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CONCEPT OPTIMAL DESIGN OF COMPOSITE FAN BLADES

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ABSTRACT

A procedure is outlined to determine the composite lay-out of a given baseline metallic fan blades. The vane material Titanium is replaced by Carbon Fiber composite materials. For the operating speed the baseline maximum strain in the vane is maintained and weight reduction is taken as the objective function. Three phases of optimization are suggested. In the first phase free size optimization is performed on the composite blades to determine the required topology. A layout of composites is then proposed. In the second phase based on free optimization results gauge optimization setup is illustrated for determining the thicknesses. In the third phase Ply-stacking optimization can be performed. By using composite materials substantial savings in the weight can be achieved without affecting the performance of engine blade.

1. INTRODUCTION

In recent times, Aerospace industry has given considerable emphasis on replacing the metallic materials by composites for weight reduction.

Structural optimization has gained importance due to its significant impact in product development processes [1, 2]. A continuum approach to structural topology optimization was first introduced by Bendsøe and Kikuchi [3]. Optimization of Finite Element-based structures is acknowledged as a useful methodology for achieving important improvements in product design and is widely used in automotive and aerospace industries. Prior to commercial code development, conventional optimization in structures was achieved by determining the strain energy density and identifying material areas where a removal can be made or it is required to be strengthened and adopt a DOE approach. This is a tedious and time consuming process requiring skilled engineers. Many commercial codes have now added optimization processes; however, topology optimization received considerable attention in recent times.

Topology Optimization capability in commercial codes such as OptiStruct has been successfully used in the recent times to obtain concept designs of aerospace e.g., A400M [4]. Topology optimization uses SIMP (Solid Isotropic Material with Penalization) method which is also called *density method*. In this method, the stiffness of the material is assumed to be linearly dependent on the density. The material density of each element is directly used as the design variable, and is normalized to have a value between 0 and 1 representing the state of void and solid, respectively.

Rao et al [5] adopted topology optimization to derive an aluminum wing structure starting from the airfoil shape itself, eliminating the intermediate steps of deriving the base line.

It is well known that composite structures are lighter in weight and give substantial weight savings compared with metallic structures. For example Poort [6] has shown the optimization of the 787 Dreamliner Horizontal Stabilizer CFRP Composite Main Box. Design and Optimization of Laminated Composite Materials were discussed by Gürdal, Haftka and Hajela [7]. Ryoo and Hajela [8] presented a novel approach for handling variable string lengths in GA-based topological design and implemented in representative algebraic problems, truss topology design, and the layout of a stiffened composite panel. Zhou and Fleury [9] discussed some of the latest capabilities in composite design and optimization. They presented Free-Size Topology Optimization, Ply-Bundle Sizing with ply-based FEA modeling and Ply Stacking Sequence Optimization. Rao and Kiran [10] adopted their Aluminum wing derived from basic airfoil to provide a concept composite wing structure with a reduction of weight by 35.7%.

In advanced military aircraft engines the weight removal can be a major objective. Till recently weight optimization was achieved by determining the strain energy density and identifying material areas where a removal can be made or it is required to be strengthened and adopt a DOE approach, see Rao et al [11]. Rao et al [12] used optimization methods to achieve removal of mass in the shank areas of blades.

Modern fans account for an increasing portion of total engine weight and employ the longest blades in the engine. The larger fans are driving the need for new lighter materials. It is estimated that ½ kg mass increase on fan blade requires ¼ kg increase each in containment case weight, rotor weight, engine structure and aircraft structure, see Crall [13]. Graphite/epoxy material properties simultaneously reduce fan weight and improve durability over metallic structures.

NOMENCLATURE

A Area

- *E* Young's modulus
- G Shear modulus
- $m \cos \theta$
- *n* $\sin \theta$
- P Load
- v volume fraction
- ε Strain
- τ Stress
- μ Poisson's ratio
- ρ Density

Subscripts f, m and c denote fiber, matrix and composite respectively

2. BACKGROUND OF COMPOSITE TECHNOLOGY

Composites consist of high strength and modulus fibers embedded in a matrix or bonded to a matrix. Both the fibers and the matrix retain their distinct properties and together they produce properties which cannot be achieved individually.

The fibers are usually the principal load carrying elements in the composite and the purpose of the matrix is essentially to keep the fibers in the desired location. The matrix also serves the purpose of protecting the fiber from heat, corrosion and other environmental damages.

The fibers may be glass, carbon, boron, silicon carbide, aluminum oxide etc. One of the popular commercial fibers is

Kevlar 49. These fibers may be embedded in the matrix either in a continuous form or in discontinuous form (chopped pieces of different lengths).

A laminate consists of several laminae (Plies) each with a fiber oriented at a particular angle. This is generally made by stacking several thin layers of fibers (Plies) at the desired locations and angles in a matrix and consolidating them to give the required thickness.

The fiber orientation in each thin layer can be arranged in a specific manner so as to achieve the required properties of the structural member. Since the fibers are of high strength in their axial direction and at the same time very light compared to conventional metals, they find many applications in aerospace, automobile industry etc. see [14].

A 0° lamina with fibers aligned in the direction of load is shown in Fig. 1a.



Fig. 1a 0º lamina with fibers aligned in the direction of load

The strains and stresses in the fiber and matrix are

$$\varepsilon_{f} = \varepsilon_{m} = \varepsilon_{c}$$

$$\tau_{f} = E_{f}\varepsilon_{f} = E_{f}\varepsilon_{c}$$

$$\tau_{m} = E_{m}\varepsilon_{m} = E_{m}\varepsilon_{c}$$
(1)

The load on the lamina is then

$$P = P_f + P_m$$

$$\tau_c A_c = \tau_f A_f + \tau_m A_m$$
(2)

Then, the stress in the composite is

$$\tau_c = \frac{1}{A_c} \left(\tau_f A_f + \tau_m A_m \right)$$

= $\tau_f v_f + \tau_m v_m = \tau_f v_f + \tau_m \left(1 - v_f \right)$ (3)

The longitudinal modulus and the composite Poisson's ratio can be written as

$$E_{11} = \frac{\tau_c}{\varepsilon_c} = E_f v_f + E_m (1 - v_f)$$

$$\mu_{12} = \mu_f v_f + \mu_m v_m$$
(4)



Fig. 1b 0° lamina with fibers transverse to the direction of load

For the load acting transverse to the fiber orientation, see Fig. 1b, we can similarly obtain the transverse modulus and minor Poisson's ratio as

Fig. 1c 0° lamina with shear load

In case of shear load as in Fig. 1c, the composite shear modulus can be shown to be



For an angle ply lamina shown in Fig. 2 we can use a transformation rule and obtain the composite moduli and Poisson's ratio as

$$\begin{aligned} \frac{1}{E_{xx}} &= \frac{m^4}{E_{11}} + \left(\frac{1}{G_{12}} - \frac{2\mu_{12}}{E_{11}}\right) m^2 n^2 + \frac{n^4}{E_{22}} \\ \frac{1}{E_{yy}} &= \frac{n^4}{E_{11}} + \left(\frac{1}{G_{12}} - \frac{2\mu_{12}}{E_{11}}\right) m^2 n^2 + \frac{m^4}{E_{22}} \\ \frac{1}{G_{xy}} &= \frac{1}{G_{12}} + 4 \left(\frac{1 + 2\mu_{12}}{E_{11}} + \frac{1}{E_{22}} - \frac{1}{G_{12}}\right) m^2 n^2 \end{aligned} \tag{7}$$

$$\mu_{xy} &= E_{xx} \left[\frac{\mu_{12}}{E_{11}} - \left(\frac{1 + 2\mu_{12}}{E_{11}} + \frac{1}{E_{22}} - \frac{1}{G_{12}}\right) m^2 n^2 \right]$$

$$\mu_{yx} &= \frac{E_{yy}}{E_{xx}} \mu_{xy}$$

The compliance relations for the general case are

$$\begin{cases} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{cases} = \begin{bmatrix} \frac{1}{E_{xx}} & \frac{-\mu_{xy}}{E_{xx}} & \frac{-\eta_{xy}}{E_{xx}} \\ & \frac{1}{E_{yy}} & \frac{-\eta_{xy}}{E_{yy}} \\ & & \frac{1}{G_{xy}} \end{bmatrix} \begin{bmatrix} \tau_{xx} \\ \tau_{yy} \\ \tau_{xy} \end{bmatrix}$$
(8)

where

$$\eta_{xy} = E_{xx} \left[-\frac{2m^3n}{E_{xx}} + \frac{2mn^3}{E_{yy}} + \left(\frac{1}{G_{xy}} - \frac{2\mu_{xy}}{E_{xx}} \right) (m^2 - n^2) mn \right]$$
$$\eta_{yx} = E_{yy} \left[-\frac{2mn^3}{E_{xx}} + \frac{2m^3n}{E_{yy}} - \left(\frac{1}{G_{xy}} - \frac{2\mu_{xy}}{E_{xx}} \right) (m^2 - n^2) mn \right]$$

The above relations can be used to build a number of suitable plies of different angles θ to carry the load most effectively by taking the advantage of directionally superior strength properties of fibers.

In this paper a procedure is outlined to determine the composite lay-out of a given baseline metallic fan vane. The vane material has been replaced by Carbon Fiber composite materials. For the operating speed the baseline maximum strain in the vane is maintained and weight reduction is taken as the objective function.

To arrive at the optimal design of the blade, three phases of optimization are needed. In the first phase free size optimization has been performed on the composite blades to determine the required topology. A layout of composites is then proposed. In the second phase based on free optimization results gauge optimization is made for determining different thicknesses. In the third phase Ply-stacking optimization is to be performed and here a procedure to do this is outlined.

3. BASE LINE MODEL

The baseline adopted for the analysis and generating concept optimal design is shown in Fig. 3. The blades are of titanium, with E = 105 GPa, v = 0.23, $\rho = 4.429 \times 10^{-9}$. The hub is taken of Stainless steel (Non Design Part). There are 18 blades and total mass excluding the hub is 3.722 Kg. The blade 200 mm long with a constant chord 65 mm and 84° pre-twist rotates at 15000 rpm.



Fig. 3 Baseline Model

The solid model is converted into equivalent shell model as shown. As the blades are symmetric, only single blade is used for optimization. (Mass = 206.8 grams)



Fig. 4 Stress Analysis of Baseline Model

3.2 Static Analysis

The blades are acted upon by air loads to be determined from a CFD analysis and centrifugal loads. Rao and Saravana Kumar [15] performed numerical simulation of the flow in a two stage turbine. The fan here is long and at 15000 rpm the stress field is very strong compared to air flow and therefore the gas loads are neglected to simplify the problem and demonstrate the design process for the fan vanes. The baseline stress field obtained is as shown in Fig. 4. The peak stress is 343 MPa and the strain is 0.00326.

3.2 Dynamic Analysis

While obtaining a composite blade vane it should be kept in mind that the natural frequency at operating speed is sufficiently away from the critical speed on the Campbell diagram. With reduction in mass and increase in stiffness the natural frequency would increase with a composite blade. This is not taken into account for the present optimization. Because of employing the composites with superior properties the stress or strain levels can be more than that allowed in metallic structure.

4. COMPOSITE OPTIMIZATION

Phase I - Free-Size or Topology Optimization

This step is to create design concepts that utilize all the potentials of a composite structure where both structure and material can be designed simultaneously. By varying the thickness of each ply with a particular fiber orientation for every element, the total laminate thickness can change continuously throughout the structure, and at the same time, the optimal composition of the laminate at every point (element) is achieved simultaneously. Manufacturing constraints like lower and upper bound thickness on the laminate, individual orientations and thickness balance between two given orientations are also defined in this stage.

Phase II – Ply-Bundle Sizing with ply-based FEA modeling

Sizing optimization is performed to control the thickness of each ply bundle, while considering all design responses and optional manufacturing constraints. Ply thicknesses are directly selected as design variables. Composite plies are shuffled to determine the optimal stacking sequence for the given design optimization problem while also satisfying manufacturing constraints like control on number of successive plies of same orientation, pairing 45° and -45° orientations together etc.

Phase III – Ply Stacking Sequence Optimization

Composite plies are shuffled to determine the optimal stacking sequence for the given design optimization problem while also satisfying additional manufacturing constraints like,

- The stacking sequence should not contain any section with more than a given number of successive plies of same orientation.
- The 45° and -45° orientations should be paired together
- The cover and/or core sections should follow a predefined stacking sequence

This detailed design is not carried out in this paper.

5. PHASE 1: FREE SIZE TOPOLOGY OPTIMIZATION

The unidirectional lamina of Carbon Fiber composite used is assumed to have the following properties:

Volume fiber fraction = 0.5

Young's modulus (in fiber direction) $E_{11} = 115$ GPa

Young's modulus (perpendicular to fiber direction) $E_{22}\ \&\ E_{33}$ = 15 GPa

Shear modulus G = 4.3 GPa

Density = 1500 Kg/mm^3

Objective function: Weight of the blade Variables:

The Free size variables can be taken into two types, the first one being the uniformly distributed thickness shell section can be taken varying from 1.75 to 5.75 mm from the blade ends to the center varying along the blade profile length and the second one as the single shell section 6 mm. The first option is adopted.

The design space is limited to the blade airfoil shape. The baseline is made of five patches (collectors) as shown in Figs. 5 and 6 and given in Tables 1a to 1e. The outer stacks 1 and 5 have a total thickness of 1.75 mm. Stacks 2 and 4 are similarly equal with 4.25 mm thick. The middle one, stack 3, has maximum thickness 5.75 mm. Carbon Fiber Reinforced Plastic with the properties given above is chosen for the laminate plies.





Ply lay-up: LAM_I Total number of plies: 4 Total thickness: 4 25

		Ply	Material	Thickness T1	Orientation Degrees
	•	1	BLADE_COMP	1.0625	0.0
<i> </i>		2	BLADE_COMP	1.0625	45.0
	•	3	BLADE_COMP	1.0625	-45.0
		4	BLADE_COMP	1.0625	90.0

Table 1c Base Laminate for STACK 3

Ply lay-up: Total number of plies:	4 4				
l otal thickness:	5.75	Ply	Material	Thickness T1	Orientation Degrees
	•	1	BLADE_COMP	1.4375	0.0
		2	BLADE_COMP	1.4375	45.0
	•	3	BLADE_COMP	1.4375	-45.0
	•	4	BLADE_COMP	1.4375	90.0



Table 1d Base Laminate for STACK 4

LAM_D13

4 BLADE_COMP 1.0625 90.0 Table 1e Base Laminate for STACK 5 Ply lay-up: LAM_E17

Total number of plies: Total thickness:	4 1.75				
		Ply	Material	Thickness T1	Orientation Degrees
	•	1	BLADE_COMP	0.4375	0.0
		2	BLADE_COMP	0.4375	45.0
	•	3	BLADE_COMP	0.4375	-45.0
	•	4	BLADE_COMP	0.4375	90.0

The maximum principal strains contour is given in Fig. 7 with the maximum value equal to 0.002855

After the Free Size Optimization, the patch locations for each Ply Bundle are found. There are 20 Ply Bundles. The results of all these plies are not shown here. The super plies are next obtained for each orientation as given in Figs. 8a to 8d.

We notice that in all the cases of the super plies, the middle stack has maximum thickness of 1.438 mm. The leading and trailing edges have minimum thickness.

Ply lay-up:



Fig. 5 Single Blade Composite FE Model



Fig. 6 Composite showing the five stacks



Fig. 7 Baseline Max Principal Strains for Composite (Baseline)





Fig. 8b Free Size Optimization for Super Ply at +45° Orientation

6. PHASE 2: SIZE OPTIMIZATION

The size optimization of the Plies obtained in Phase 1 with Free Size or Topology optimization was next performed to determine the thickness and laminate family within all the shell sections chosen in the blade.

Manufacturing constraints are defined that include ply percentage, ply thickness balancing, ply thickness, and laminate thickness, which can be defined at the concept level free sizing stage.



Fig. 8c Free Size Optimization for Super Ply at -45° Orientation



Fig. 8d Free Size Optimization for Super Ply at 90° Orientation

For the composite blade optimization discussed in this paper, two manufacturing constraints were incorporated:

- Ply percentage for the 0's and 90's such that no less than 1. 10% and no more than 60% exist.
- 2. A balance constraint that ensures an equal thickness distribution for the +45's and -45's.

The optimization results for total thickness are given in Figs. 9a to 9d for all 0° , $+45^{\circ}$, -45° and 90° plies respectively; with maximum thicknesses in the middle patch at the root given by 1.509, 1.344, 1.344 and 1.474 mm respectively

For the structural loading under consideration, weight saving obtained over the baseline metallic blades is 27%.

A review of the design process up to now reveals that we established the optimum ply shape and patch locations in phase 1 (free size optimization) and subsequently optimized the ply bundle thicknesses in phase 2 (ply bundle sizing optimization), allowing us to determine the required number of plies. These ply bundles represent the Optimal Ply Shapes (Coverage Zones).



Fig. 9b Total Thickness of 45° Ply after Size Optimization



Fig. 9c Total Thickness of -45° Ply after Size Optimization



Fig. 9d Total Thickness of 90° Super Ply

7. PHASE 3: PLY STACK OPTIMIZATION

A detailed design for Ply Stacking Sequence Optimization can be adopted by using algorithm HyperShuffle, see [10], aimed at providing a 'Global view' of what the optimal stacking sequence could be. While shuffling the stacking sequence it is important that behavioral and design constraints are preserved. Further, it is required that certain ply book rules be applied to guide the stacking of plies based on specific requirements

Some of the ply book rules that control the stacking sequence are:

- Symmetric Stack Required
- Number of plies in any one direction placed sequentially in the stack is limited
- Stack is balanced, i.e. the number of 45° and -45° plies is the same.
- Outer plies for the laminate should contain a particular ply (i.e. ±45°)
- Minimize the number of occurrences of the 0° to 90° (or 90° to 0°) change in any two adjacent plies.
- Minimize the number of occurrences of 45° to -45° (or -45° to 45°) change inside the stack by putting one 0° or 90° ply between them

Table 2a

Stacking sequence for STACK 1

Stacking sequence for STACK 2

Iteration 0	Iteration 1	Iteration 2	Iteration 3	Iteration 4
11101	12101	11201	12301	12301
12101	13101	11301	13301	13301
13101	11101	11401	11201	11201
14101	11201	12201	11301	11301
11201	12201	13201	11401	11401
12201	13201	12301	12201	12201
13201	11301	13301	13201	13201
11301	12301	12401	12401	12401
12301	13301	13401	13401	13401
13301	11401	14101	14101	14101
11401	12401	12101	12101	12101
12401	13401	13101	13101	13101
13401	14101	11101	11101	11101

Table 2b

Iteration 0	Iteration 1	Iteration 2	Iteration 3	Iteration 4
21101	22101	21401	21401	21401
22101	23101	21301	21301	21301
23101	21101	21201	21201	21201
24101	24101	22201	22201	22201
21201	22201	23201	23201	23201
22201	23201	22401	22401	22401
23201	21201	23401	23401	23401
24201	24201	22301	22301	22301
21301	22301	23301	23301	23301
22301	23301	24301	24301	24301
23301	21301	24201	24201	24201
24301	24301	22101	24401	24401
21401	22401	23101	22101	22101
22401	23401	24401	23101	23101
23401	21401	21101	21101	21101
	24401	24101	24101	24101

Stacking sequence for STACK 3

Table 2c

Iteration 0	Iteration 1	Iteration 2	Iteration 3	Iteration 4	Legend
31101	32101	31401	34401	32101	90.0 degree
32101	33101	34401	32101	33101	45.0 degree
33101	31101	32101	33101	31401	0.0 degree
34101	34101	33101	31401	31101	-45.0 degre
31201	32201	31101	31101	34401	
32201	33201	34301	34301	34301	
33201	31201	34201	34201	34201	
34201	34201	34101	34101	34101	
31301	32301	32201	32201	32201	
32301	33301	33201	33201	33201	
33301	31301	32301	32301	32301	
34301	34301	33301	33301	33301	
31401	32401	32401	32401	32401	
32401	33401	33401	33401	33401	
33401	31401	31301	31301	31301	
34401	34401	31201	31201	31201	

Legend

15 0 de

		Г	Table 2d		
acking se	quence for ST	ACK 4			
				-	
Iteration 0	Iteration 1	Iteration 2	Iteration 3	Iteration 4	Legend
41101	42101	41101	41101	41101	90.0 degrees
42101	43101	42101	42101	42101	45.0 degrees
43101	41101	43101	43101	43101	0.0 degrees
44101	44101	44101]	44101 Table 2e	44101	-45.0 degrees
⁴⁴¹⁰¹ tacking se	44101 quence for ST	44101] [ACK 5	⁴⁴¹⁰¹ Table 2e	44101	-45.0 degrees
44101 tacking se Iteration 0	44101 quence for ST	44101	44101 Fable 2e	44101 Iteration 4	-45.0 degrees
44101 tacking se Iteration 0 51101	44101 quence for ST Reration 1 52101	44101	44101 Fable 2e Reration 3 51101	44101 Reration 4 51101	-45.0 degrees
44101 tacking se Iteration 0 51101 52101	44101 quence for ST Iteration 1 52101 53101	44101	44101 Fable 2e Reration 3 51101 52101	44101 Iteration 4 51101 52101	-45.0 degrees Legend 90.0 degrees 45.0 degrees
44101 tacking se Iteration 0 51101 52101 53101	44101 quence for ST Reration 1 52101 53101 51101	44101	44101 Fable 2e Reration 3 51101 52101 53101	44101 Iteration 4 51101 52101 53101	Legend 90.0 degrees 45.0 degrees 0.0 degrees

The stacking sequences obtained after four iterations of shuffling optimization are given in Tables 2a to 2e for the five stacks used.

For the middle stack 3 which is thickest and having 16 plies as in Table 2c, the ply thicknesses in mm are given in Table 3. The maximum thickness in this stack is then 5.664 mm.

	Table 3
PLY	ТНК
31101	1.262
31201	0.095
31301	0.137
31401	0.013
32101	1.097
32201	0.084
32301	0.144
32401	0.017
33101	1.097
33201	0.084
33301	0.144
33401	0.0172
34101	0.925
34201	0.246
34301	0.092
34401	0.21

Here no restriction was prescribed on the minimum ply thickness and also sizes available to the stacker. These constraints can be included.

Maximum principal strain of the optimized composite blade obtained in Fig. 10 is 0.00364 same order as baseline metallic blade 0.00326. Note that there is still considerable margin for a composite because of its strength.



Fig. 10 Max Principal Strains of Optimized Composite Blade

8. CONCLUSION

A procedure for obtaining a composite fan blade from the given metallic blade is presented.

The steps in Free Sizing or Topology optimization of the baseline composite are presented. Five stacks are adopted here. The super plies and drop off plies required are shown.

Then a sizing optimization is performed for minimum weight. Manufacturing constraints are included in the sizing optimization. Total thicknesses of 0° , $+45^{\circ}$, -45° and 90° plies are presented in all five stacks.

Finally a Ply-Stacking optimization is performed taking into account manufacturing constraints. The stacking in all five stacks is shown.

The weight savings of 27% was achieved for the structural load case considered. The maximum strain is kept to be of the same order in the final optimized vane as that in the metallic blade baseline, though the composite can take much higher value.

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