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Ideal Turbo charger Modeling and Simulation using Bond Graph Approach

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Abstract - In this paper one turbocharger for simulating its performance has been modeled in unsteady state.Our model subject to find out state space differential equations of system which contains different subsystems such as compressor, IC engine, turbine, fuel injection system, nozzle and shaft dynamics. We use bond graph method for modeling this system especially because our system is complex of components and stages additionally each subsystem has nonlinear performance. Whereas this system is base on Energy distribution in all elements .First we drew our system bond graph then according to bond graph we have found our state equations that will be simulated with use of initial conditions. In the end of this paper variability of pressure, temperature, rotational speed and pressure history in each stage according to time will be showed. In validation of results we have used references.

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1. INTRODUCTION

The invention of bond graph was driven by the need for a common language to model systems involving different energetic domains. Bond Graph is a graphical representation of a physical system model [1,2]. It is based on the representation of the flow of the different types of energy that are involved. It consists of a set of nodes, called multi-port elements, which are linked by edges that are called bonds. These bonds represent flows of energy (power) by means of oriented half arrows.With each bond, a set of power conjugate variables is associated, the effort variable e_k and the flow variables f_k. As far as f_k is the flux of the extensive variable q_k, the product $e_k f_k$ is an energy flux or power associated with the k^{th} energy form or occurrence [1,2]. The nodes, multi-port elements, of the bond graph define relations between these variables and are defined according to the fundamental principles of the dynamics of the systems. Until now the bond graph language has been mainly applied to finite dimensional mechanical, hydraulic, thermal and electrical systems within the context of transient state modelling and control [1]. As far as far as finite dimensional systems are concerned, the bond graph language is supported by commercially available softwares (20-sim® developed by controllab products, MS1 by Lorenz Simulation, CAMP-G from Cadsim Engineering or MTT, a free software licensed under the GNU General Public License) [3]. By using this approach, one can manage libraries of dynamical models by manipulating reusable sub-models through a graphical interface. The connections between the sub-models are organized by using the power conjugate variables as port variables. The causality assignment is performed on the basis of the interconnections and the nature of the inputs: at the boundaries of a system and through a given port, one can impose either the effort variables either the flow variable. In this paper the bond graph model of the ideal turbocharger is modeled and after deriving state space from the model, the results are compared with the experimental results in refrences.

2.Nomenclatures

m	$\frac{Kg}{Sec}$	**Mass Flow Rate**
P_u	Pa	**Pressure**
A_R	m^2	**Area**
ω	rad/sec	**Rotational Speed**
S_f		**Sourse of flow**
H	Kj	**enthalpy**
T	K	**Temperature**
$\boldsymbol{V}$	m/s	**Velocity**

Table.1.

3.Turbocharger Performance

In figure 1 the schematic model of a turbocharger is shown. As illustrated in the figure, the high temperature, high pressure outlet gas from IC engine causes rotation of the rotors of turbine, which produces torque and rotational speed (τ, ω). These elements are trasformed to the rotors of compressor by shaft and the rotation in compressor make pressure difference at the engine inlet. An result of this process is that inlet air flow rate is increased and compressor compresses the air and transfers it to the IC. In this section the air entered engine combines with fuel and causes combustion and the process is repeats.

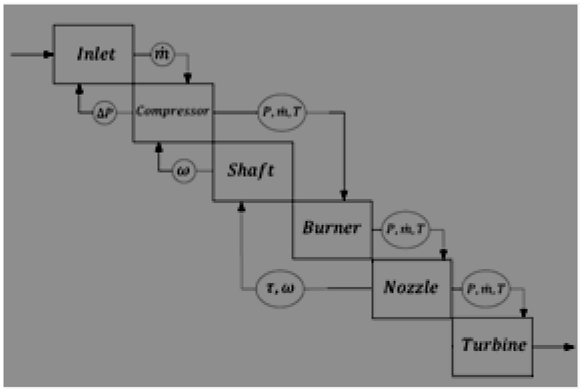


Fig1. Schematic model of a turbocharger

4 . The Bond Graph Language

Particularly table 1 illustrates how the different types of bonds are shown. As an example the bond graph representation of a simple mass and spring system is shown on figure 2.

Table 2. Different types of bond

Number of Bonds	Element Type	Representation	Energy and Flow Equations
Single	Inertia	$\xrightarrow[f]{e} I$	
	Capacity	$\xrightarrow[f]{e} C$	
	Resistance	$\xrightarrow[f]{e} R$	
	Energy Source	$S_e \xrightarrow{e}$	
	Flow Source	$S_f \xrightarrow{f}$	
Two	Transformer	$\xrightarrow[f_1]{e_1} \overset{m}{TF} \xrightarrow[f_2]{e_2}$	$e_1 = me_2$ $f_1 = mf_2$
	Gyrator	$\xrightarrow[f_1]{e_1} \overset{n}{GY} \xrightarrow[f_2]{e_2}$	$e_1 = nf_2$ $f_1 = \frac{1}{n} e_2$
Several	Zero	$e_3 \uparrow f_3$ $\xrightarrow[f_1]{e_1} 0 \xrightarrow[f_2]{e_2}$ $e_4 \downarrow f_4$	$e_1 = e_2 = e_3 = \cdots$ $f_1 + f_2 + f_3 + \cdots = 0$
	one	$e_3 \uparrow f_3$ $\xrightarrow[f_1]{e_1} 1 \xrightarrow[f_2]{e_2}$ $e_4 \downarrow f_4$	$f_1 = f_2 = f_3 = \cdots$ $e_1 + e_2 + e_3 + \cdots = 0$

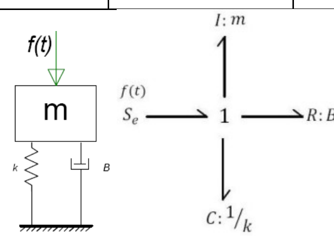


Fig 2. Mass and Spring System and its Bond Graph

4.1. Causality

The energetic coupling between two interconnected physical components is represented by a bond as developed before. Such model is acausual. Each component is described by its constitutive equation). The physical component must behave according to its constraint defined in the terms of the conjugate power variables. But if we need to simulate the physical phenomena (the model), we have to decide in which order the variables (efforts or flows) will be computed. Thus we will introduce the block diagram simulation which is casual. Consequently we need to make a series of cause and effect decisions, alternatively called casual assignment. This concept of causality is central to any computational model development. To make the bond graph model casual, the founders of this theory introduced a perpendicular stroke on each bond (fig3). This single mark on bond, called a casual stroke, indicates how e and f are simultaneously determined on a bond. [4].

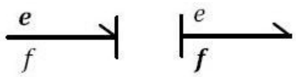


Fig3. Causality

The table 2 shows the causality assignment for the different kind of elements in every system.Determining where to put causality stroke is based on two aspects,

1) Causality rules
2) Physical aspect

Fig.4 shows the bond graph of fig.2 after putting causality stokes.

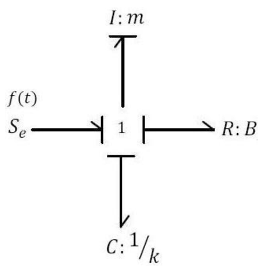
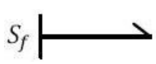
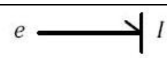
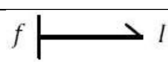
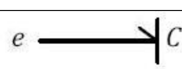
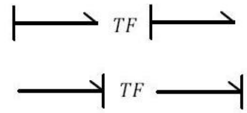
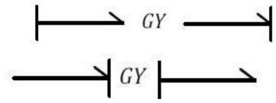
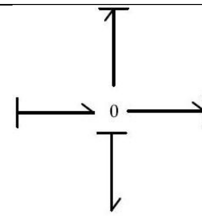
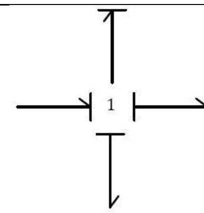


Fig4. Bond Graph of Fig4

Table 3. Causality Assignment for Different kind of Elements

Sources		
		
Inertia Element	Integral	
	Differential	
Capacity Element	Integral	
	Differential	
Transformer		
Gyrator		
Zero		
One		

5 . State Space Deriving From Bond Graph

5.1 State variables

As in equation (1) Energy can be transferred by effort and flow parameters.

(1) $P(t) = e(t)f(t)$

The two state variables are:

$p(t)$ Momentum and $q(t)$ transformation

5.2 Basic Equations

Defining the basic equations in deriving the state space equations are important and is one of the main steps of the procedure of derivation. The basic equations (2) , (3) are for inertia and capacity elements respectedly.

$$(2) \qquad \begin{cases} e = \dot{p} \\ e = I\dot{f} \end{cases} \rightarrow \quad p = If$$

$$(3) \qquad \begin{cases} f = \dot{q} \\ f = c\dot{e} \end{cases} \rightarrow \quad q = ce$$

5.3 State Space Deriving Sessions

According to the above sections the sessions of deriving state space equations from bond graph is as follows:

1) Putting causalities
2) Labeling the bonds
3) Determining the integral elements
4) Writing the situation variable for every integral element
5) Writing the basic equations of every element

After all the sessions above, we start to write the differential of situation parameters from one of the integral element and using the joint, elements rules with reference to other situation variables.

6. Word Bond Graph of Turbocharger

Word Bond Graph help us to consider how energy transit between sub systems. Figure 5 shows the word bond graph of turbocharger.

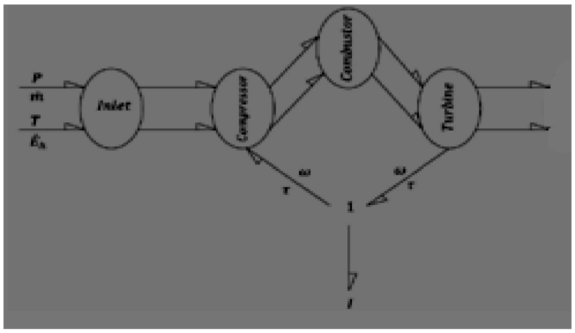


Fig5. Word bond graph of turbocharger

7.Thermo fluid Multiport elements

Whereas this process is a thermo fluid, the multiport bond graph elements ought to be used [1,5].Two multiport elements are known in thermo fluid process. In all of the models of physical systems that have been studied so far C- and I- fields were constrained to conserve energy and R-fields were arranged to dissipate power.

7.1 The thermodynamic accumulator

Consider the control volume with the internal gas at instantaneous pressure P, absolute temperature T, density ρ , and volume V , moving at velocity v and containing mass m and energy E.The control volume can transport mass through the "in" port and the out port. Finally we can obtain work from the control volume by the volume expansion. The one dimensional energy, mass can be written in the following forms:

$$\begin{cases} \frac{dE}{dt} = \left(h_i + {v_i^2}/{2}\right)\dot{m}_i - \left(h_o + {v_o^2}/{2}\right)\dot{m}_o - work & (4) \\ \frac{dE}{dt} = \left(h_i + {v_i^2}/{2}\right)\dot{m} - \left(h_o + {v_o^2}/{2}\right)\dot{m} - P\frac{dv}{dt} & (5) \end{cases}$$

$$\frac{d}{dt}m = \dot{m}_i - \dot{m}_0 \quad (mass) \quad (6)$$

$$\frac{dv}{dt} = \dot{v} \qquad (7)$$

ZEquations 4, 5, 6 and 7 look like first-order state equations, and they motivate the construction of the bond graph of figure.6.The 3-port C-field in this figure has one true bond and two pseudo bonds. The true bond has pressure P as its efforts variable and volume rate $\dot{V}$ as its flow variable. The P$\dot{V}$ is power. One of the bonds has energy flow $\dot{E}$ as the flow variable and temperature T as the effort. The other bonds uses pressure P as the effort and mass flow $\dot{m}$ as the flow. As the causality indicates in figure.6, the C-field possesses all integral causality and therefore accepts flow inputs on all three bonds ($\dot{E}, \dot{m}$ and $\dot{V}$).It then integrates these flows to produce the state variables E, m and V. And finally the C-field operates on these state variables through appropriate constitutive laws to produce the outputs P and T.

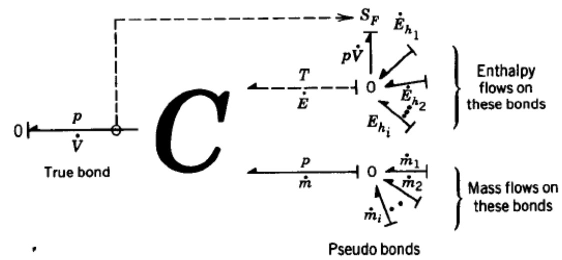


Fig.6

In turbo machinery, power is added to or removed from the fluid by the rotating components. These rotating components exert forces on the fluid which change both the energy and tangential momentum of the fluid. By applying Euler's equations for turbo machinery that relate the change in energy to the change in tangential momentum we write equation (4) in a new form.

$$\tau_A = \dot{m}(r_e v_e - r_i v_i) \qquad (8)$$

The input power is $\dot{W}_c = \omega \tau_A$ (9)

$$\frac{dE}{dt} = \left(h_i + {v_i^2}/{2}\right)\dot{m}_i - \left(h_o + {v_o^2}/{2}\right)\dot{m}_o - rotational\ work \ (10)$$

pattern for our equation getting out in function of rotational speed. Then we can write rotational work as following form.

$$Rotational\ work = \dot{m}\frac{U_t v_2}{g_c} \ (11)$$

7.2 The Isentropic Nozzle or Diffuser

We now consider the nozzle with upstream pressure and temperature P_u ,T_u , downstream pressure and temperature P_d ,T_d , exit Area A, and mass flow $\dot{m}$. If we assume that isentropic flow exists , then $\dot{m}$ depends upon the pressure ratio

$$P_r = \frac{P_d}{P_u} \qquad (12)$$

Rather than the pressure drop $P_u - P_d$.It can be arrived [1] that $\dot{m}$ is given by

$$\dot{\boldsymbol{m}} = \frac{\boldsymbol{A}\boldsymbol{P_u}}{\sqrt{\boldsymbol{T_u}}}\sqrt{\frac{\boldsymbol{2\gamma}}{\boldsymbol{R(\gamma-1)}}}\sqrt{\boldsymbol{P_r}^{2/\gamma} - \boldsymbol{P_r}^{\frac{\gamma+1}{\gamma}}} \qquad (13)$$

And for

$$P_r \le P_{r\,cri} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} \qquad (14)$$

The flow is "choked" and $\dot{m}$ is independent of the downstream pressure, P_d and the transported energy

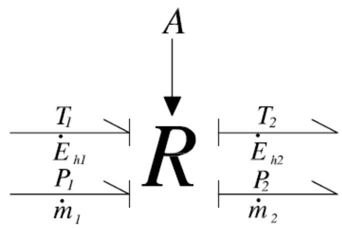


Fig.7

Consider the 4-port R-element of figure.7.Where all bonds have "effort in" causality. This R-element operates on the input efforts P_a, P_b, T_a, T_b and delivers the output flows$\dot{m}_a, \dot{m}_b, \dot{E_{ha}}, \dot{E_{hb}}$. This can be expressed functionally as

$$\begin{aligned} \dot{m}_a &= \dot{m}_a(P_a, P_b, T_a, T_b) \\ \dot{m}_b &= \dot{m}_b(P_a, P_b, T_a, T_b) \\ E_{ha} &= \dot{E}_{ha}(P_a, P_b, T_a, T_b) \\ \dot{E}_{hb} &= \dot{E}_{hb}(P_a, P_b, T_a, T_b) \end{aligned} \qquad (15)$$

We now define the computational procedures that allow
The R-field of figure.7. to represent the isentropic nozzle :

If $P_a > P_b$, $then\ P_u = P_a$, $T_u = T_a$, $P_d = P_b$ (16)

If $P_a < P_b$, $then\ P_u = P_b$, $T_u = T_b$, $P_d = P_a$ (17)

And $P_r = \frac{P_d}{P_u}$ (18)

Calculate $\dot{m}$ with using equation (13)

Now obtain outputs :

If $P_a > P_b$ $\quad then \quad \dot{m}_a = \dot{m}_b = m$ (19)

If $P_a < P_b$ $\quad then \quad \dot{m}_a = \dot{m}_b = -\dot{m}$ (20)

And in either case

$$\dot{E}_{ha} = \dot{E}_{hb} = C_p T_u \dot{m}_a \qquad (21)$$

These elements are used to show the turbocharger bond graph model, but they are not complete enough to show the turbocharger subsystems like compressor, turbine and etc, thus for modeling each subsystem we should create new multiport elements or using previous multiport elements.

8. Turbocharger elements in Bond Graph

8.1. Compressor

Centrifugal-flow compressors are used in gas turbine engines when the corrected mass flow rate is small. For small corrected mass flow rates the rotor blades of axial flow compressors are very short and the losses are high. The performance of a centrifugal compressor for low flow rates can be as good as or better that that of the axial flow compressor. Flow passes through the annulus between r_{1h} and

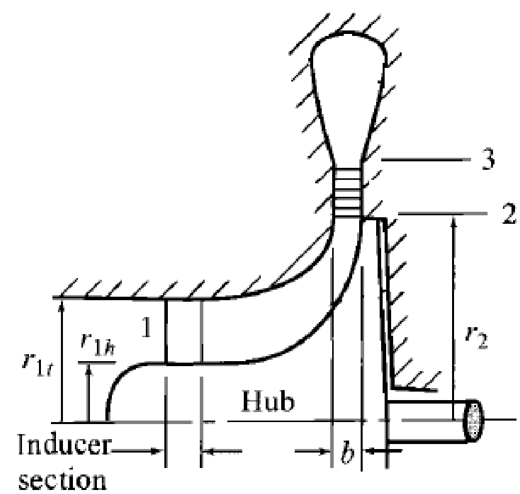


Fig.8.Centrifugal Compressor

r_{1t} at station 1[Fig.8] and enters the inducer section of the rotor. Flow leaves the rotor at station 2 through the cylindrical area of radius A_i and width b. The flow then passes through the diffuser, where it is slowed and then enters the collector scroll at a station 3.The flow then passes through the diffuser where it is slowed and then enters the collector scroll at station 3.The inlet flow is assumed to be axial of uniform velocity u_1.The flow leaves the rotor with a radial component of velocity w_2 that is approximately equal to the inlet axial velocity u_1 and a swirl component of velocity v_2 that is about 90 percent of the rotor velocity U_t. Since total pressure and total temperature depend upon the speed of the gas, they have different values " traveling with rotor " than for an observer not riding on the rotor. Consequently, the total pressure and total temperature do not change relative to an observer on the rotor as the gas passes through the rotor. An observer not on the rotor sees the force F (rotor on gas) .
moving at the rate ωr. Hence to the stationary observer, work is done on the gas passing through the rotor and the total temperature and total pressure increase.

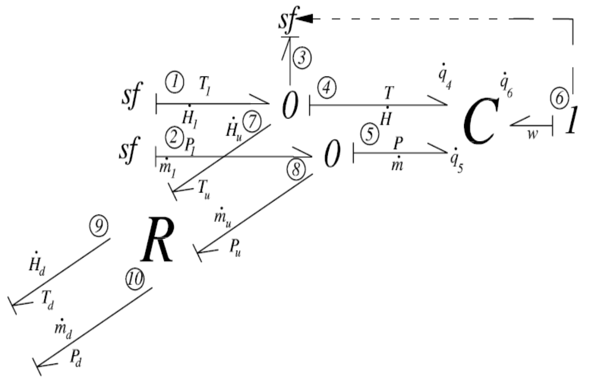


Fig.9.Compressor Bond Graph Model

For modeling a stage in axial compressor we used two Bond graph multi port elements which were studied. Capacity (C) and Resistance (R).

The Rotor is modeled with multiport element-C
And stator-diffuser was modeled with multiport element-R.
states variables in modeling compressor stage are $q_1 = H, q_2 = M \ and \ q_3 = w$
In Fig.9. Bonds 2, 5, 8, 10 show mass flows and relative pressure as effort but Bonds 1,4,7,9 show Enthalpy flows and relative temperature as their efforts. Bonds 1, 2 show the air entering the rotor, Bonds 4 and 5 shows the air in a control volume which is the space between the blades and Bonds 7 and 8 show air exiting from the rotor conditions.
0-joint shows bonds are connected to this joint have equal efforts and summation of their flows is 0.For example Bonds 2, 5, 8 have equal efforts. These 3 bonds have equal relative total pressure but summation of their mass flow rate is 0.Bond 6 shows w as its flow and τ is it's effort.
The flow source, SF← associated with the T ,$\dot{E}$ bond is necessary to properly account for the work done by the fluid in our control volume. The remaining external bonds associated with T ,$\dot{E}$ bond tell the Rotor and Stator control volumes the enthalpy flows $h_i\dot{m}_i$.Similarly the $P, \dot{m}$ bonds has external bonds which casually prescribe the mass flows from the linked subsystems.
For finding space state equations we consider transporting energy flows in each 0- joint.

$$\dot{E}_h = \sum_i \dot{E}_{hi} \tag{22}$$

$$\dot{m} = \sum_i \dot{m}_i \tag{23}$$

$$\dot{q}_5 = f_5 = f_2 - f_8 \ \Rightarrow \ \dot{q}_5 = S_f - f(P_u, P_d, T_u, A_R) \tag{24}$$

$$\dot{q}_6 = f_6 = \omega \tag{25}$$

$$\dot{q}_4 = f_4 = f_1 - f_3 - f_7 \ \Rightarrow \ \dot{q}_4 = S_f - f_6 - f(P_u, P_d, T_u, A_R)c_pT_u \tag{26}$$

$$e_6 = \tau = \dot{m}(V_1 \cos\beta_1 - V_2 \cos\beta_2) \tag{28}$$

8.2. Turbine

The Turbines' stators accelerate the flow and increase its tangential velocity. The rotor decreases the tangential velocity of the flow as it removes energy from the flow. Figure 10 shows the station numbering used in the analysis of the centrifugal flow turbine The flow enters the stators at station 1 and leaves at station 2.It then passes through the rotor and leaves at station 3. Normally the flow leaving the rotor is axial ($V_3 = u_3$).Application of the Euler turbine equation to the flow through the stator and rotor assuming stator gives

$$h_{t1} - h_{t2} = \frac{U_t v_2}{g_c} \tag{29}$$

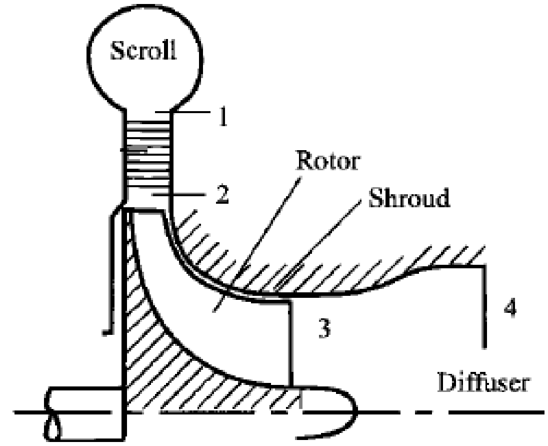


Fig.10.Centrifugal Turbine

The Relative velocity V_{2R} at the entrance to the rotor is designed to be radial ($v_{2R} = 0, V_{2R} = w_2$); thus the tangential velocity at station 2 (v_2) equals the rotor speed at its tip U_t.For this case Eq.

$$h_{t1} - h_{t3} = \frac{{U_t}^2}{g_c} \tag{30}$$

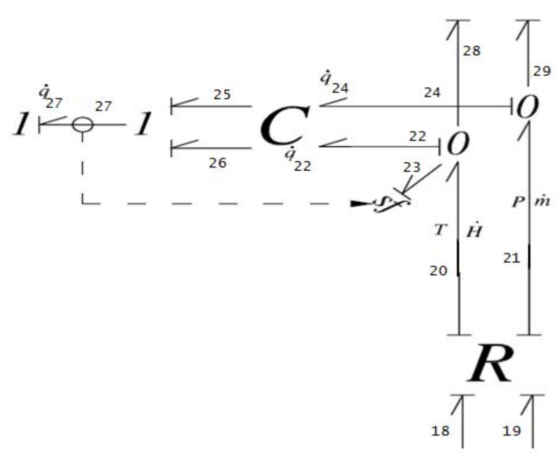


Fig.11.

Once the bonds have energy flow $\dot{E}$ as the flow variable and relative total temperature as the effort. The other bonds use relative total pressure as the effort and mass flow $\dot{m}$ as the flow. Thus if $\dot{E}, P\ and\ \omega$ are prescribed then the turbine will out put ω. The flow source SF◄— associated with the $T, \dot{E}$ bond is necessary to properly account for the work done by the fluid in our control volume. The remaining external bonds associated with the $T, \dot{E}$ bond tell the turbine the enthalpy flows $h_i \dot{m}_i$ from whatever system is attached to the turbine and as the 0-junction enforces the attached system learns of the control volume. Similarly the $P, \dot{m}$ bond has external bonds which causally prescribe the mass flow from the attached system ,while these external bonds also prescribe the control volume rotational speed as an input to the attached system.

We need to describe the origin of the transported energy flow so we start to write state equations in each 0-joint as below:

$$e_{27} = \tau = \dot{m}(V_1 \cos\beta_1 - V_2 \cos\beta_2) \qquad (30)$$

$$\dot{q}_{22} = \dot{q}_{20}(P_{22}, P_{20}, T_{20})T_{20}C_P - \dot{q}_{28}(P_{22}, P_{28}, T_{28})C_P T_{28} + \omega * \dot{m}(V_1 \cos\beta - V_2 \cos\alpha) \qquad (31)$$

$$\dot{q}_{22} = f_{22} = f_{20} - f_{23} - f_{28} \qquad (32)$$

$$\dot{q}_{27} = \omega = f_{27} \qquad (34)$$

Table 4. Results & Assumptions in Compressor

Properties	Results	Reference[3]
Number of stages		*1*
MFR	*13.8*	*8*
pressure ratio	*5.1*	*4.93*
polytropic efficiency		*0.85*
Inlet root diameter		*15 cm*
Outlet diameter imp		*50 cm*
Exit velocity	*86 m/sec*	*90 m/sec*
P total at inlet		*101.3 KPa*
T total at inlet		*288.16 k*
P total at exit	*500 KPa*	
T total at exit	*550 k*	
RPM	*145000*	
Slip factor		*0.9*
Inlet tip diameter		*30 mm*

Graphs & Discussion

The various responses of the simulated system are obtained by applying the step input as the supply inlet pressure and temperature. Figs. 12 and 13 show the dynamical response of the total exit pressure during first 16 seconds in compressor and turbine. The first case indicates the increasing of total exit pressure from 101Kpa to 510 KPa. On the other hand, Fig. 12 represents the profile of adjusting pressure, while the exit pressure is increased in 15 seconds our system convert from transient to steady state which shows our shaft rotational speed after 10 seconds is going to be constant and Fig.16 prove this. Observing these figures, it is demonstrated that variations of the exit pressure and temperature spend its transient period in reaching to steady state. The resulting adjusted pressure and temperature through bond graph are compared with experimental data in and [3] and [6].

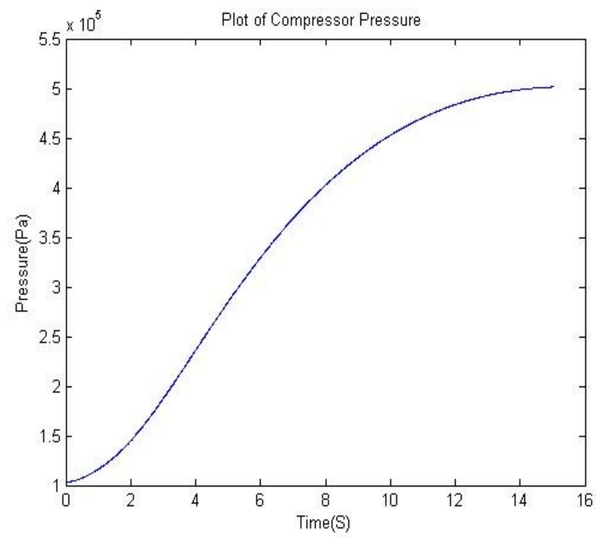


Fig12.Compressor Total pressure history

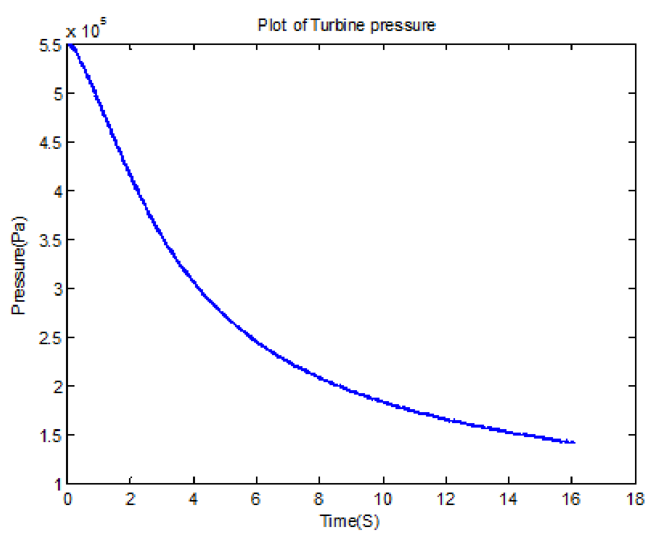


Fig13.Turbine exit Total pressure history

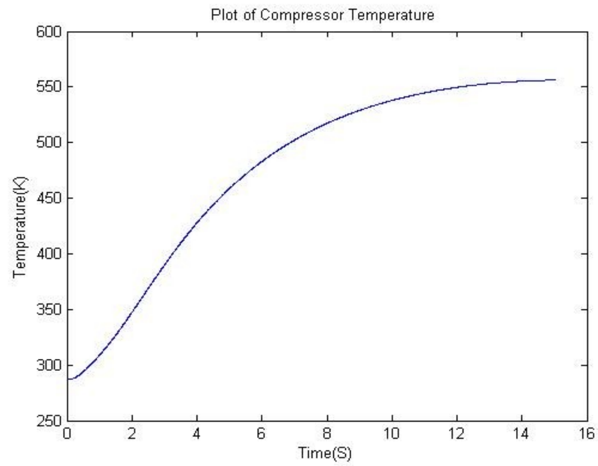


Fig14.Compressor exit total temperature

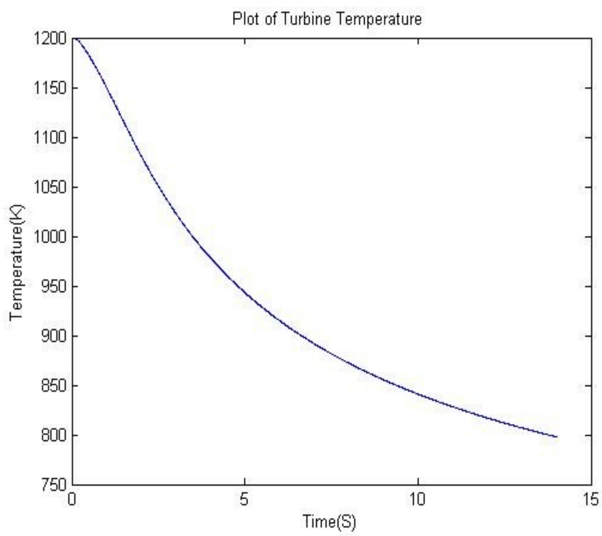


Fig15.Turbine exit Total temperature

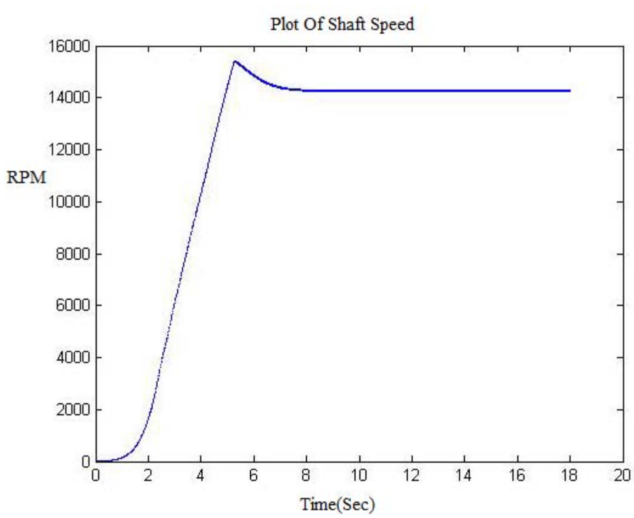


Fig16.Shaft velocity

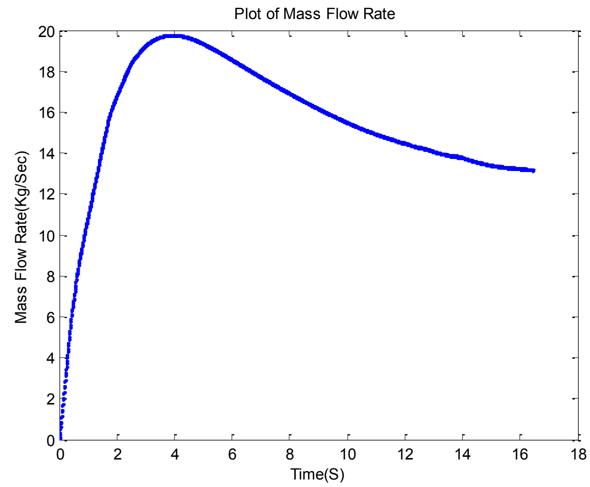


Fig17.Compressor mass flow rate

Conclusion

A dynamic model of a Turbo charger by utilizing the bond graph method is achieved in this study. While solving to state-space differential equations the basis of state equations and the effects of various design parameters on the overall response of the system are investigated. The verification of the simulation results with the experimental studies justifies the proposed dynamic model. The presentation of state variables profiles imply operation of Turbo charger to settle the changes on the inlet pressure and flow rate in the appropriate time. Several various nonlinearities of the system are taking into account through the modeling process. The effects of variations in four significant parameters include the Compressor pressure and Temperature, Turbine pressure and temperature main shaft inertia and velocity and compressor mass flow rate have been presented. It implies that Total Pressure and Temperature Total and shaft velocity are more important than the other parameters affecting the dynamic characteristics of the system. It is further concluded that this work provides a good base upon which an interaction study between other components and the IC engine. An important advantage of this modeling process is its simplicity lends itself to be used for wider variation of system parameters. Before employing bond graph methodology, the other attempts caused to inaccurate results

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