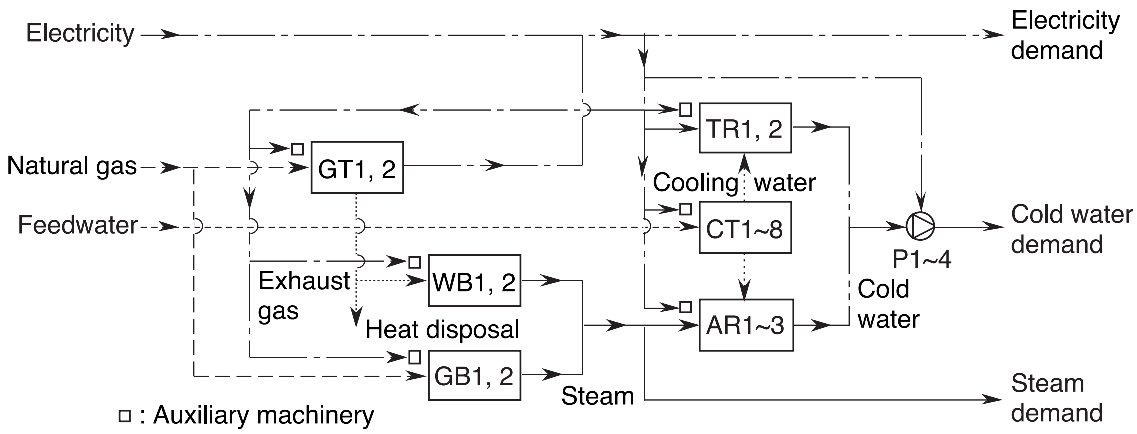


Fig. 1 Configuration of gas turbine cogeneration plant for district energy supply

into Eqs. (4) and (6). Namely, Eqs. (4) and (6) are replaced by

$$\{t_{\mathrm{up}}(k-1) - \underline{t}_{\mathrm{up}}\}\{\delta(k-1) - \delta(k)\} \geq -P_{\mathrm{up}}(k) \quad (k = 1, 2, \cdots, K) \qquad (13)$$

and

$$\{t_{\mathrm{down}}(k-1) - \underline{t}_{\mathrm{down}}\}\{\delta(k-1) - \delta(k)\} \leq P_{\mathrm{down}}(k) \quad (k = 1, 2, \cdots, K) \qquad (14)$$

respectively, where $P_{\mathrm{up}}(k)$ and $P_{\mathrm{down}}(k)$ are the variables expressing the degrees with which Eqs. (4) and (6) are not satisfied, respectively. With this replacement, the objective function of the first equation in Eqs. (8) and (12) is replaced by

$$J' = \sum_{k=1}^{K} \{f_k(\boldsymbol{x}(k)) + \psi_{\mathrm{up}}P_{\mathrm{up}}(k) + \psi_{\mathrm{down}}P_{\mathrm{down}}(k)\}\Delta t \qquad (15)$$

where ψ_{up} and ψ_{down} are the unit costs for penalty corresponding to $P_{\mathrm{up}}(k)$ and $P_{\mathrm{down}}(k)$, respectively, and their values should be large enough so that $P_{\mathrm{up}}(k)$ and $P_{\mathrm{down}}(k)$ become positive only if Eqs. (4) and (6) are not satisfied, respectively.

NUMERICAL STUDY

Plant Configuration

Figure 1 shows the configuration of a gas turbine cogeneration plant for district energy supply considered in this numerical study. This plant is composed of two gas turbine generators (GT1, 2), two waste heat recovery boilers (WB1, 2), two gas-fired auxiliary boilers (GB1, 2), two electric compression refrigerators (TR1, 2), three steam absorption refrigerators (AR1~3), eight cooling towers (CT1~8), and four pumps (P1~4). Each block means that multiple units are connected in parallel, except that one of the gas turbine generators is connected with one of the waste heat recovery boilers in series.

In this plant, electricity is supplied to users by operating gas turbine generators and by purchasing electricity from an outside electric power company. Electricity is also used to drive electric compression refrigerators, cooling towers, pumps, and other auxiliary machinery in the plant. Waste heat of exhaust gas generated from gas turbines is recovered by waste heat recovery boilers, and steam generated is used for thermal energy supply. Surplus waste heat is disposed of through exhaust gas dumpers. Shortage of steam is supplemented by gas-fired auxiliary boilers. Both electric compression and steam absorption refrigerators are installed to supply cold water for space cooling. Steam is used for space heating and other miscellaneous purposes.

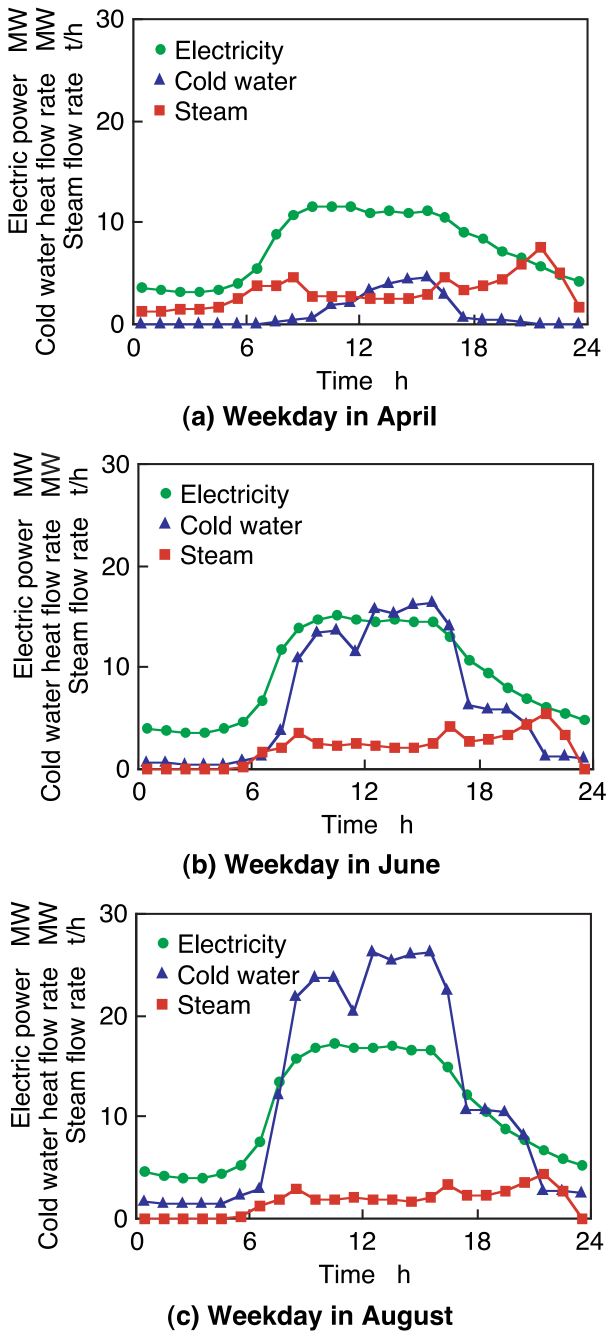
The formulation of the optimal operational planning problem for this cogeneration plant is partly given in the appendix.

Input Data

A weekday, a Saturday, and a holiday in each month are selected as representative days, and the daily operational strategy is investigated on each day, i.e., T = 24 h. Each day is discretized into 24 identical sampling time intervals, i.e., K = 24 and Δt = 1 h. As examples, Figs. 2 (a) to (c) show the energy demands estimated on the weekdays in April, June, and August, respectively, based on historical data.

Performance characteristics of equipment are identified on the basis of data measured. One binary variable is used to express performance characteristics of each piece of equipment except cooling towers. Therefore, the number of binary variables at each sampling time interval is N = 15, and their total number is $NK = 15 \times 24 = 360$. Table 1 shows capacities and performance characteristic values of equipment, and contract demands of utilities purchased. Table 2 shows unit costs for energy charge of utilities.

Table 3 shows minimum up and down times of equipment set in this study. The same value is used for minimum up and down times of each piece of equipment. In addition, the initial



(a) Weekday in April

(b) Weekday in June

(c) Weekday in August

Fig. 2 Estimated energy demands

value of on/off status $\delta(k)$ is set as $\delta(0) = 0$ for all the pieces of equipment. In relation to this condition, the initial value of continuous up time $t_{\mathrm{up}}(k)$ is set as $t_{\mathrm{up}}(0) = 0$, and that of continuous down time $t_{\mathrm{down}}(k)$ is set as $t_{\mathrm{down}}(0) = \underline{t}_{\mathrm{down}}$ for all the pieces of equipment, so that they can be started up at the first sampling time interval. The upper bounds for $t_{\mathrm{up}}(k-1)$ and $t_{\mathrm{down}}(k-1)$ are set as $\tilde{t}_{\mathrm{up}} = T$ and $\tilde{t}_{\mathrm{down}} = T + \underline{t}_{\mathrm{down}}$, respectively, so that they can be operated or stopped continuously during the period T.

Table 1 Capacities and performance characteristic values of equipment and contract demands of utilities

Equipment		Capacity	Performance [*1]	
Gas turbine	GT1	3.6 MW [*2]	Efficiency	0.25 [*2]
generator plus	WB1	10.9 t/h [*2]	Efficiency	0.49 [*2]
waste heat	GT2	3.8 MW [*2]	Efficiency	0.26 [*2]
recovery boiler	WB2	11.3 t/h [*2]	Efficiency	0.51 [*2]
Gas-fired	GB1	9.6 t/h	Efficiency	0.91
auxiliary boiler	GB2	11.3 t/h	Efficiency	0.90
Electric compression	TR1	3.2 MW	COP [*3]	4.42
refrigerator	TR2	3.4 MW	COP [*3]	5.02
Steam absorption refrigerator	AR1	6.8 MW	COP [*3]	1.14
	AR2	7.0 MW	COP [*3]	1.17
	AR3	7.0 MW	COP [*3]	1.18
Utility		**Contract demand**		
Electricity		14.5 MW		
Natural gas		3.9 $\times 10^3$ m^3/h		
Feedwater		–		

[*1] Rated load status
[*2] Intake air temperature 17.5°C
[*3] COP: Coefficient of performance

Table 2 Unit costs for energy charge of utilities

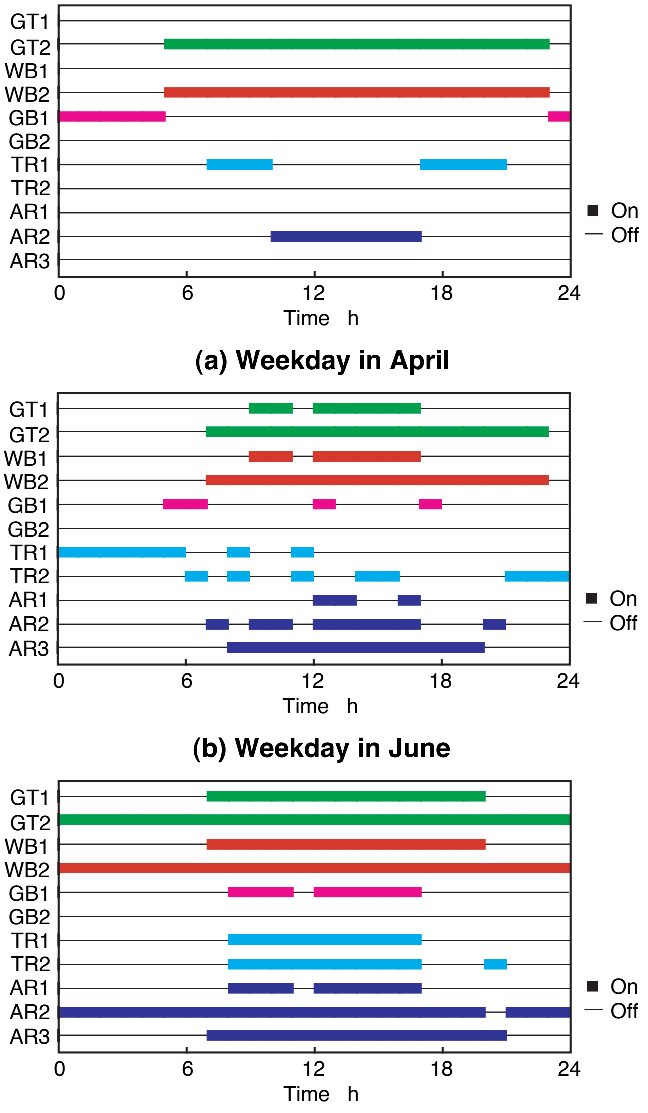
Utility	Unit cost	
Electricity	10.77	yen/kWh (Jul.~Sep.)
	9.79	yen/kWh (Other months)
Natural gas	30.88	yen/m^3
Feedwater	360.00	yen/m^3

Table 3 Minimum up and down times of equipment

Equipment		Minimum up time h	Minimum down time h
Gas turbine	GT1	5.0	5.0
generator	GT2	5.0	5.0
Waste heat	WB1	5.0	5.0
recovery boiler	WB2	5.0	5.0
Gas-fired	GB1	3.0	3.0
auxiliary boiler	GB2	3.0	3.0
Electric compression	TR1	2.0	2.0
refrigerator	TR2	2.0	2.0
Steam absorption refrigerator	AR1	4.0	4.0
	AR2	4.0	4.0
	AR3	4.0	4.0

Calculation Methods

In order to investigate the validity and effectiveness of the proposed method, the optimization calculation is carried out as

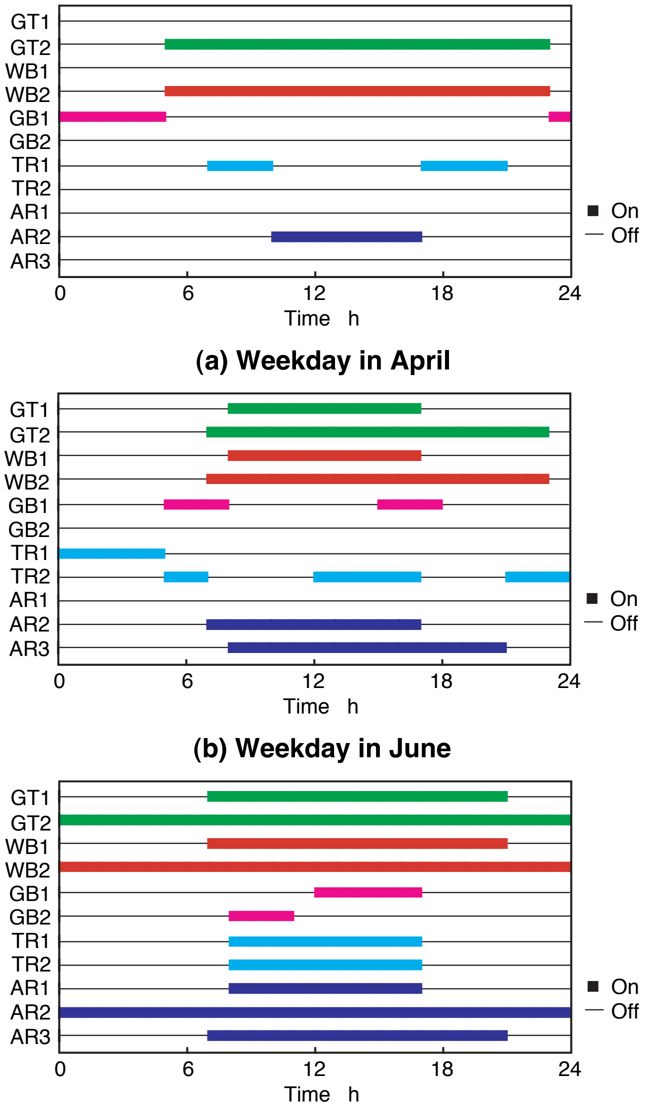


(a) Weekday in April

(b) Weekday in June

(c) Weekday in August

Fig. 3 On/off status of equipment (case A)



(a) Weekday in April

(b) Weekday in June

(c) Weekday in August

Fig. 4 On/off status of equipment (case B)

follows:

- Case A: The operational strategy is derived independently at each sampling time interval by neglecting the constraints for minimum up and down times.
- Case B: The operational strategy is derived in consideration of the constraints for minimum up and down times by the proposed method.

The comparison of cases A and B clarifies the influence of the constraints for minimum up and down times on the operational strategy and cost.

Results and Discussion

Operational Strategy. All the constraints for minimum up and down times are satisfied on every representative day under the condition shown in Table 3, and do not need to be relaxed by Eqs. (13) and (14). Figures 3 (a) to (c) show the on/off status of equipment for the optimal operational strategy on the weekdays in April, June, and August, respectively, obtained in case A. Figures 4 (a) to (c) show the corresponding results obtained in case B.

On the weekday in April, the operational strategy of all the pieces of equipment satisfies the constraints for minimum up and down times even in case A. Therefore, the operational strategy in case B is the same as that in case A. On the weekday in June, the operational strategy in case B is quite different from that in case A. The first gas turbine cogeneration unit is

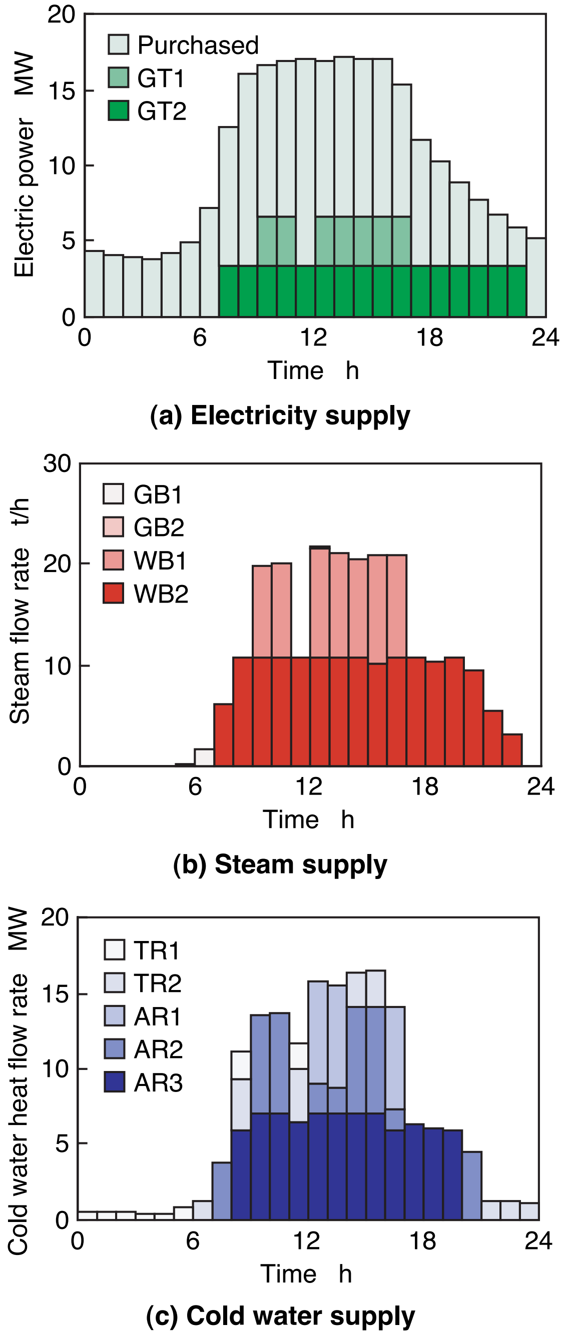


Fig. 5 Energy allocation (weekday in June, case A)

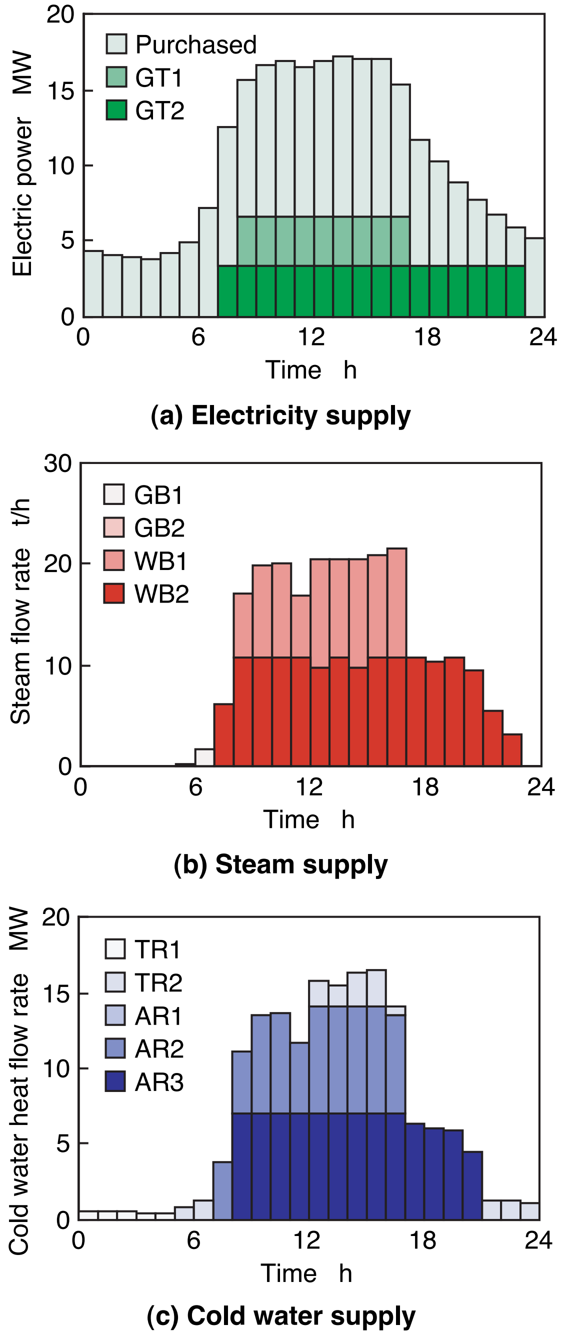


Fig. 6 Energy allocation (weekday in June, case B)

started up later, and it is shut down and started up again on the way of operation in case A, while both the gas turbine cogeneration units are operated continuously in case B. Both the electric compression and steam absorption refrigerators repeat their startups and shutdowns frequently in case A. The numbers of their startups and shutdowns decrease drastically by considering the constraints for minimum up and down times in case B. On the weekday in August, the operational strategy in case B is similar to that in case A. The first gas turbine cogeneration unit is shut down later in case B. In addition, the operational strategy of the gas-fired auxiliary boilers and both the electric compression and steam absorption refrigerators in case B is slightly different from that in case A. The numbers of startups and shutdowns of the refrigerators decrease slightly by considering the constraints for minimum up and down times in case B.

As aforementioned, the conventional method makes transition of on/off status of equipment sensitive to variations in energy demands and increases the numbers of startups and shutdowns of equipment excessively. On the other hand, the proposed method makes transition of on/off status of equipment less sensitive, and can reduce the numbers of startups and shutdowns of equipment. Therefore, the proposed method can assess the operational strategy suitable from the viewpoint of continuous operation.

Figures 5 (a) to (c) show the energy allocation for electricity, steam, and cold water supplies, respectively, for the optimal operational strategy on the weekday in June obtained in case A. Figures 6 (a) to (c) show the corresponding results obtained in case B.

According to (a) and (b), although both the gas turbine generators are operated at the rated load status when they are at the on status, the flow rates of steam generated by both the waste heat recovery boilers change with time. This is because the power generating efficiencies of the gas turbine generators decrease significantly at part load status, and it is cost-effective for them to be operated at the rated load status although a part of exhaust heat generated by the gas turbines must be disposed of. According to (c), the cooling load to each refrigerator changes significantly with time in case A. Such an operational strategy is not suitable from the viewpoint of smooth operation. On the other hand, the cooling load to each refrigerator changes smoothly with time in case B.

Operational Cost. Table 4 shows the comparison of cases A and B in terms of the daily operational cost as well as the increases in the daily operational cost and their rates on all the representative days. All the increase rates in the daily operational cost are only less than 0.4 %. These results show that the consideration of the constraints for minimum up and down times can assess the operational strategy suitable from the viewpoint of smooth operation at the sacrifice of a small increase in the operational cost.

Table 4 Comparison of cases in operational cost

Day		Cost in case A $\times 10^3$ yen/d	Cost in case B $\times 10^3$ yen/d	Increase $\times 10^3$ yen/d	Increase rate %
Jan.	w	2126.04	2127.59	1.55	0.073
	s	1903.54	1909.61	6.07	0.319
	h	1642.88	1646.31	3.43	0.209
Feb.	w	2115.65	2117.58	1.93	0.091
	s	1899.00	1905.02	6.02	0.317
	h	1642.07	1646.19	4.12	0.251
Mar.	w	2077.07	2083.81	6.74	0.324
	s	1862.09	1864.26	2.17	0.117
	h	1642.95	1642.95	0.00	0.000
Apr.	w	1940.33	1940.33	0.00	0.000
	s	1730.58	1733.49	2.91	0.168
	h	1562.73	1563.62	0.89	0.057
May	w	2362.70	2364.70	2.00	0.085
	s	2070.14	2072.68	2.54	0.123
	h	1834.44	1835.38	0.94	0.051
Jun.	w	2703.25	2706.70	3.45	0.128
	s	2327.38	2331.10	3.72	0.160
	h	2011.29	2014.10	2.81	0.140
Jul.	w	3485.57	3489.06	3.49	0.100
	s	2957.07	2960.21	3.14	0.106
	h	2503.52	2504.47	0.95	0.038
Aug.	w	3601.56	3603.80	2.24	0.062
	s	3028.85	3035.26	6.41	0.212
	h	2537.04	2538.72	1.68	0.066
Sep.	w	3272.70	3274.30	1.60	0.049
	s	2807.69	2810.56	2.87	0.102
	h	2410.66	2411.18	0.52	0.022
Oct.	w	2299.62	2301.10	1.48	0.064
	s	2055.72	2061.31	5.59	0.272
	h	1825.94	1828.61	2.67	0.146
Nov.	w	1894.67	1894.67	0.00	0.000
	s	1711.89	1714.61	2.72	0.159
	h	1552.32	1552.58	0.26	0.017
Dec.	w	2084.89	2086.52	1.63	0.078
	s	1868.56	1869.77	1.21	0.065
	h	1631.36	1632.77	1.41	0.086

w: weekday, s: Saturday, h: holiday

CONCLUSIONS

An optimal operational planning method based on the mixed-integer linear programming has been developed to determine the operational strategy of equipment so as to minimize the operational cost, in consideration of equipment minimum up and down times for each piece of equipment to be operated with appropriate numbers of startups and shutdowns. Constraints for minimum up and down times have first been formulated as quadratic equations, and then they have been transformed into linear equations by adding new variables and constraints. In addition, a penalty method has also been introduced so that the constraints for minimum up and down times are relaxed in case that they are not perfectly satisfied even by any combinations of on/off status of equipment.

In the numerical study, the proposed method has been applied to the daily operational planning of a gas turbine cogeneration plant for district energy supply. It has been clarified how the constraints for minimum up and down times affect the operational strategy and cost. The proposed method makes transition of on/off status of equipment less sensitive to variations in energy demands, and can reduce the numbers of startups and shutdowns of equipment at the sacrifice of a small increase in the operational cost. Through the study, the validity and effectiveness of the proposed method has been ascertained. Therefore, the proposed method can be applied to a real-time operational advisory system from the viewpoints of operational strategy and cost.

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APPENDIX

Concrete forms of the part of objective function f_k and the constraints $\boldsymbol{h}_k$ are presented for the gas turbine cogeneration plant in the numerical study.

The constraints $\boldsymbol{h}_k$ are composed of performance characteristics of equipment and energy balance relationships. As an example, performance characteristics of gas turbine generators are formulated as follows:

$$\left.\begin{aligned} E_{\mathrm{GT}m}(k) &= a_{\mathrm{GT}m}F_{\mathrm{GT}m}(k) + b_{\mathrm{GT}m}\delta_{\mathrm{GT}m}(k) \\ Q^{\mathrm{e}}_{\mathrm{GT}m}(k) &= a^{\mathrm{e}}_{\mathrm{GT}m}F_{\mathrm{GT}m}(k) + b^{\mathrm{e}}_{\mathrm{GT}m}\delta_{\mathrm{GT}m}(k) \\ E^{\mathrm{a}}_{\mathrm{GT}m}(k) &= a^{\mathrm{a}}_{\mathrm{GT}m}F_{\mathrm{GT}m}(k) + b^{\mathrm{a}}_{\mathrm{GT}m}\delta_{\mathrm{GT}m}(k) \\ \underline{F}_{\mathrm{GT}m}\delta_{\mathrm{GT}m}(k) &\le F_{\mathrm{GT}m}(k) \le \overline{F}_{\mathrm{GT}m}\delta_{\mathrm{GT}m}(k) \end{aligned}\right\} (m = 1,\ 2) \quad \text{(A1)}$$

where E is the electric power generated, Q^{e} is the exhaust heat generated, E^{a} is the electric power consumed for auxiliary machinery, F is the natural gas consumption, and δ is the binary variable for on/off status. These are treated as variables in the optimization problem. In addition, a and b are performance characteristic values, and $\underline{(\ )}$ and $\overline{(\ )}$ are lower and upper limits, respectively. These are treated as parameters. The subscript GTm denotes the mth gas turbine generator, and the superscripts e and a denote exhaust heat and auxiliary machinery, respectively. On the other hand, as an example, the electric power balance relationship is formulated as follows:

$$\begin{aligned} E_{\mathrm{elec}}(k) + \sum_{m=1}^{2} E_{\mathrm{GT}m}(k) &= E_{\mathrm{dem}}(k) + \sum_{m=1}^{2} E_{\mathrm{TR}m}(k) \\ &+ \sum_{m=1}^{2} E^{\mathrm{a}}_{\mathrm{GT}m}(k) + \sum_{m=1}^{2} E^{\mathrm{a}}_{\mathrm{WB}m}(k) + \sum_{m=1}^{2} E^{\mathrm{a}}_{\mathrm{GB}m}(k) \\ &+ \sum_{m=1}^{2} E^{\mathrm{a}}_{\mathrm{TR}m}(k) + \sum_{m=1}^{3} E^{\mathrm{a}}_{\mathrm{AR}m}(k) + \sum_{m=1}^{8} E^{\mathrm{a}}_{\mathrm{CT}m}(k) \\ &+ \sum_{m=1}^{4} E^{\mathrm{a}}_{\mathrm{P}m}(k) \end{aligned} \quad \text{(A2)}$$

where E_{dem} is the electric power demand, and is treated as a parameter. In addition, E_{elec} is the electric power purchased. All the quantities except E_{dem} are treated as variables.

The hourly operational cost f_k is formulated as follows:

$$\begin{aligned} f_k = \varphi_{\mathrm{elec}}E_{\mathrm{elec}}(k) &+ \varphi_{\mathrm{gas}}\left(\sum_{m=1}^{2} F_{\mathrm{GT}m}(k) + \sum_{m=1}^{2} F_{\mathrm{GB}m}(k)\right) \\ &+ \varphi_{\mathrm{water}}\sum_{m=1}^{8} W_{\mathrm{CT}m}(k) \end{aligned} \quad \text{(A3)}$$

where W is the water consumption, and is treated as a variable. In addition, φ_{elec}, φ_{gas}, and φ_{water} are the unit costs for energy charge of electricity, natural gas, and water, respectively, and are treated as parameters.