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Single vs. Dual Recycle System Requirements in the Design of High Pressure Ratio, Low Inertia Centrifugal Compressor Stations

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ABSTRACT

Compression systems are designed and operated in a manner to eliminate or minimize the potential for surge, which is a dynamic instability that is very detrimental to the integrity of the compressor unit. Compressor surge can occur when compressors are subjected to rapid transients such as those occurring following an emergency shutdown (ESD) or a power failure, which in turn, requires fast reaction. To prevent this from occurring, compressor stations are designed with single or dual recycle systems with recycle valves, which are required to open upon ESD. There has been extensive debate and confusion as to whether a single recycle or a dual recycle system is required and the circumstances and the conditions under which one system or the other must be used. This paper discusses this crucial design issue in detail and highlights the parameters affecting the decision to employ either system, particularly for high pressure ratio, low inertia compressors. Parameters such as gas volume capacitance (V) in the recycle path, compressor power train inertia, compressor performance characteristics, the recycle valve coefficient (Cv), pre-stroke and stroke time, and check valve dynamic characteristic are crucial in determining the conditions for dynamic instabilities. A simple analytical methodology based on the perturbation theory is developed that provides a first-cut analysis to determine if a single recycle system is adequate for a given compression system. The concept of an inertia number is then introduced with a threshold value that determines which recycle system to use. Techniques to circumvent compressor surge following ESD are discussed and their respective effectiveness are highlighted including when and if a delay in the fuel cut-off will be effective. An example of a Case study with actual field data of a high pressure ratio centrifugal compressor employed in a natural gas compressor station is presented to illustrate the fundamental concept of single vs. dual recycle systems.

INTRODUCTION

Compression systems employed in gas processing plants and in gas pipeline transmission systems provide a vital function to the overall operation of both systems, and therefore, must be vigilantly attended to in order to ensure a high level of operational reliability. The majority of these compression systems employ centrifugal compressors, either single- or multi-staged, driven by either gas turbines or electric motors with/without gearboxes. These compression systems are required to not only withstand uninterrupted operation for extended periods of time but also be able to cope with flow and pressure transients associated with part-load operation, startup and emergency shutdown (ESD) [1-8]. During these transients, the anti-surge system may be activated and centrifugal compressors interact dynamically with system components around them, i.e. piping, fittings and equipment, drivers, as well as the associated control protocols [9-10]. Fluid inertias and compressor/driver rotor inertias play an important role in either stabilizing or destabilizing the system dynamics [11]. The compressors' performance characteristics have also an important role in the system dynamics behaviour [12]. Ensuring reliable and safe operation of the various aspects of these compression systems requires a good understanding of their dynamic behavior, which enables sound system design, operation and control.

Several experimental and numerical investigations aimed at analyzing the dynamic interactions that take place between compression system components, particularly during ESD, have been reported, e.g. [13-16]. In these investigations, the surge model proposed by Greitzer and Moore [17,18] has been extended to centrifugal compressors. The method of characteristics for the solution of the governing one-dimensional equations of gas flow [19] has been proven to be adequate and correlates well with field measurements [12].

The recycle system around the compressor unit is an essential component in the operation of the centrifugal compressor. It is necessary for startup, shutdown, surge protection and flow control (turndown capability). As these operations are transient in nature, all dynamic parameters of gas flow, equipment and control play an important role and impact system's instabilities, performance and safety.

Of particular concern are the dynamics occurring following ESD as they represent the most severe and fast transients that could be damaging to the compressor unit and surrounding equipment. Parameters that affect the potential for the compressor to undergo surge during ESD are the recycle system gas volume, recycle valve characteristics such as maximum capacity, flow vs. opening characteristics, opening delay (i.e. the time between valve open solenoid drop out and the start of the valve stem movement on the valve – often called 'pre-stroke' delay), and valve travel time (i.e. the time taken for the valve to travel from closed to open positions – often called 'stroke' time) [14,15]. Additionally, timing of the compressor ESD signal, the fuel gas shutoff signal, fuel gas manifold size (in the case of gas turbine drivers), power train inertia, and the compressor's aerodynamic characteristics close to the surge point, all contribute to the complexity of the problem [11,12,16].

In the case of high pressure ratio (high head) multi-stage compressors, after coolers (typically aerial type) are often employed to bring the discharge gas temperature down to a level accepted for continuous operation of the downstream pipeline (set by the external coating maximum temperature limit). Two recycle systems are then contemplated: a) a single recycle system where the recycle path includes the aerial cooler as shown in Fig. 1 (top); and b) a dual recycle system where in addition to the aforementioned recycle system, another short-circuited system is employed specifically to deal with compressor surge control, as shown in Fig. 1 (bottom). In the dual recycle system, the longer recycle system is often called the cold recycle, as it re-circulates cooler gas downstream of the aerial cooler back to the compressor suction. The shorter recycle system is also called a hot recycle system for the opposite reason.

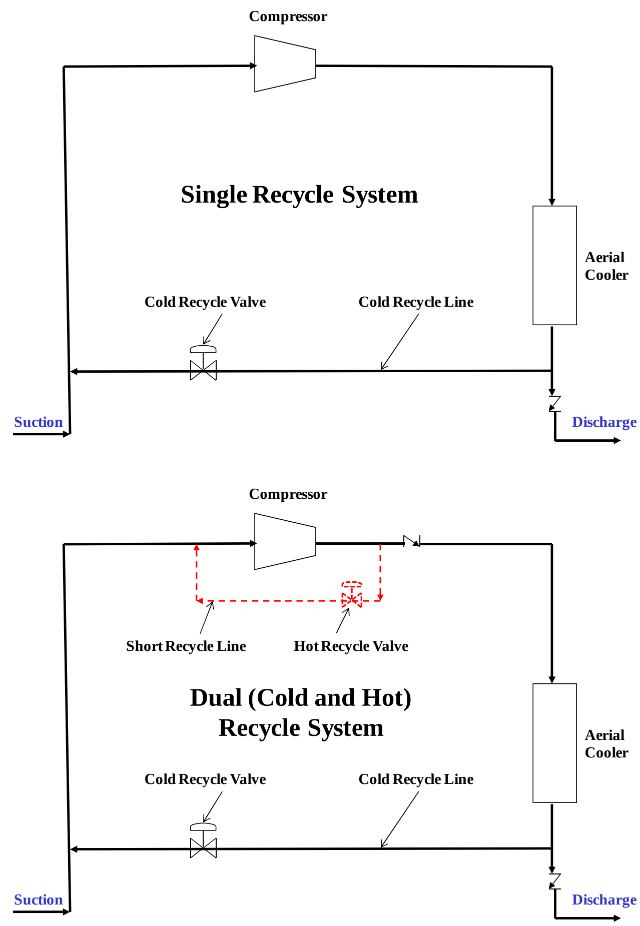

FIGURE 1: SINGLE VS. DUAL RECYCLE SYSTEMS.

The present paper addresses the criteria for selecting a single vs. dual recycle system, particularly in the case of high head, low inertia compressors involving after coolers that add volume capacitance to the recycle path of the cold recycle system. It attempts to quantify the effects of the aforementioned parameters on the potential for the compressor to undergo surge, specifically upon ESD, which is considered the fastest expected transient to occur in any compression system involving centrifugal compressors. The paper first presents a simple analytical methodology based on the perturbation theory which provides a first-cut analysis to determine if a single recycle system is adequate for a given compression system. The concept of an inertia number introduced in [20] is elaborated on to provide a threshold value that determines which recycle system to use. Full dynamic simulations are conducted on a single unit compressor station used on an existing high pressure natural gas transmission system in North America to demonstrate the interactions between the various parameters involved.

Techniques to circumvent compressor surge following ESD are discussed and their respective effectiveness are highlighted, including when and if a delay in the fuel cut-off will be effective.

DESCRIPTION OF AN ESD PROCESS

The process of compressor station ESD is schematically depicted on a head-flow diagram in Fig. 2, following a trend observed both experimentally and numerically [11-13]. Six phases are identified as the compressor decelerates from a steady state point (S.S.) to zero flow and zero head across the compressor. Following an ESD, the operating point of the compressor follows approximately a straight line characterized by the slope (S) for a period of time before any expansion or pressure waves arrive at the compressor outlet or inlet, respectively, as a result of opening the recycle valve. This period corresponds to the recycle valve pre-opening (pre-stroke) delay, which is a combined effect of a process signal delay and inherent mechanical delay in opening of the recycle valve once an ESD signal is issued. During this phase (Phase I), although the driver power is assumed to be completely shut-off, the compressor continues to rotate due to the combined inertia of its shaft, impeller and driver. The compressor decelerates due to the head across it, windage, friction, etc., according to the balance of the following equation:

$$\dot{W}_{driver} = I_{driver} \,.\, N_{driver} \,.\, \frac{dN_{driver}}{dt} + I_c \,.\, N_c \,.\, \frac{dN_c}{dt} + \frac{\dot{m} H_a}{\eta_a \, \eta_m} + Windage \,\&\, Losses \tag{1}$$

where:

- $\dot{W}_{driver}$ - driver power
- I_{driver} - driver inertia
- N_{driver} - driver speed
- I_c - compressor inertia
- N_c - compressor speed
- t - time
- H_a - compressor adiabatic head
- $\dot{m}$ - gas mass flow rate through the compressor
- η_a - compressor adiabatic efficiency
- η_m - mechanical efficiency

In the above equation, it is assumed that the driver power was set to zero instantaneously at the instant of ESD. While this is correct for cases where electric motors are used as drivers, it is not absolutely correct for cases with gas turbine drivers. White and Kurz [16] have shown that one of the key problems is that there is residual power from the gas turbine even after the fuel is shut off. This is due to two effects: one is that there is always some fuel gas remaining in the fuel gas manifold system which will continue to feed the gas turbine combustor and hence sustain power for a few hundred milliseconds; and secondly, the rotor inertia of the gas generator itself will continue to provide hot gas to the power turbine even at a decreasing temperature. These effects can be mathematically represented by describing the power term on the left hand side of Eq. (1), $\dot{W}_{driver}$, as a declining function of time instead of setting it to zero at the instant of ESD. Similar treatment can be adopted to steam turbine drivers.

Once the recycle valve opens, a pressure wave travels downstream of the valve along the low pressure part of the recycle line and along the main suction line, while an expansion wave travels upstream of the valve along the high pressure part of the recycle line and along the main discharge line. The first wave to arrive at the compressor suction or discharge sides depends on the distance, which either wave needs to travel, and the local speed of sound along the corresponding path. The time taken for either wave to arrive first to the compressor determines the duration of Phase II shown in Fig. 2. Once this wave arrives at the compressor, the flow starts to increase through the compressor and head decreases, and hence the beginning of Phase III. It can be shown that the slope (S) of the line identified in Fig. 2 for phases I and II can be expressed as [13]:

$$S = \frac{dH}{dQ} = \left(\frac{k-1}{k}\right)(H_o + \xi)\frac{\rho_1 C_1}{P_1 A}\left[1 + \frac{C_2 P_1}{C_1 P_2}\right] \tag{2}$$

where:

- A - pipe cross-section area
- C_1 - sound speed of the gas at suction condition
- C_2 - sound speed of the gas at discharge condition
- H_o - compressor head at steady state point
- k - isentropic exponent of the gas
- P_1 - suction static pressure
- P_2 - discharge static pressure
- R - gas constant
- T_1 - suction gas temperature
- Z_{av} - average gas compressibility factor
- ρ_1 - gas density at compressor inlet
- ξ - parameter defined as $Z_{av}RT_1 / [\,(k - 1) / k\,]$

In Equation (2) above, when all parameters are in SI units, the units of the slope S will be (J.s/kg.m^3). The recycle valve continues to open to the maximum open position resulting in further pressure waves and expansion waves arriving at the compressor suction and discharge sides, respectively. However, due to gas inertia in the recycle line and mainline, the flow through the compressor tends to overshoot as is manifested by Phase IV, followed by a short period of undershoot (Phase V) around the recycle system resistance line shown in Fig. 1. The final Phase VI is compressor wind down in which small over- and under-shootings around the recycle system resistance line occur until zero flow and zero head are reached. Note that the recycle system resistant line is not the surge control line [20,21].

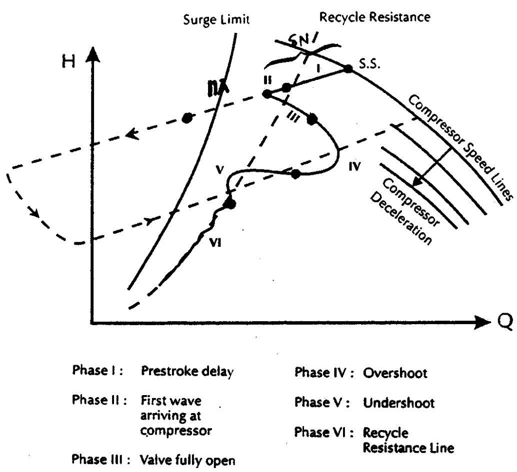

FIGURE 2: A SCHEMATIC OF THE DIFFERENT PHASES OF HEAD-FLOW THROUGH THE COMPRESSOR DURING AN ESD PROCESS [11].

SIMPLE METHODOLOGY TO DETERMINE WHETHER A SINGLE OR A DUAL RECYCLE SYSTEM IS REQUIRED

The perturbation characteristic of Eq. (2) represents the relationship between changes in the adiabatic head and actual inlet flow to the compressor for cases of no reflections from either ends to the compressor (i.e., recycle valve is still closed and both suction and discharge lines are non-reflective). The maximum drop in the compressor speed during this period following an ESD and before the compressor undergoes surge can be determined by running a tangent from the initial operating point with a slope '*S*' to the compressor characteristic at speed N_o - δN_{max}, as shown in Fig. 3. If the compressor speed drops below this limiting speed during ESD operation (as a result of e.g., low compressor shaft inertia, an initial condition of high head, a late arrival of the first expansion wave to the compressor discharge or the pressure wave to the suction of the compressor due to opening of the recycle valve), the compressor will undergo reverse flow (surge). This is because the compressor impeller at this speed cannot sustain a positive flow against the prevailing high differential pressure (head) across it. In this case, the high differential pressure will drive reverse flow through the impeller while it is spinning forward, which is defined by the intersection of the characteristic line with the full compressor characteristic at the prevailing speed at this instant (point B in Fig 3), hence the first surge cycle. In summary:

if $\delta N > \delta N_{max}$, surge will occur, and

if $\delta N < \delta N_{max}$, surge will not occur.

Here, δN is determined from ESD equation (1), which can be simplified for the case of gas turbine driven compressors as follows:

$$-I\,N_o \cdot \frac{dN}{dt} = \frac{\dot{m}_o\,H_o}{\eta_a\,\eta_m} \tag{3}$$

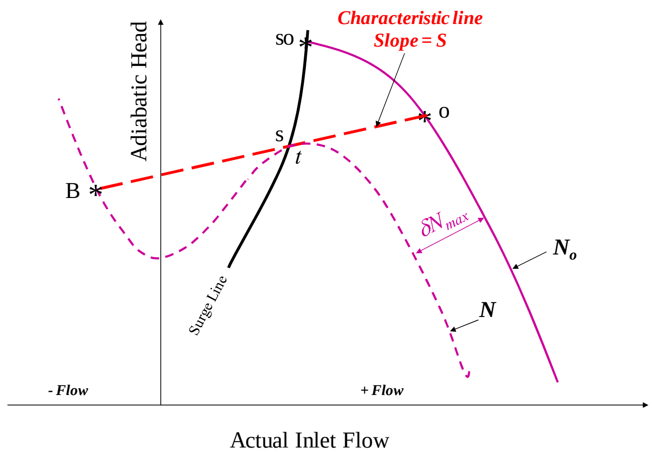

FIGURE 3: DETERMINATION OF MAXIMUM DROP IN COMPRESSOR SPEED BEFORE COMPRESSOR SURGING FOLLOWING ESD.

Theoretically, if the characteristic slope '*S*' is known, and the initial condition of the compressor (point 'o') is known, the maximum speed drop can be determined from geometrical algebra established by Fig. 3 and the fan laws of the compressor characteristics, including the cubic representation of the full compressor characteristics to the left of the surge point [19]. One approximation is to assume that (N_o - δN_{max}) corresponds to the mainline characteristics line meeting the compressor speed line at the surge point (s) at speed = N_o - δN_{max} instead of being tangent to it at point (t) as shown in Fig. 3. This approximation is, in fact, more realistic as the surge point (s) defines the surge limit. Following the fan laws the relation between adiabatic heads, actual inlet flows and compressor speeds at surge points can be correlated as follows:

$$\frac{Q_s}{N} = \frac{Q_{so}}{N_o} = K_1\,; \qquad \frac{H_s}{N^2} = \frac{H_{so}}{N_o^2} = K_2 \tag{4}$$

According to the above discussion and referring to Fig. 3, the following relation can be written:

$$S = \frac{H_o - H_s}{Q_o - Q_s} \tag{5}$$

where 'S' is the slope of the characteristic line defined by Eq. (2).

Introducing Eq. (4) into Eq. (5), the following equations can be developed:

$$
\begin{aligned}
& H_o - H_s = S\left(Q_o - Q_s\right) \\
& H_o - K_2\left(N_o - \delta N\right)^2 = S\left[Q_o - K_1\left(N - \delta N\right)\right] \\
& H_o - K_2 N_o^2 + 2K_2 N_o \delta N = S\left[Q_o - K_1 N_o + K_1 \delta N\right] \\
& H_o - H_{so} + 2H_{so}\frac{\delta N}{N_0} = S\left[Q_o - Q_{so} + Q_{so}\frac{\delta N}{N_o}\right]
\end{aligned} \tag{6}
$$

and finally,

$$\frac{\delta N_{\max}}{N_o} = \frac{S\left(Q_o - Q_{so}\right) + \left(H_{so} - H_o\right)}{2H_{so} - S\,Q_{so}} \tag{7}$$

Now combining Eq. (7) and Eq. (3) and integrating, we arrive at the simple equation that determines the maximum (longest) time that the compressor can withstand before going into surge, i.e. before the arrival of the relief expansion or pressure waves resulting from opening the recycle valve, as follows:

$$\delta t_{\max} = I\,N_o^2\left[\frac{S\left(Q_o - Q_{so}\right) + \left(H_{so} - H_o\right)}{2H_{so} - S\,Q_{so}}\right] / \left[\frac{\dot{m}_o\,H_o}{\eta_a\,\eta_m}\right] \tag{8}$$

The above equation, though simple and easy to evaluate, is significant. The $\delta t_{\max}$ thus calculated by this equation can be compared to the time it will take for the first relief expansion or pressure wave to arrive at the compressor discharge or suction side, respectively. This time of arrival can be estimated from the sum of the recycle valve pre-stroke delay and the travel time of either the expansion or the pressure wave to arrive at the compressor. The latter is calculated from the distance along the corresponding piping between the recycle valve and the compressor and the local speed of sound in the gas, either on the discharge or the suction side, respectively.

Fig. 4 shows an example of head-flow characteristics of a high head, low compressor/driver rotor inertia unit employed in a natural gas compression system depicted schematically in Fig. 5. This characteristic map is specific for typical natural gas composition flowing in gas transmission systems. A steady state operating point was assumed close to the surge control line as shown in the Figure at Q_o = 4.363 m^3/s and H_o = 37.072 kJ/kg. Other relevant parameters required for the calculation of $\delta t_{\max}$ in Eq. (8) are given in Table 1. The resulting $\delta t_{\max}$ is calculated as 116 ms.

According to the axial distances along the suction and discharge piping separating the cold recycle valve from the compressor (see Fig. 5), the arrival time of the expansion and pressure waves, along with valve pre-stroke delay are calculated in Table 2. These times are 296 ms and 288 ms, respectively. Clearly, the compressor in this case will undergo surge following an ESD operation since the time of arrival of either wave is much longer than $\delta t_{\max}$ of 116 ms calculated above. In fact, the pre-stroke delay alone of this valve (=200 ms) is obviously too long to prevent the compressor unit from surging. The obvious solution is not only to employ a recycle valve that has a shorter pre-stroke delay, but to have it located very close to either compressor discharge (preferable) or suction sides. This means, a dual recycle system is required, i.e. the addition of a short (hot) recycle system as shown in Fig. 5 (dotted line). Again, the function of the short recycle system is for ESD and surge protection, while the cold recycle system which includes the aerial cooler in the recycle path, is for unit startup and part load operation.

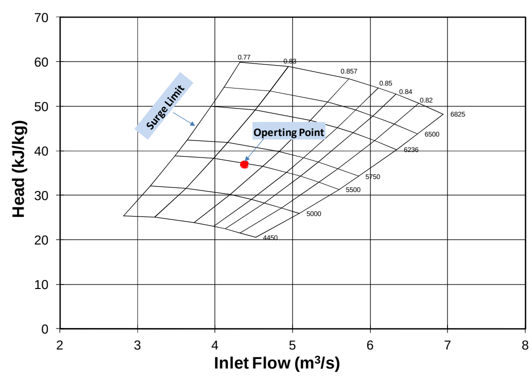

FIGURE 4: EXAMPLE OF A HIGH HEAD LOW INERTIA COMPRESSOR CHARACTERISTICS.

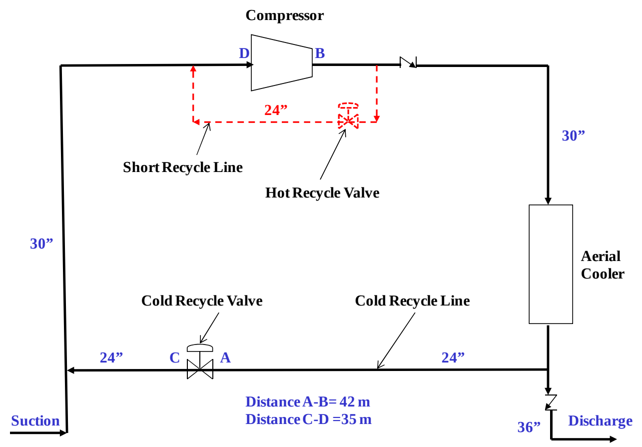

FIGURE 5: SCHEMATIC OF THE EXAMPLE STATION LAYOUT SHOWING THE AXIAL PIPING DISTANCE BETWEEN THE COLD RECYCLE VALVE AND COMPRESSOR SUCTION AND DISCHARGE SIDES OF THE COMPRESSOR UNIT.

If the compressor/power turbine combined inertia was 300 kg.m^2 (instead of the 117 kg.m^2), the time to surge would have been 297 ms (rather than 116 ms). Additionally, $\delta t_{\max}$ also depends on the initial operating point on the compressor characteristics (i.e. head and flow values) with respect to the surge line, as well as gas dynamic parameters involved in the equation, as mentioned earlier. The farther the initial operating point from the surge line is, the longer the value of $\delta t_{\max}$. This is due to the term (Q_o-Q_s) on the numerator of Eq. (8). This will be discussed further in Section 6.0.

TABLE 1: EXAMPLE OF COMPRESSOR OPERATING PARAMETERS.

Flow Conditions:		
Suction Pressure	8202	kPa-a
Suction Temperature	283	K
Discharge Pressure	11352	kPa-a
Discharge Temperature	314	K
Average Compressibility	0.817	
Molecular weight	17.953	kg/kmol
Suction Density	76.560	kg/m3
Isentropic Exponent	1.482	
Isentropic Efficiency	0.8	
Mechanical Efficiency	0.96	
Gas Constant	463.098	J/kg.K
C_1	398.390	m/s
C_2	436.7	m/s
Pipe Size (30" Nominal)		
Internal Diameter	0.737	m
X-Sectional Area	0.426	m2
Operating Point		
Qo	4.363	m3/s
Ho	37072	J/kg
Qso	3.482	m3/s
Hso	38863	J/kg
No	5500	RPM
ω	575.959	rad/s
Inertia	117	kg.m2
Results of Simple Approach :		
(k-1)/k	0.325	
ξ	329625.404	J/kg
S	1863.713	(J.s/kg.m^3)
dN_{max}/No	0.048	
dN_{max}	265.048	RPM
$d\,\omega$	27.756	rad/s
	16124.062	kW
$\delta tmax$	0.116	s
$\delta tmax$	116	ms

TABLE 2: CALCULATION OF ARRIVAL TIME OF THE EXPANSION AND PRESSURE WAVES TO COMPRESSOR UNIT DISCHARGE AND SUCTION SIDES, RESPECTIVELY (COLD RECYCLE SYSTEM).

Valve Pre-stroke delay	200	ms
Discharge Piping Length	42	m
Expansion Wave Arrival Time	96.18	ms
Combined Time	296.18	ms
Suction Piping Length	35	m
Pressure Wave Arrival Time	87.85	ms
Combined Time	287.85	ms

It should be emphasized that if the time to surge ($\delta t_{\max}$) is longer than the combined pre-stroke delay time and the perturbation wave time, the compressor will not go into the first surge cycle. This does not mean that the compressor will be entirely surge free. The other important parameter that then comes to play is the piping capacitance (volume of gas) contained between the station check valve, the cold recycle valve and the compressor discharge side due to the presence of the aerial cooler and associated piping. This high pressure gas volume needs to be relieved via the cold recycle valve to the lower pressure on the suction side. With the inclusion of the aerial cooler in the recycle path, the recycle valve size/capacity has to be large enough to relieve this pressure otherwise the compressor will surge, now due to this volume capacitance instead of the timing issues described earlier. This will be illustrated further in Section 7.0.

Therefore, it can be said that the above simple methodology applies to compression systems that do not involve large volume capacitance along the wave travelling path. The absence of large volume capacitance system allows the perturbation wave to travel axially along the linear piping system without been dampened by a large capacitance that would act as a 'muffler' and substantially weaken its effects on the compressor.

INERTIA NUMBER

Following numerous analyses conducted on different design philosophies of compression systems, and incorporating different types of centrifugal compressors, the decision to employ a short recycle system around the compressor unit to overcome the possibility of surging the unit during ESD operation lies in the balance between the following parameters:

- Effective compressor/driver rotor inertia defined at the compressor end, (I).
- Maximum compressor speed.
- The delay time before the recycle valve starts its opening stroke (that is the time associated with Phases I and II in the ESD process described above), (τ).
- The maximum fluid energy extracted from the compressor/driver power trains; which can be approximated by the product $\dot{m}_{so} H_{so}$, where subscript (so) refers to conditions at the surge point (see Fig. 3), and at maximum compressor speed.

With the aid of Equation (8), a non-dimensional number can be derived that includes all of the above independent parameters. This dimensionless number, herein referred to as the Inertia number (N_I), is defined as:

$$N_I = \frac{I\, N_{\max}^2}{\dot{m}_{so}\, H_{so}\, \tau} \qquad (9)$$

A threshold value of the Inertia number was found from actual installations and dynamic analyses conducted on 17 industrial compression systems employing different compressor models and station design. This threshold value was found to be ~30, below which a shorter recycle system would definitely be needed to prevent the compressor unit from undergoing surge during ESD operation. When the Inertia number is greater than 100, a single recycle system would be adequate. For an Inertia number (N_I) in the range of 30-100, detailed dynamic simulation on the station should be conducted in a manner similar to the examples which will be presented in the next Section. Table 3 gives the various operating and design parameters for those 17 industrial compression systems analyzed and the respective Inertia number (N_I) based on a value of τ corresponding to the cold recycle system. Note that consistent SI units should be used to calculate (N_I). The comment column in Table 3 indicates whether the design incorporated a short (hot) recycle or not.

TABLE 3: OPERATING AND DESIGN PARAMETERS TO DERIVE THE INERTIA NUMBERS FOR THE VARIOUS INDUSTRIAL COMPRESSION SYSTEM ANALYZED.

Station	Compressor Model	I ($kg.m^2$)	N (RPM)	Flow (kg/s)	Hso (J/kg)	$\tau (ms)$	No of Stages	Cooler	Inertia Number	Comment
1	Cooper RFBB20	36.1	6800	250	28000	200	1	No	13.08	Delay in Fuel Gas by 100 ms
2	Solar C652	33.7	8856	143	80600	200	2	Yes	12.62	Hot & Cold Recycle Installed
3	Mitsubishi 5V-2	32.2	7780	125	64500	200	2	Yes	13.25	Hot & Cold Recycle Installed
4	Cooper DR555P2	56.5	6500	180	52000	200	2	Yes	13.98	Hot & Cold Recycle Installed
5	Cooper RFBB30	41.5	6671	150	40000	200	2	Yes	16.89	Hot & Cold Recycle Installed
6	Nuovo Pignone (PGT25+, PCL802)	243.6	6100	299	68772	200	2	Yes	24.17	Hot & Cold Recycle Installed
7	Cooper RFBB36	259.6	4250	380	26220	200	1	Yes	25.80	Hot & Cold Recycle Installed
8	Nuovo Pignone (PGT25,PCL603)	116.8	6500	244	52625	200	2	Yes	21.07	Hot & Cold Recycle Installed
9	Elliot (48M6)	102.5	6000	35	160000	215	6	Yes	33.61	Hot & Cold Recycle Installed
10	Nuovo Pigone (PGT25,PCL603-1)	113.5	6825	420	31000	588	1	Yes	7.57	Hot & Cold Recycle Installed
11	Hitachi (BCH354)	3.9	11967	3	117000	337	4	Yes	54.00	Hot & Cold Recycle Installed
12	Hitachi (BCH354)	6.1	11970	9	124386	324	4	Yes	27.61	Hot & Cold Recycle Installed
13	Hitachi (BCH605)	104.8	7850	51	160612	368	5	Yes	23.43	Hot & Cold Recycle Installed
14	Hitachi (BCH506)	94.3	8311	50	152825	368	6	Yes	25.30	Hot & Cold Recycle Installed
15	Solar-C16	0.2	20000	20	32000	185	1	No	6.48	Hot Recycle
16	Cooper Bessemer	128.6	5194	480	32000	200	1	No	12.38	Hot Recycle
17	Dresser Rand 75P	870.0	5775	350	39000	200	2	Yes	116.55	Only Cold Recycle

The above threshold value of the Inertia number is useful for station design engineers, which allows a quick check to determine whether a short (hot) recycle system would be required for a type and model of compressor unit and the neighbouring equipment, particularly cooler and suction separators being in the recycle loop. If the calculation of the Inertia number (N_I) reveals a value less than or equal to 30, a shorter recycle system would be needed, and a detailed dynamic simulation should be conducted. If on the other hand, the Inertia number was approximately 100 or greater, a single recycle system is acceptable, and a detailed dynamic simulation may not be required.

However, when a short (hot) recycle system is employed, often the compressor suction and discharge temperatures gradually climb during ESD, as warmer gas is recycled back to the suction of the compressor, which in turn, increases the discharge temperature. Although throttling of the gas through the recycle valve will reduce the temperature to some extent, and the continually decreasing head will moderate the temperature rise across the compressor, it is important to ascertain that the compressor discharge temperature does not increase to a level that could affect the integrity of the compressor internals, e.g. O-rings and seals. Higher pressure-ratio compressors would be more prone to such a problem, and therefore a dynamic analysis should be conducted to determine the temporal temperature profile during this process. In order to mitigate higher temperature rise in such scenarios, the cold recycle system is also opened simultaneously with the short recycle, such that gas from the cold recycle (having gone through the unit after coolers) would mix with the warmer gas from the short recycle and result in a moderated gas temperature at the compressor suction. Examples of such scenarios are reported in [20].

FULL DYNAMIC SIMULATION

Compression systems in process plants and in pipeline transmission systems are not as simple as those depicted in Fig. 1. By and large, in addition to the compressor units, compression systems are composed of suction equipment such as scrubbers or separators, control valves, upstream process equipment that could either be close (like in gas plants) or separated by a long section of pipe (like in a pipeline system), a number of bays of coolers on the discharge side, check valves, unit block valves, recycle system with recycle valves, blow-down or vent lines and valves leading to a flare header system, etc. Although, in principle, the dynamic equation that describes such a system can still be reduced to a second-order ordinary differential equation in nature, it simply cannot be derived analytically due to the complexity of the system and the sheer number of elements comprising it. The problem could be doubly or triply accentuated if there are two or three compressor units operating in a parallel/series configuration. For this reason, numerical simulations are resorted to, where the fundamental governing equations describing the gas and equipment interaction dynamics between all of the piping and control systems are employed.

In any numerical dynamic simulation of such compression systems, it is important to include the temporal-spatial dependence terms in all three governing equations for the gas flow in each pipe element. In many of the commercial codes, only the time gradients are considered in dynamic simulations, which amounts to describing the dynamics of the system using Ordinary Differential Equations (ODEs) that are much less rigorous than the Partial Differential Equations (PDEs). This is referred to as the "lumped parameter" method, which gives a solution that is a reasonable approximation of the distributed model solution.

The lumped parameter approach is not adequate for dealing with compression dynamics involving recycle systems and phenomena of compressors going into, and out of, surge. The spatial gradients along the length of the pipe segments are crucial, as they describe the time required for perturbations in pressure, flow and temperature to propagate from one point in the system to another, akin to those discussed in Section 3.0. A good simulation model would retain all terms in the following one-dimensional governing PDEs:

Continuity

$$\frac{\partial \rho}{\partial t} + \rho \frac{\partial v}{\partial x} + v \frac{\partial \rho}{\partial x} = 0 \qquad (10)$$

Momentum

$$\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} + \frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{f_{DW}}{2D} v|v| = 0 \qquad (11)$$

Energy

$$\frac{\partial p}{\partial t} + v \frac{\partial p}{\partial x} - C^2\left(\frac{\partial \rho}{\partial t} + v\frac{\partial \rho}{\partial x}\right) - E = 0 \qquad (12)$$

where

$$E = \frac{f_{DW}}{2DC_v}\left(\frac{\partial p}{\partial T}\right)_v v^2|v| + \frac{4k}{DC_v\rho}\left(\frac{\partial p}{\partial T}\right)_v (T_a - T) \qquad (13)$$

where:

- C - speed of sound
- C_v - gas specific heat at constant volume
- D - pipe internal diameter
- E - friction and heat transfer term defined in Eq. 8
- f_{DW} - Darcy Weisbach friction factor
- k - overall heat transfer coefficient between fluid in pipe and surrounding temperature
- p - pressure
- v - gas mean flow velocity
- x - spatial length along a pipe
- t - time
- T - gas temperature
- T_a - surrounding (ambient) temperature
- ρ - gas density
- $(\ )_v$ - derivative at constant specific volume

Note that both pipe wall friction and heat transfer with the surroundings are taken into account based on Equation (13). Using the method of characteristics, the above hyperbolic partial differential equations are transformed into total differential equations, which lead to a set of algebraic compatibility equations along two characteristic lines and a particle path line [22-26]. These compatibility equations, together with the respective characteristic lines are integrated in the time-space domain. In the derivation of the finite-difference compatibility equations, the real gas assumption is introduced and either AGA-8 (American Gas Association) [27] or Peng-Robinson [28] equations of state were employed, depending on the nature of the gas mixtures, to specify the relation between the gas density, temperature and pressure, as well as to determine the physical and thermodynamic properties at each node in the system.

Similar governing equations describing the transient flows through physical components in the system are also formulated and combined with Eq. (12-15). These elements are throttle or pressure loss elements, combining and dividing tees, reducers or expanders, capacitance (plenum, volume, vessels), choked and un-choked valves, heat exchangers with a set duty or a set outlet temperature, adiabatic or isothermal flash separators, etc. A full account of the governing equations for these elements is given in [21]. Throttle elements and capacitance elements are modeled based on quasi-steady state equations describing pressure changes as well as energy and mass balances across the element. These equations, when combined with the three compatibility equations for the pipes connected upstream and downstream of the element, are solved simultaneously to give the unknown variables on both sides of the element at each time step. Different formulations of the equations are used to account for reversed flow situations.

The compressor itself is assumed to respond to any perturbation in a quasi-steady manner following its full characteristic curve, including that to the left of the surge limit [2,3,17,18]. Compressor/driver dynamics are governed by Eq. (1); relating the driver power to the gas power and the inertias of both driver and compressor.

It should be noted that the driver and compressor inertias should also include the inertia of the elements of the gearbox and couplings connected to either side, respectively. Additionally, in the case of a two-shaft gas turbine driver, the applicable driver inertia in Eq. (1) above is only the power turbine (i.e. the driver rotor) and coupling inertias and is not inclusive of the gas generator inertia.

Generally, a compression system consists of most of the elements described above, whether it is a pipe element, a connecting element or boundary condition. These constitute sets of highly non-linear equations, which must be solved simultaneously to determine the unknowns (pressure, mass flow rate and temperature). Each set of equations requires solution at each Δx location along each pipe element at each Δt time step. To achieve numerical stability, the Courant stability condition [29] is applied which stipulates that:

$$\Delta t \leq \frac{\Delta x}{C+v} \tag{16}$$

where:

C = local speed of sound of the flow medium
v = instantaneous mean gas flow velocity

The Newton-Raphson method for the solution of non-linear equations is used because of its convergence speed and efficiency. The method is iterative in nature and solves all equations simultaneously. The starting point for a variable at a given time step is the value obtained as a solution from the previous time step. With small time steps (required by the stability condition), parameter changes longer than this time step will be captured and therefore transients occurring over several time steps will certainly be accounted for. Variables calculated at a given time step represent a good starting point for the next time step calculation. The iteration process within each time step is continued until the desired solution tolerance is achieved. Generally, the required calculation accuracy is obtained in under ten iterations at each time step.

Finally, at each time step, a set of Mixed Algebraic and ODE's equations (constituting a MAD set) which describe the governing equations of the connected control system. These equations are solved simultaneously at each time step in a different solver routine. The output from the control system solver is then fed to the gas dynamic solver described above and act as set points, constraints or boundary conditions.

EXAMPLE RESULTS

An example of a typical dynamic analysis conducted on a natural gas compressor station is briefly described in this section. The schematic layout of this station is that depicted in Fig. 5, and the compressor performance characteristic is that shown in Fig. 4. The compressor is Nuovo Pignone PCL 603 and is driven by LM2500 gas generator driving a PGT25 power turbine ISO rated at 23.3 MW. The unit check valve is 30" in size while the station check valve is 36". Suction Pressure ranges between 7-8 MPag while discharge pressure ranges between 11-12 MPag. The adiabatic head of these units is in the order of 60-80 kJ/kg, and the pressure ratio is ~ 1.6, hence each compressor unit is equipped with two centrifugal impellers. Due to such high head and high pressure ratio, the discharge gas temperature from the compressor unit could reach 60 ^{o}C, which requires an aerial cooler to cool the gas before leaving the station to the mainline. As mentioned earlier, these aerial coolers are placed within the recycle system to permit startup and gas recycling around the compressor unit for flow control and part loads. The power turbine and booster rotor combined inertia is relatively low (117 kg.m^2).

The Inertia number based on Eq. (9) was calculated for this station and was found to be equal to 10.36, which is below the threshold value of 30 discussed above. Secondly, the $\delta t_{\max}$ of 116 ms calculated in Section 3.0 was found to be less than the arrival time of the expansion and pressure waves, along with valve pre-stroke delay, which were calculated to be 296.18 ms and 287.85 ms, respectively. Therefore, and at the outset, a short recycle system is required.

In order to ascertain this finding, field measurements were conducted on the station where a fast-stop (equivalent to an ESD) was initiated and only the cold recycle system was active (i.e. the short recycle system was suppressed). The compressor initial operating point before the fast stop was initiated corresponded to minimum speed and minimum flow possible so as to mitigate any damage should the compressor go into surge. The measured data from this test are shown in indicative plots in Fig. 6, along with vibration measurements taken by an accelerometer placed on the flange of the unit check valve, which is a nozzle type valve. It is shown that the compressor in this case has undergone several surge cycles following a fast-stop operation.

The above test scenario was also simulated using the full dynamic simulation model described earlier. The initial operating point on the compressor characteristic map is shown in Fig. 4, while all other geometrical, operating and control conditions are given in Table 1. The results of this simulation are shown in several plots in Fig. 7. Again, it is evident that the compressor has undergone several surge cycles each of duration of about 0.86 seconds (7 surge cycles in 6 seconds). It is important to note that the station check valve closes immediately following the fast-stop, while the unit check valve is experiencing multiple cycles of opening and closing. The unit and station check valves are nozzle type valves with very little inherent damping.

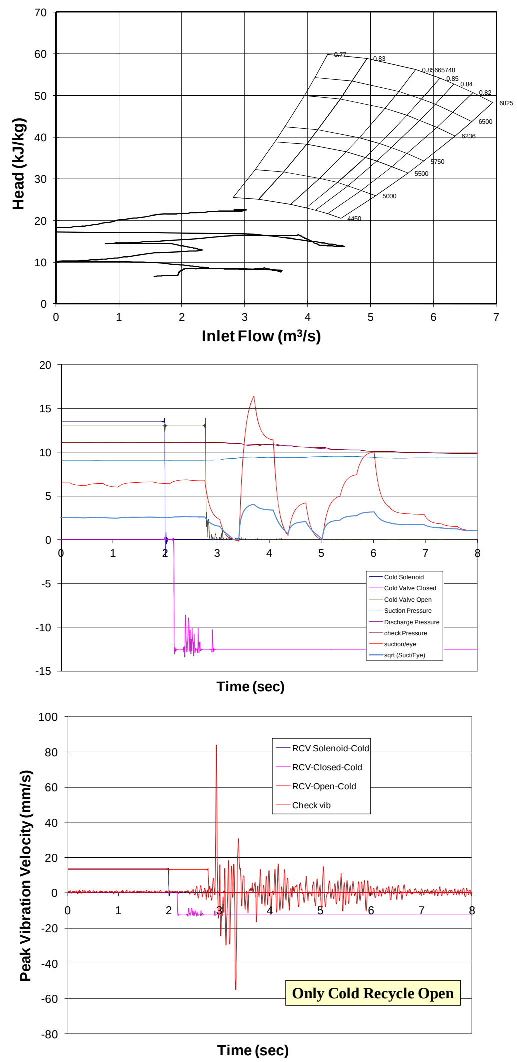

FIGURE 6: MEASURED DATA DURING ESD OF THE COMPRESSOR STATION SHOWN IN FIG. 5 USING ONLY THE COLD RECYCLE SYSTEM.

As indicated earlier in Section 3.0, the obvious solution is not only to employ a recycle valve that has a shorter pre-stroke delay, but to locate it very close to either compressor discharge (preferable) or suction sides. This means the installation of a dual recycle system, i.e. the addition of a short (hot) recycle system as shown in Fig. 5 (dotted line). In fact this compressor has dual recycle systems, which allowed another test similar to the previous one but with activation of both recycle systems: the cold and the hot ones. In this second test, higher compressor speed was permitted, and the initial condition before a fast stop was initiated is that of Fig. 4 and Table 1. The results of this test are shown in Fig. 8. It appears that the compressor has undergone one mild surge (to the left of the surge line), though not fully to a deep surge characterized by a reverse flow through the unit. Following this one mild surge, the compressor is shown to have recovered to a normal shutdown scenario without subsequently going in and out of surge as in the previous test when only the cold recycle was activated.

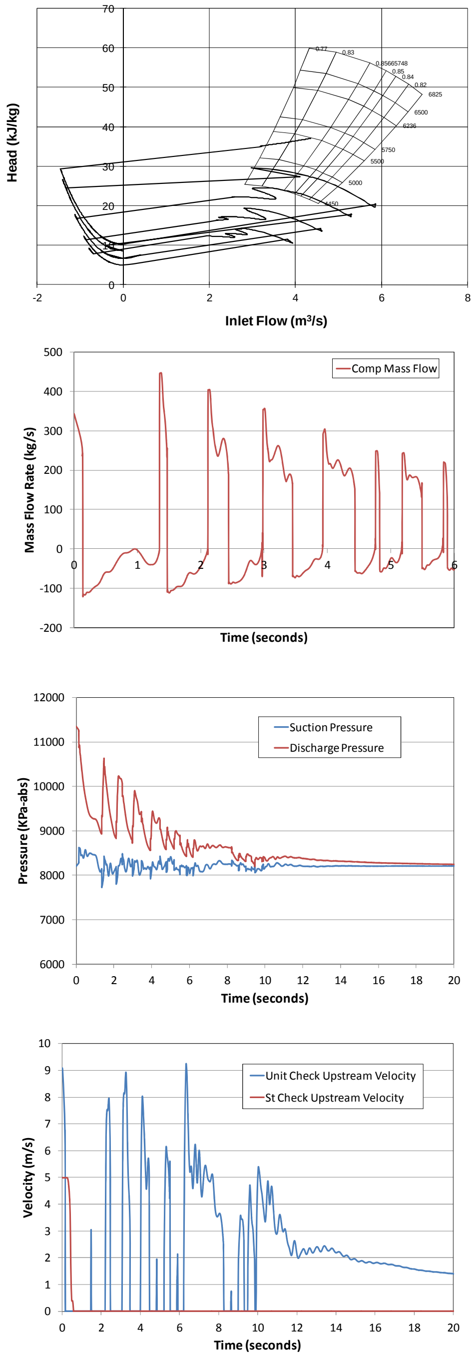

FIGURE 7: DYNAMIC SIMULATION RESULTS OF THE ESD OPERATION OF FIG. 6.

This second test was also numerically simulated and the results are shown in Fig. 9. It is indicated that the compressor has also undergone one surge cycle initially although for a very short duration (about 0.2 seconds) unlike the first surge cycle in Fig. 7 (which is approx. 1.3 seconds). The compressor is also shown to have recovered subsequently to a normal shutdown scenario without surging.

The question then arises as to why the compressor has undergone one mild (or short lived) surge cycle even with both the hot and cold recycle systems activated.

In order to answer this question, it would be helpful to revert to the simple methodology described in Section 3.0 as applied to the short (hot) recycle system. The hot recycle valve has a pre-stroke delay of 147 ms, which despite being shorter than the cold recycle valve (which is 200 ms) is still longer than δt_{max} when combined with the time of wave arrival calculated for the short recycle in Table 4. Recall that δt_{max} calculated before for this flow condition is found to be ~ 116 ms.

Had the initial flow condition been slightly to the right on the flow characteristics map, δt_{max} would have been longer, and the compressor would not undergo any surge cycles following ESD or fast-stop. To demonstrate this, a dynamic simulation was carried out similar to the previous one but from an initial operating flow at 4.8 m^3/s (i.e. slightly to the right of the previous operating point of 4.363 m^3/s). For this new initial operating point, a similar calculation revealed that δt_{max} is 206 ms, which is higher than the hot recycle pre-stroke delay plus the wave arrival time given in Table 4. Dynamic simulation was then conducted for this case and the results are shown in Fig. 10 indicating that the compressor has wound down following a fast-stop without any signs of surge.

It is clear from the above results that in the present case of a high head, low inertia compressor along with an aerial cooler being placed in the cold recycle path, a short recycle was necessary, and that could have been determined earlier in the design without even resorting to a full dynamic simulation, primarily by means of the simple methodology described in Section 3.0 and by the criterion of the Inertia number threshold given in Section 4.0.

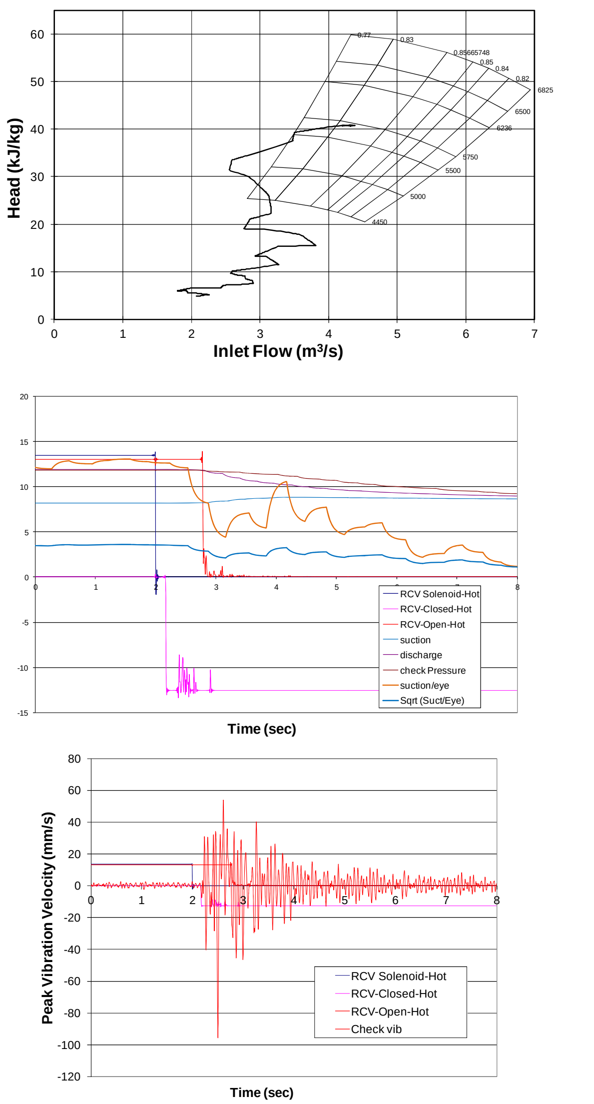

FIGURE 8: MEASURED DATA DURING ESD OF THE COMPRESSOR STATION SHOWN IN FIG. 5 USING BOTH HOT AND COLD RECYCLE SYSTEMS.

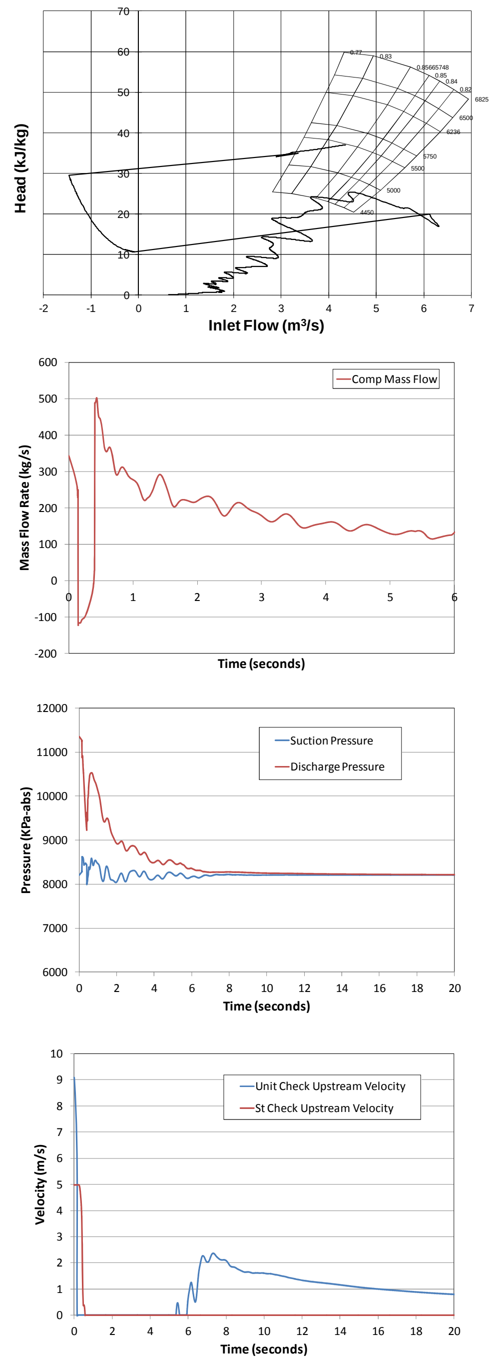

FIGURE 9: DYNAMIC SIMULATION RESULTS OF THE ESD OPERATION OF FIG. 8.

TABLE 4: CALCULATION OF ARRIVAL TIME OF THE EXPANSION AND PRESSURE WAVES TO COMPRESSOR UNIT DISCHARGE AND SUCTION SIDES, RESPECTIVELY (HOT RECYCLE SYSTEM).

Valve Pre-stroke delay	147	ms
Discharge Piping Length	6	m
Expansion Wave Arrival Time	13.74	ms
Combined Time	160.74	ms
Suction Piping Length	15	m
Pressure Wave Arrival Time	37.65	ms
Combined Time	184.65	ms

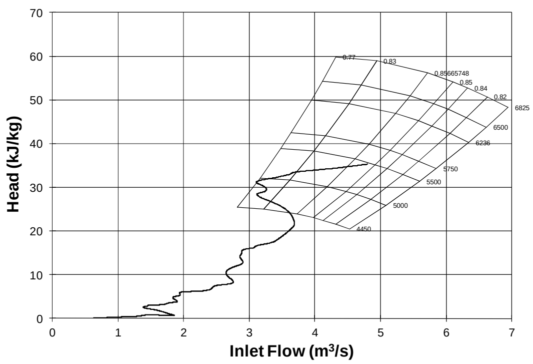

FIGURE 10: RESULTS OF DYNAMIC SIMULATION OF AN ESD OPERATION FROM SLIGHTLY HIGHER INITIAL FLOW CONDITIONS, AND USING BOTH HOT AND COLD RECYCLE SYSTEMS.

DELAY IN FUEL SHUT-OFF – WHEN IT CAN WORK

If it appears that the combined pre-stroke time and the time it takes for the expansion or pressure waves to the arrive at the compressor could be longer than $\delta t_{\max}$, a suggestion is often made to institute a delay in the fuel shut-off valve in the case of a turbine driven compressor for a period to allow for the recycle valve to start to stroke and for the waves to arrive at the compressor. That is to establish full flow through the compressor from a full recycle before the fuel valve is shut-off. While this has actually worked in stations without extra capacitance due to the presence of an aerial cooler in the recycle path [11], it will not work if such extra capacitance exists. It was alluded to in Section 3.0 that the simple methodology of comparing $\delta t_{\max}$ to the combined pre-stroke time and wave arrival time is strictly based on no extra capacitance being present in the recycle path. The latter was said to cause wave dampening and act as a muffler which isolates the compressor from the benefits of such expansion or the pressure waves that would ‘rescue’ the compressor from surging. In short, the simple methodology and the Inertia number threshold, for that matter, work only on a linear piping configuration, i.e. free from extra capacitance that cannot be accounted for in the axial distance required to calculate (τ) – see Eq. (9).

In order to demonstrate this, a dynamic simulation was conducted for the same operating condition of Fig. 4 and data of Table 1. Here, a considerably longer delay is imposed on the fuel shut-off of 2 seconds, which is much longer than the cold recycle valve would need not only to pre-stroke but to be fully open. In fact, this cold recycle valve has a 200 ms pre-stroke delay and takes an additional 1.04 seconds to stroke from a fully closed to a fully open position. That is, the 358 ms combined pre-stroke and wave arrival time does not apply any more in this scenario. The results of this simulation are shown in Fig. 11, which clearly show that the compressor continued to undergo surge cycles. The only difference between these results and that of Fig. 7 is that the first surge cycle in Fig. 7 is eliminated, but not the remainders. The reason for this is solely due to the large capacitance in the recycle path which impairs quick pressure relief of the discharge piping. In fact, such a large volume of gas at such high pressure would require a significantly large capacity recycle valve to handle the relief flow. Such large capacity valves do not exist for a recycle size of 20”.

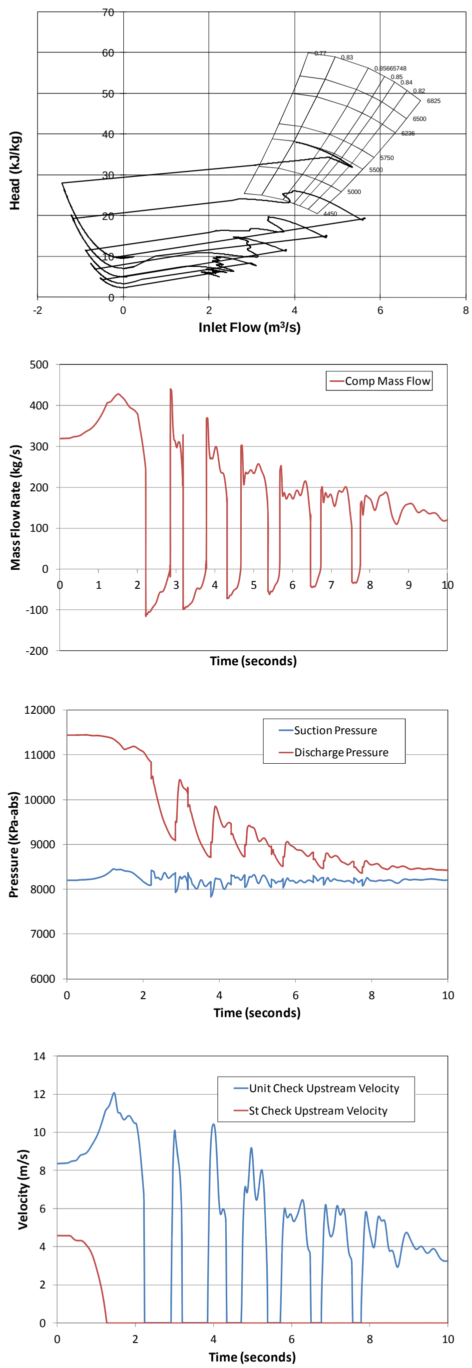

FIGURE 11: RESULTS OF DYNAMIC SIMULATION OF AN ESD OPERATION USING ONLY THE COLD RECYCLE SYSTEM WITH 2 SECONDS DELAY IN THE FUEL SHUT-OFF.

CONCLUDING REMARKS

The old belief that “a single combined surge/recycle line is acceptable and is commonly used in the industry in the design of compressor stations” should be re-examined vigilantly. A simple methodology and an Inertia number are introduced to help the designer of compression stations make the right decision as to whether a single or a dual recycle system is required for a given compression system and under given operating conditions.

It is also important to recognize the influence of salient parameters that play a key role in the dynamics of the recycle systems in compressor stations for all intended purposes and functions, namely: startup, part loads, surge control, and surge protection following ESD or fast-stop. These very key parameters are:

1. compressor/power turbine rotor inertia;
2. compressor pressure ratio (i.e. compressor adiabatic head);
3. volumetric capacitance of recycle system;
4. recycle system response time (i.e. recycle valve pre-stroke delay, stroke time, time of arrival of expansion and pressure wave from the recycle valve to the compressor outlet and inlet, respectively);
5. compressor performance characteristics; and
6. flow capacity of the recycle system.

The designer should always consider all of the above parameters and with the aid of the methodology presented in this paper, as well as the aid of dynamic simulations should be able to make the right decision in designing an adequate recycle system for the benefit of equipment integrity, performance and safety.

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