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LIMITS ON MORROW MEAN STRESS CORRECTION OF MANSON-COFFIN LIFE PREDICTION MODELS

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ABSTRACT

Accurate prediction of crack initiation life is of critical importance in designing turbo-machinery. To improve this accuracy, more sophisticated prediction techniques are required. A large number of materials have shown a correlation between low cycle fatigue initiation life and strain range, as represented by the well-known Manson-Coffin equation. Testing has shown that tensile mean stress has a negative impact on life. This effect has been noted for many years when applied to high-cycle fatigue, resulting in the use of Goodman or Haigh diagrams to account for the impact of both stress range (and therefore strain range) and mean stress.

Morrow proposed a methodology for accounting for mean stress in low cycle fatigue. Noting that as plasticity increases, the effect of mean stress decreases, the correction was applied only to the elastic strain versus life line. Use of the Morrow mean stress corrections improves the accuracy of life predictions, but there are limitations. The most significant of these limitations are situations in which the correction may be non conservative for high compressive mean stresses or very high tensile mean stresses. While a benefit from compressive mean stress is to be expected, at some point further increasing the compressive mean stress should have a negative impact on life. At very high tensile mean stresses near the material yield, the calculated impact of mean stress on life is non conservative.

To overcome these limitations, the analyst may place limits on the acceptable range of R-ratios used based on actual test data, but this would do little more than highlight when the user is outside of his database limits. Alternatively a life system may be made in which mean stresses are conservatively expected to be tensile. Neither of these methods are useful for calculating different lives in compressive or tensile regimes. This difficulty is especially seen in bolted joints, which are subjected to high mean stresses and small stress amplitudes. A technique is proposed here in-which limits are placed on the mean stress correction, directly analogous to those used in the creation of so-called modified Goodman diagrams. This technique has been successfully applied at PSM to improve the accuracy of life prediction without increasing the risk of non conservatism. A review of some literature is made to show examples where this effect may be taking place. A small number of tests provide additional validation.

INTRODUCTION

The high cost of electrical power puts pressure on plant operators to run their units at the extreme edge of component capability. Operators increase firing temperatures until any excess capability is consumed. This aggressive usage makes it especially critical that accurate low cycle fatigue, LCF, predictions be made during component design. Without accurate life predictions it is not possible to optimize the aero, mechanical, and thermal aspects of the design. Furthermore, it is not sufficient to calculate life at discreet locations on a component. To avoid escapes, the analyst must be able to codify his life calculation procedure so it can be applied accurately to every node in a finite element model, Fig 1.

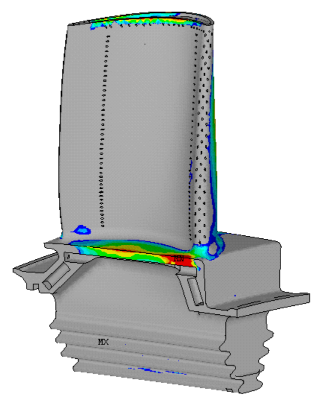

Figure 1. Turbine Blade FEA w/Life Prediction

Numerous empirical relationships have been published between the initiation life and variables such as strain range, temperature and the stress-state. This paper will discuss the evolution of one popular model, the Manson-Coffin model, with emphasis on the Morrow mean stress correction. While this method has received widespread usage, it has difficulties when either extremely high or low values of mean stress are applied. A method will be proposed for placing limits on this mean stress correction, to prevent non conservative predictions. Methods used for predicting the state of stress and strain are of equal importance to the prediction of life. Some of these methods will be discussed in general terms, but are too numerous, complex, and proprietary to be discussed in this paper.

NOMENCLATURE

A	*A-ratio,* $\frac{\sigma_a}{\sigma_m} = \frac{1-R}{1+R}$
b	Fatigue Strength Exponent
c	Fatigue Ductility Exponent
e	Strain From Linear Analysis
E	Static Modulus
F_{MM}	Modified Morrow Factor
HCF	High Cycle Fatig*ue*
Ksi, ksi	klb/in^2
LCF	Low Cycle Fatigue
N	Number of cycles
Ni	Number of cycles to initiation
Nf	Number of cycles to failure
Nr	Number of reversals, 2N
R	R-ratio, $\frac{\sigma_{\min}}{\sigma_{\max}} = \frac{1-A}{1+A}$
S	Stress or Strain as specified in context
ε_a	Strain Amplitude
$\varepsilon_{endurance}$	Strain Amplitude at endurance limit
ε'_f	Fatigue Ductility Coefficient
ε'_{yield}	Strain Amplitude at cyclic yield
$\Delta\varepsilon$	Strain Range
$\Delta\varepsilon_{elastic}$	Elastic Strain Range
$\Delta\varepsilon_{plastic}$	Plastic Strain Range
σ	Stress
σ'_f	Fatigue strength Coefficient
σ_a	Stress Amplitude
σ_{al}	Vibratory Stress Allowable
$\sigma_{cintersect}$	Compressive Mean Stress Intersect
σ_e	Endurance Limit
σ_m	Mean Stress
σ_{m_ult}	Cyclic Ultimate
$\sigma_{tintersect}$	Tensile Mean Stress Intersect
σ_u	Ultimate Strength
σ_y	Yield Strength
σ'_{yield}	Cyclic Yield Strength

MANSON-COFFIN MODEL BACKGROUND

A century ago, Basquin, proposed that the stress, S, should be plotted against the number of cycles, N, as a log-log relationship, [1]. Thus the SN curve was born. This methodology was primarily applied in a higher life regime, so no plasticity was accounted for. Since the behavior was elastic, in this context an SN curve could refer to either stress or strain.

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma'_f}{E}\left(2N_f\right)^b \quad (1)$$

In 1953 Manson and Coffin independently proposed a relationship between the plastic strain and initiation life, [2][3]. This relationship is referred to as either the Manson-Coffin or the Coffin-Manson model. It relates the number of cycles to the plastic strain as a straight line on a log-log scale.

$$\frac{\Delta\varepsilon_{plastic}}{2} = \varepsilon'_f\left(2N_f\right)^c \quad (2)$$

Manson and Coffin later combined the elastic term from the Basquin relationship with the plastic terms to produce the Manson-Coffin-Basquin relationship written here in terms of stress amplitude and cycles instead of reversals (3).

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma'_f}{E}\left(2N\right)^b + \varepsilon'_f\left(2N\right)^c \quad (3)$$

Because the fatigue strength exponent and the fatigue ductility exponent are dramatically different, the resulting relationship is no-longer a straight line in log-log, Figure 2. The combined expression is accurate over a much wider strain range.

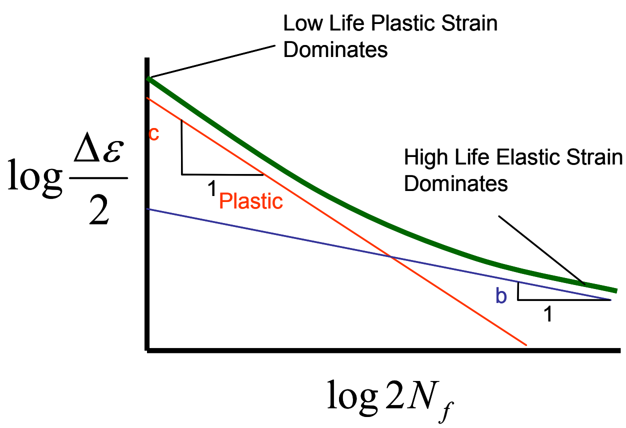

Figure 2. Manson-Coffin Curve

The coefficients and the exponents must be determined by material testing and regression. The definition of total strain (4) would indicate that the coefficients in equations (1) and (2) be directly related to those in equation 3.

$$\Delta\varepsilon = \Delta\varepsilon_{elastic} + \Delta\varepsilon_{plastic} \qquad (4)$$

Some references [4] assume that the elastic and plastic portions of the strain vs life curve are exactly an addition of (2) and (3) and will therefore relate the coefficients to the terms of the Ramberg-Osgood equation. Although, these relationships provide trends that are helpful in estimating curves at temperatures where data is absent, this author has not observed a exact partitioning of the plastic and elastic portions of strain.

MEAN STRESS CORRECTIONS FOR HCF

The effects of non-zero mean stress, or R-ratio effects, have been observed for decades in relation to HCF. As mean stress increases the fatigue stress capability degrades. This has led to the Goodman equation (5).

$$\sigma_{al} = \left(1 - \frac{\sigma_m}{\sigma_u}\right)\sigma_e \qquad (5)$$

The less conservative Gerber equation (6)

$$\sigma_{al} = \left(1 - \left(\frac{\sigma_m}{\sigma_u}\right)^2\right)\sigma_e \qquad (6)$$

The more conservative Soderberg equation (7)

$$\sigma_{al} = \left(1 - \frac{\sigma_m}{\sigma_y}\right)\sigma_e \qquad (7)$$

These failure theories are most commonly presented graphically in the form of a Goodman Diagram, Figure 3., although this name applies only to (5). A more generalized description of this figure would be called a Haigh Diagram, wherein the curve does not necessarily adhere to equation 5, and multiple lines of constant life may be plotted. The Haigh Diagram is sometimes refered to as a stress-range diagram. Numerous examples of mean stress effecting fatigue capability are documented, [5]

It is worth mentioning that the yield and ultimate strength quoted in these equations are determined from monotonic tests. When these equations are applied to ductile alloys, one may question the validity of a monotonic value. For example, consider that Inco625 might have an average RT tensile yield of 414 MPA (60Ksi) [6], and have a 10E7 Cycle Endurance limit of 454 MPA (455Ksi), [7]. This alloy cyclically hardens to a higher cyclic yield.

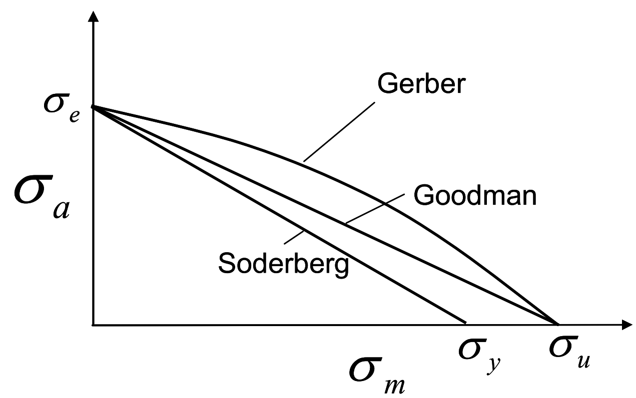

Figure 3. Goodman Diagram, with Gerber and Soderberg

Of the 3 equations, the Goodman equation is the most widely accepted relationship. The Goodman equation is sometime modified at high mean stresses by truncating the allowable stress above the intercept of a line from yield on the alternating axis to yield on the mean axis. This does not indicate that failure occurs when the elastic mean stress exceeds cyclic yield. Rather plasticity will result in the max stress equaling the yield stress for mean stresses beyond the intersection of the endurance-ultimate line and the yield-yield lines, [8]. This becomes a Modified Goodman Diagram, Figure 4.

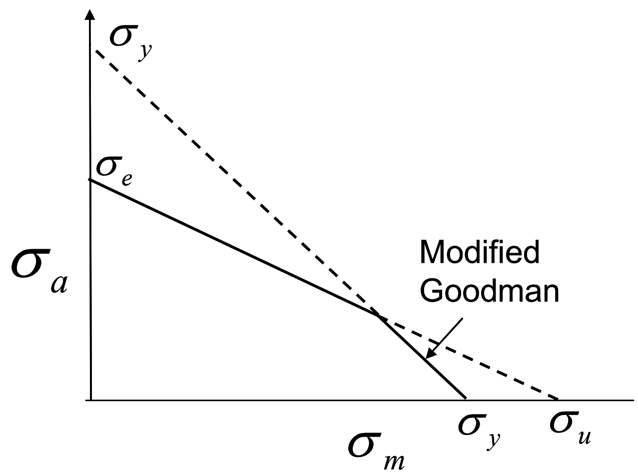

Figure 4. Modified Goodman Diagram

Considering that positive mean stresses decrease life, leads to the obvious questioning of whether compressive mean stresses increases fatigue capability. Should the Goodman diagram be conservatively reflected in the direction of negative mean stress or should a credit be assumed? Figure 5 shows some of the possible alternatives. The influence of compressive mean stress on life is greater in LCF than HCF, [8]. In HCF, common practice is to not take a benefit for compressive mean stresses. Therefore the blue curve is not generally assumed. This may be based more on conservatism rather than on data. Either the middle (red) or lower (green) curve is generally assumed.

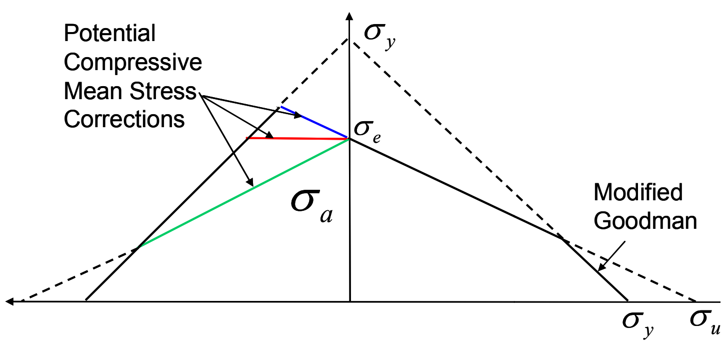

Figure 5. Modified Goodman Diagram w/Compression

MORROW MEAN STRESS CORRECTION

It was observed that strain controlled fatigue tests operated at A=1 (or R=0, positive mean strain) conditions resulted in lower lives than fully reversed (A=∞, or R=-1) tests. Conversely, tests in compression gave higher lives, Figure 6.

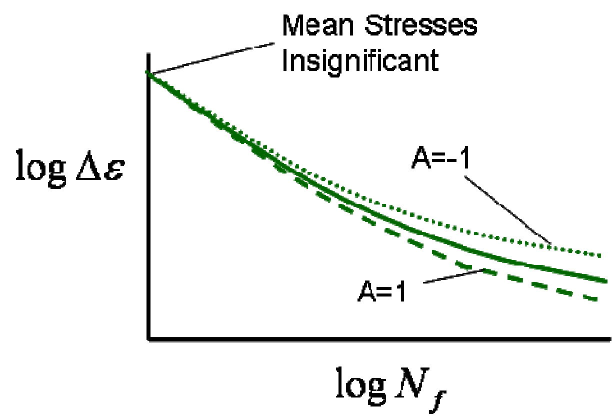

Figure 6. Modified Goodman Diagram w/Compression

Morrow observed that the effect of mean stress decreased as strain increased. At high strain levels, mean stresses are expected to shake-out [9]. Morrow modified just the elastic portion of the Manson-Coffin equation (8) in a manner directly analogous to the Goodman equation.

$$\varepsilon_a = \frac{\Delta\varepsilon}{2} = \frac{\sigma'_f - \sigma_m}{E}\left(2N_i\right)^b + \varepsilon'_f\left(2N_i\right)^c \quad (8)$$

At high values of life the plastic portion of strain amplitude goes to zero. As the mean stress approaches σ'_f, the cyclic strain amplitude goes to zero. Plotting this expression with a line of constant life, we see the similarity to the Goodman equation, Figure 7.

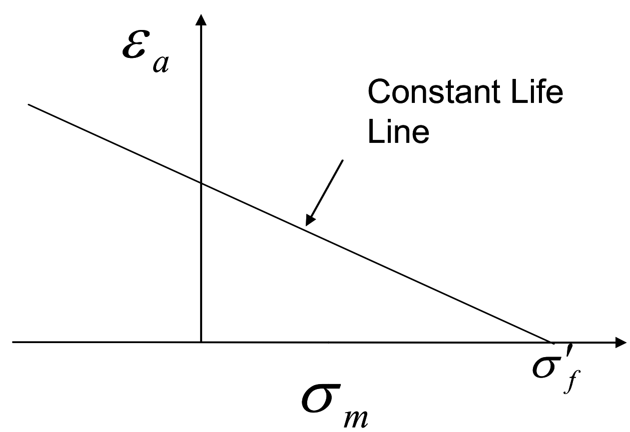

Figure 7. Morrow Mean Stress Correction

When the number of cycles is high, the constant life line will intercept the mean axis at σ'_f. It will intercept the alternating axis at an endurance strain amplitude that can be derived by assuming zero mean stress. This strain endurance limit is calculated based on the strain amplitude which will result in 1E7 cycles with zero mean stress (9)

$$\varepsilon_{endurance} = \frac{\sigma'_f}{E}\left(2E7\right)^b + \varepsilon'_f\left(2E7\right)^c \approx \frac{\sigma'_f}{E}\left(2E7\right)^b \quad (9)$$

Other methods exist to account for the mean stress debit. For example, the Smith-Watson-Topper approach uses the strain range and the max stress (10).

$$\sigma_{\max}\varepsilon_a = \frac{{\sigma'_f}^2}{E}\left(2N_i\right)^{2b} + \sigma'_f\varepsilon'_f\left(2N_i\right)^{b+c} \quad (10)$$

However, the Smith-Watson-Topper approach is only valid when the max stress is positive. The Morrow correction produces rational results over a wider range. However, this paper addresses the need for further constraints.

NEED FOR MEAN STRESS CORRECTION

The necessity for a mean stress correction is not universally accepted. At high temperatures and low lives, one can reasonably expect mean stresses to relax leaving negligible mean stresses if the loading is predominantly strain controlled. Most material test literature is for fully reversed R=-1 data. If mean stress must be accounted for, a methodology for calculating mean stress must be agreed upon. One could argue that mean stresses should be neglected or a conservative assumption should be made. For example, one could base life predictions upon testing with a compressive hold time. As a result of creep, a tensile mean stress will be included in the allowable curve.

Computationally it is simpler to not consider mean stress, both from the perspective of calculating the mean stress and for calculating the life itself. If mean stress is not considered, a curve fit of the strain vs. N relationship may be used for the life calculation. If mean stress is to be included, N must be expressed as a mathematical function that includes mean stress. If the data does not fit the function well, the accuracy is degraded. The argument to include mean stress is therefore not completely one-sided. However, there are some compelling cases were one should consider mean stress effects.

A prediction that does not include mean stress will result in the same life regardless of the sign of the mean stress. Assuming the mean stress is positive will at times result in conservative predictions. A conservative methodology may be desirable for robust designs but are not suitable for root cause analysis or when a high level of accuracy is required.

Fatigue testing is most commonly performed at isothermal conditions. Assuming that the tensile and compressive stresses are nearly the same, for a ductile material in strain control, it is not surprising that the mean stress shakes down to nearly zero. In TMF, the temperature as well as the stress is changing. Since yield and modulus at low temperature may be much higher, one cannot expect the max stress to equal |min stress|. The unequal yield strengths will result in tensile or compressive mean stresses not seen in the LCF testing.

Some applications will inherently have high mean stresses. Any bolted joint will have a significant mean stress. Most bolts are torqued to a considerable percentage of yield at assembly. Companies that already consider mean stress corrections may place limitations based on the test data that the curves are based on (for example R-ratios < .6). Generally this would take the form of calculating the R-ratio of the location being analyzed and calling an error if it exceeded the valid range. This practice is unhelpful to the analyst, unless an alternative technique is provided when this occurs. Bolted joints in particular operate under extremely high R-ratios. A joint having a high mean stress but very small cyclic stresses, would be presumed to have an R-ratio approaching 1 and would be considered invalid since R=+1. is not possible.

Although less common in heavy gas turbine applications, there may also be instance in which the component is Load controlled. Perhaps this would apply to control system actuators.

High temperature applications are subject to creep. Tensile stresses will relax resulting in compressive mean stress and compressive stresses at temperature will relax into tensile mean. If mean stress is to be considered, creep effects must be taken into account. Assuming that the analyst has a method to accurately calculate mean stress, there are valid reasons to consider mean stress.

LIMITATIONS OF MORROW CORRECTION

The Morrow correction is valid over a range of mean stresses, but there are some limitations both at low and high mean stresses. Continuing the analogy to the Goodman equation we can compare equivalent terms. The σ_u term in the (5) is approximately equal to σ'_f as shown in (11)

$$\sigma_{m\_ult} = E\varepsilon'_f\left(2E7\right)^{c-b} + \sigma'_f \approx \sigma'_f \quad (11)$$

It should always exceed the yield strength of the material. In practice, mean stresses in excess of yield after shake-down show a substantial reduction in life. The Morrow correction is non conservative in this region. At negative mean stresses, an increase in life is expected. However, mathematically the benefit increases indefinitely. In practice, components exceeding compressive yield will be subject to buckling, or cracking during the unload. A limitation put on compressive mean stress may not be strictly accurate, but may be prudent anyway.

PROPOSED LIMITS ON MORROW CORRECTION

It is proposed that the analogy between the Morrow correction and Goodman be extended to produce a Modified-Morrow correction similar to the Modified Goodman correction. Unlike the Goodman equation, only cyclic properties will be considered. The strain amplitude is maintained as the y-axis. So the endurance limit must be defined to correspond to a particular number of cycles (1E7). The yield-yield line is a mixture of strain and stress were the values assume a .2% strain offset and cyclic values as defined in (12).

$$\varepsilon'_{yield} = \sigma'_{yield}E + .002 \quad (12)$$

Plotting the Morrow correction one sees the opportunity to apply similar constraints, Figure 8.

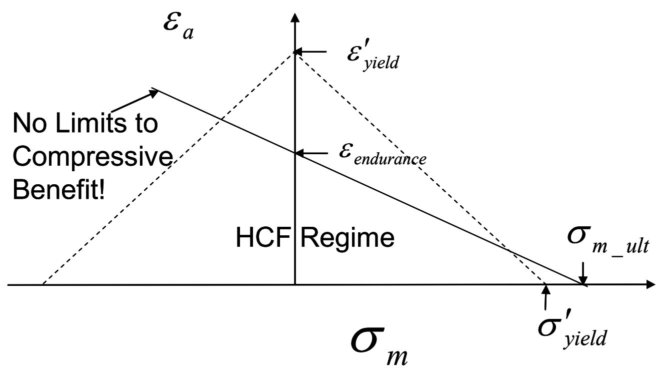

Figure 8. Potential Morrow Mean Stress Correction

Figure 8 shows a diagram at a single value of N. Considering the correction over multiple possible constant lives we get the following plot, Figure 9.

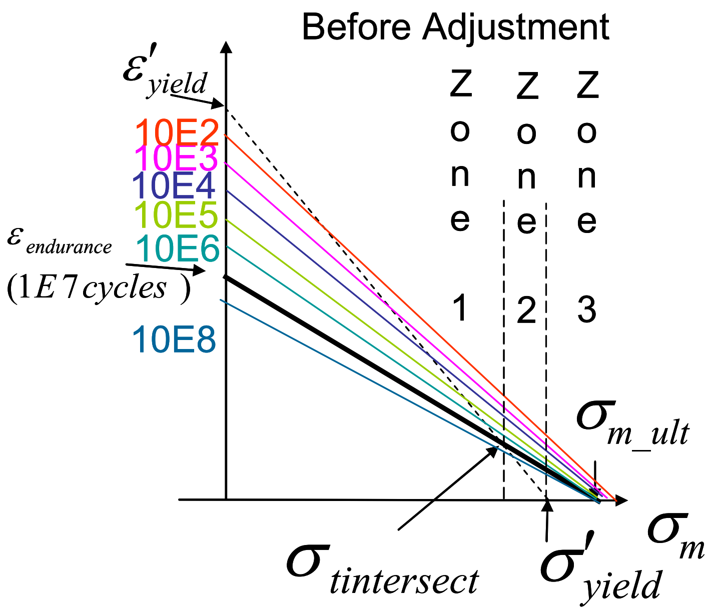

Figure 9. Tensile Morrow Mean Stress Correction

At lower lives the x intercept will exceed σ'_f due to plasticity. It is proposed at a modification be place on the strain-life curve based on zones in as shown in Figure 10.

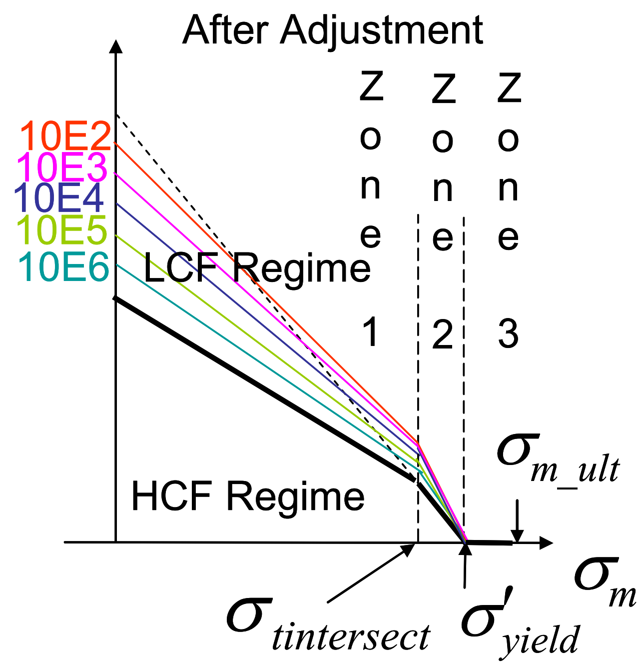

Figure 10. Tensile MODIFIED Morrow Mean Stress Correction

A strain debit factor is included to the life equation (13).

$$\varepsilon_a = \frac{\Delta\varepsilon}{2} = F_{MM}\left[\frac{\sigma'_f - \sigma_m}{E}(2N_i)^b + \varepsilon'_f(2N_i)^c\right] \quad (13)$$

Where F_{MM} is a modified Morrow Factor to reduce the strain capability depending on the zone of the mean stress (14).

$$\sigma_{\text{tintersect}} = \frac{(\sigma'_{yield} - \sigma_e)}{(\sigma_{m\_ult} - \sigma_e)}\sigma_{m\_ult} \quad (14)$$

Zone 1, $0 < \sigma_m \le \sigma_{tintersect}$

$$F_{MM} = 1 \quad (14a)$$

Zone 2, $\sigma_{tintersect} < \sigma_m < \sigma'_y$

$$F_{MM} = \frac{(\sigma'_{yield} - \sigma_m)}{(\sigma'_{yield} - \sigma_{tintersect})} \quad (14b)$$

Zone 3, $\sigma'_y \le \sigma_m$

$$F_{MM} = 0 \quad (14c)$$

A similar methodology is applied to the compressive regime as shown in Figure 11.

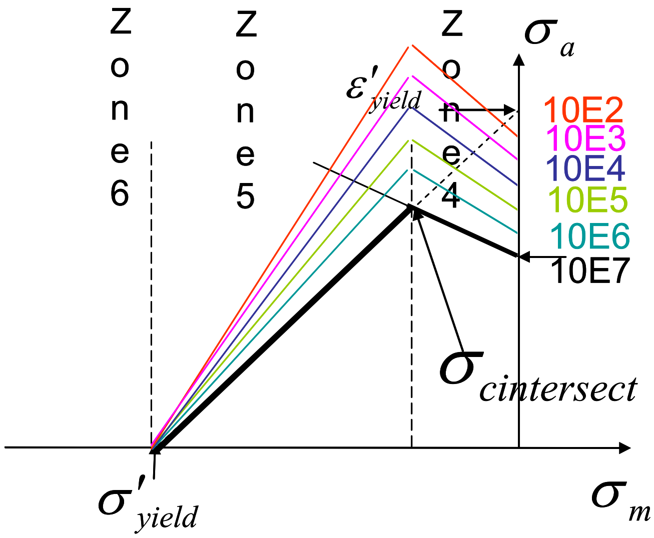

Figure 11. Compressive MODIFIED Morrow Mean Stress Correction

The compressive intersect is calculated in (15).

$$\sigma_{cintersect} = -\frac{\left(\sigma'_{yield} - \sigma_e\right)}{\left(\sigma_{m\_ult} + \sigma_e\right)}\sigma_{m\_ult} \qquad (15)$$

Zone 4, $\sigma_{cintersect} \le \sigma_m \le 0$

$$F_{MM} = 1 \qquad (16a)$$

Zone 5, $-\sigma'_y < \sigma_m < \sigma_{cintersect}$

$$F_{MM} = \frac{\left(\sigma'_{yield} + \sigma_m\right)}{\left(\sigma'_{yield} + \sigma_{tintersect}\right)} \qquad (16b)$$

Zone 6, $\sigma_m \le \sigma'_y$

$$F_{MM} = 0 \qquad (16c)$$

The preceding equations can be programmed with relative ease, resulting in reasonable life predictions over a very wide range of mean stresses.

VALIDATION

Currently most smooth fatigue tests are conducted under strain control. As a result, relatively small values of mean stress are expected. However, before improvements were made to servo-hydraulic control systems, most testing was load controlled. Therefore an older reference was used to provide test validation of this concept [10]. A clear drop-off in fatigue capability can be seen, in Figure 2., at high mean stresses, presumably near cyclic yield. Note that at A=0 (0 alternating stress), the difference in the curves is based on time to stress rupture rather than number of cycles

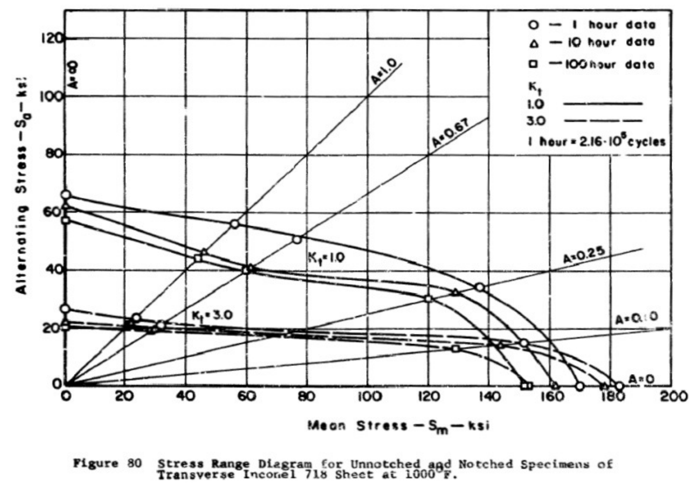

Figure 12. IN718 Stress Range Diagram [10]

The author conducted a small number of tests at high mean stresses on samples of a martensitic, precipitation-hardening stainless steel used for compressor blades. The tests were load controlled fatigue tests, primarily intended for HCF information. The sudden decrease in fatigue capability at high mean stress was demonstrated. Two specimens were run at R=0.67 for an extended number of cycles as shown in Table 1., After running out, the load amplitude was increased until the mean stress reached a significant percentage of cyclic yield. Both tests resulted in failures in a low number of cycles after the mean stress approached the calculated value of cyclic yield. The first specimen, 24-1, ran 20M cycles with a mean stress at 76% of cyclic yield without failure. When the load was increased to 95% of cyclic yield the specimen went only 48 more cycles before failure. The second specimen, 37-5, ran 10M cycles at 76% cyclic yield. Load amplitude was increased to 84% for 10M more cycles, and finally to 92% of cyclic yield for only 101 cycles before failure. Table 1. shows the results of the tests which indicate a sudden change in LCF capability near the cyclic yield. All of these tests were conducted at mean stress levels well within the region depicted as "zone 2" in Figure 10, where the mean stress exceeds $\sigma_{\mathrm{tintersect}}$.

Table 1. Compressor Alloy Test Results with High Mean Stress

Specimen	R-ratio	Mean Stress (%of Cyclic Yield)	Cycles @Loading	Result
24-1	0.67	76%	20,000,000	runout
24-1	0.67	95%	**48**	failure
37-5	0.67	76%	10,000,000	runout
37-5	0.67	84%	10,000,000	runout
37-5	0.67	92%	**101**	failure

DETERMINATION OF MEAN STRESS AND STRAIN RANGE

This paper is not intended to discuss methods for calculating strain range or mean stress. However, it is important to mention that the accuracy with which strain and stress are calculated is of great importance. Techniques can vary from overly simplistic elastic-perfectly plastic assumptions, shake-down relationships such as Neuber or Glinka, [11], to sophisticated constitutive time-dependent visco-plastic models. Further complexity is introduced in deciding what temperature to use for calculations, equivalent strain formulation, rotating stress fields, non-homogeneous and even non-isotropic behavior [12][13]. Without a reliable method for predicting strain range and mean stress, mean stress correction is pointless.

CONCLUSIONS

Accurate LCF predictions are of critical importance in order to optimize the design of turbo-machinery. Mean stress has been demonstrated to effect the fatigue capability of a component. Mean stresses should be considered in order to increase the accuracy of life predictions. The Morrow mean stress correction of the Manson-Coffin life model is a useful method for accounting for mean stress; however, limitations should be placed on the correction in order to avoid non conservative predictions. The methods presented in this paper have been incorporated into codes at PSM that have successfully predicted cracking on a number of existing designs. By accurately calculating life over the entire component, more highly optimized designs have been produced.

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