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DEVELOPMENT OF STATE OF THE ART DYNAMIC STRESS PREDICTION METHODOLOGY FOR STEAM TURBINES

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ABSTRACT

To predict the dynamic stresses due to forced response of steam turbine blades, a commercial FE solver ABAQUS has been linked with an in-house CFD solver TF3D-VIB, in the time domain in both one-way and two-way coupling. Both methods have been applied to analyse a freestanding subsonic turbine stage excited by upstream flow perturbations. Over a frequency range the peak responses are very similar, but the peak response of two-way coupling is shifted to a lower frequency, due to the aerodynamic coupling effect of fluid-structure interaction. That means a speed / frequency sweep is necessary to search for the peak response in two-way coupling. However, in one-way coupling, the frequency shift can be derived from the vibration induced modal force, and only one calculation is needed to predict the response over a range of frequency ratio using the classic single degree-of-freedom equation. One calculation using two-way coupling typically takes seven times more computing time than one-way coupling. The total computing time for two-way coupling to define the response characteristic is therefore much higher; more calculations are needed and each calculation takes much longer. Thus a one-way coupling method including the frequency shift correction is much more practical and suitable for blade design iterations.

The blade forced response is also limited by damping. In the case of low damping such as material only damping, this can be well represented in the harmonic ABAQUS calculation. However, high values of nonlinear damping can be deliberately introduced by managing the friction forces at blade root attachment. The nonlinear damping can be simulated directly by ABAQUS/Explicit method, convergence criteria often lead to excessive runtimes. Therefore a simple mass/spring model has been developed, which applies an exciting force to a system comprising two masses and springs to represent the blade and the root respectively and includes modelling of both the stick and slip forces of the root due to friction. Both the masses and their spring stiffness are chosen to produce either the sticking natural frequency (with infinite friction) or sliding natural frequency (zero friction). Using the simple two-mass model, the significant nonlinear response pattern is demonstrated. The resulting pattern has been verified against the ABAQUS/Explicit method. This allows the blade forced response prediction from the one way coupling to be further corrected to account for the nonlinear friction damping effect.

INTRODUCTION

The flow in turbo-machines is inherently unsteady due to the relative rotation of the blade rows. Strong periodic flow perturbations from adjacent blade rows can cause high levels of blade excitation and result in dangerous levels of vibration. In the quest for higher efficiency, modern designs of steam turbine are pushing towards smaller axial spacing and narrower blade chords to allow more stages to be put into the axial extent of the design envelope. However, to enable narrower blades to be applied safely it is necessary to be able to more accurately predict the dynamic stresses due to unsteady steam loading and manage the mechanical integrity of the blade through its design life.

Forced response is the complex interaction between the unsteady fluid flow and the blade structure. The unsteady flow can be caused by the periodic disturbances from passing adjacent blade-rows, and from non-symmetrical features creating distortions in the flow such as inlet, control valve, exhaust or extraction geometries. The blade vibration can be further complicated by contacts at a shroud, snubber, root or pins, which can have nonlinear contributions. The vibration of the blade induces a local unsteady pressure field around the blade surface. This results in aerodynamic damping that will, along with mechanical damping and material damping, limit the vibration amplitude.

Approaches to predict forced response have been actively developed in the last two decades. They can be generally categorized into either two-way or one-way coupling, depending on the level of interaction between fluid and structure. Two-way coupling solves both fluid equations and structure equations simultaneously and exchanges pressure and displacement at the interface at each time step (Breard *et al.* [1]; Moyroud *et al.* [2]; and Vahdati *et al.* [3]). One-way coupling solves the two sets of equations separately based on the linear assumption that there is negligible interaction between them (Kielb [4]; Chiang and Kielb [5]; Manwaring and Kirkeng [6]; Green and Marshall [7]). Two-way coupling includes more interaction physics than one-way coupling, and can be implemented much more simply, but one-way coupling can reduce the computing cost considerably.

CFD/FE based forced response calculations are still prohibitively computationally expensive for use in the design of order specific steam turbine cylinders comprising of perhaps 40 rows of blades. Furthermore validation is sparse. Most of the computing cost is attributed to solving the fluid equations (CFD). There are various commercial or proprietary CFD codes to solve the periodic unsteady flows, such as inlet distortion, upstream wake, downstream potential or exhaust perturbation. The most comprehensive way is to solve the periodic unsteady flow using multiple passages. It is simple to implement but very expensive to execute. However, there are other methods to improve computational efficiency by using a single passage. In the time domain, the Shape Correction method stores the harmonic coefficients at both periodic boundaries and rotor-stator interfaces (Dewhurst *et al.* [8]; Li *et al.* [9]). In the frequency domain, the linear harmonic or harmonic balance method solves the linearised fluid equations for each harmonic disturbance (Giles [10]; Hall *et al.* [11]. Nonlinearity can be included by calculating the deterministic stresses from linear variables (Chen *et al.* [12]; Vasanthakumar *et al.* [13]; Moffatt *et al.* [14]). Both methods can reduce computing time by one or two orders of magnitude depending on the physical blade count ratios.

Two-way coupling and one-way coupling approaches have been developed fairly independently. Only a few publications have compared them. Moffatt and He [15] have given a comprehensive review. They have compared rigorously the "fully coupled method" and "decoupled method". They found that the vibration induced unsteady pressure produces not only aero damping but also a shift of response frequency due to aerodynamic coupling. Thus a resonance tracking scheme has been proposed to find the peak response in the "fully coupled method".

At near resonant condition, the forced response is largely limited by the aggregated damping, including aero damping, material damping and friction damping. The aero damping can be calculated by TF3D-VIB [16], and the material damping can be user specified. The friction damping at the contacting areas can be modelled using an explicit solver, but computation can be very expensive due to its nonlinear nature. Others have shown that it can also be calculated using a "multiharmonic" approach (Petrov *et al.* [17]; Peng *et al.* [18]). However a simpler and faster method is needed for practical design applications.

This paper describes a state of the art forced response prediction method, combining a one-way coupling tool with a simple friction damping correction. Both one-way coupling and two-way coupling tools have been developed for blade forced response. They are compared on a freestanding subsonic turbine stage to verify their relative accuracy and computational efficiency. Furthermore, a simple two-mass model has been developed to simulate the nonlinear friction damping. It is verified against ABAQUS/Explicit method on a T-root turbine blade to demonstrate both accuracy and efficiency.

NOMENCLATURE

m	Blade mass
m'	Mass change due to aerodynamic coupling
ω_n	Blade natural frequency
ω	Aero excitation frequency
ω'	Shifted natural frequency due to aerodynamic coupling
ξ_M	Mechanical damping (normalised by critical damping, including both friction damping and material damping)
ξ_A	Aero damping (normalised by kinetic engergy)
ξ	Aggregate damping ($\xi=\xi_M+\xi_A$)
k	Modal stiffness
q	Complex modal displacement
$\tilde{q}$	Complex amplitude of modal displacement
f	Complex modal excitation force
$\tilde{f}$	Complex amplitude of modal excitation force
d	Complex modal damping force
$\tilde{d}$	Complex amplitude of modal damping force
$\{\Phi\}$	Complex eigenvector, modal shape of displacement
$\{F\}$	Complex excitation force vector, induced by vane-blade interaction
$\{D\}$	Complex damping force vector, induced by blade vibration
μ	Friction coefficient
CFD	Computational fluid dynamics
FE	Finite element
FS	Frequency shift

HCF High cycle fatigue
C Equivalent damping coefficient
F Excitation force
F_R Friction force
M_1 Equivalent blade mass
M_2 Equivalent root mass
K_1 Equivalent blade stiffness
K_2 Equivalent root stiffness
X_1 Blade displacement
X_2 Root displacement

TWO-WAY COUPLING

The two-way coupling tool has been developed by linking a commercial FE code ABAQUS and an in-house CFD code TF3D-VIB. The dynamic response of the system is solved using the "Direct Integration" method and the Navier-Stokes unsteady fluid equations are solved using the Shape Correction method [8,9,16] with the Baldwin and Lomax mixing length turbulence model. At each time step, both pressure and displacement are exchanged at the interface using a commercial code MPCCI (Figs. 1-3). Then the blade moves according to the unsteady pressure and the CFD mesh deforms respectively. At the end of the calculation, blade vibration reaches an energy balance with constant amplitude. Displacement of chosen points on the CFD mesh is monitored (Fig. 4). By post-processing the displacement history, the mean and dynamic displacement can be calculated, as well as the mean and dynamic stresses. Alternatively the stress can be monitored directly on the FE mesh.

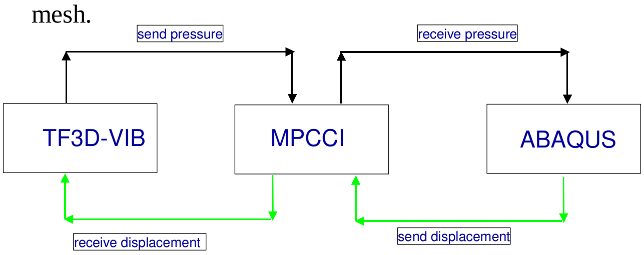


Figure 1. Flow chart for two-way coupling method

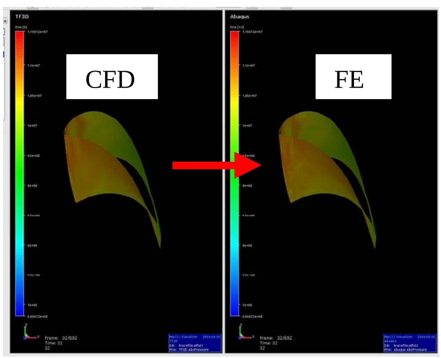


Figure 2. Pressure interpolation from CFD to FE mesh

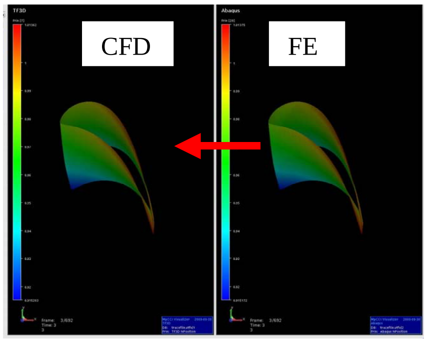


Figure 3. Displacement interpolation from FE to CFD mesh

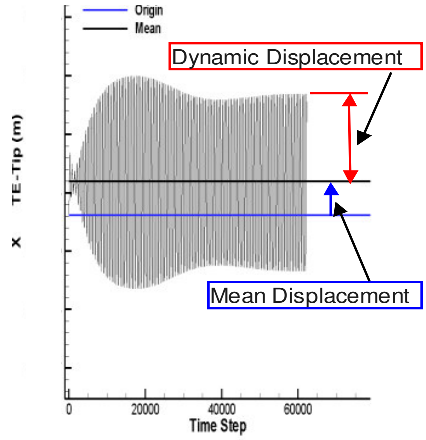


Figure 4. Monitor of displacement at blade tip-Trailing Edge

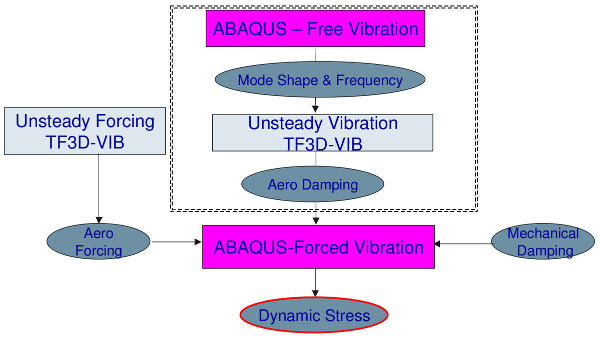


Figure 5. Flow chart for one-way coupling method

ONE-WAY COUPLING

A one-way coupling tool has been developed by linking ABAQUS and TF3D-VIB in several steps (Fig. 5). First, the aero excitation forcing is calculated by TF3D-VIB and interpolated from the CFD mesh to the FE mesh (Fig. 6). Secondly, the mode shape obtained from ABAQUS is interpolated to the CFD mesh (Fig. 7). The mode shape and the given amplitude and phase are used to deform the CFD mesh at the blade passing frequency. The damping unsteady pressure induced by the vibration is calculated by subtracting the steam forcing unsteady pressure from the resultant unsteady pressure. The imaginary part of the multiplication of the complex mode shape and the complex damping unsteady pressure is derived to provide the aero damping, which is normalised by kinetic energy to give an equivalent critical damping ratio. Thus the aero damping is independent of both vibration amplitude and relative phase. Similarly the real part is calculated and normalised to derive the frequency shift (see section Frequency Shift).

Using both the calculated aero forcing and aero damping and the specified mechanical damping, the dynamic stress is calculated by ABAQUS. The FE model to calculate the forced response for the coupled row of blades in this example consists of a single blade and a corresponding sector of disc, with cyclic symmetry constraints applied to the sector boundaries. Mechanical damping includes both material damping and friction damping. Material damping is a conservative value based on both a vast company database and operating experience. Friction damping is covered in section Friction Damping At Contact Interfaces.

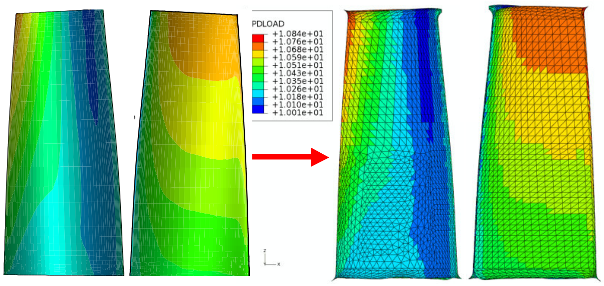


Figure 6. Pressure interpolation from CFD to FE mesh in one-way coupling

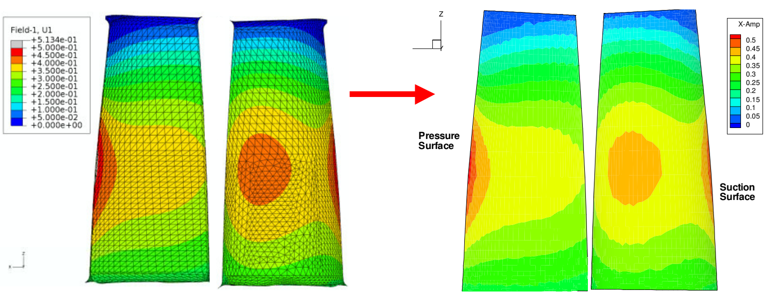


Figure 7. Displacement interpolation from FE to CFD mesh in one-way coupling

COMPARISON AND DISCUSSION BETWEEN TWO-WAY AND ONE-WAY COUPLING

Both one-way coupling and two-way coupling tools have been used to simulate the forced response of a turbine stage (Fig. 8). The blade is subject to a periodic disturbance from the vane wake. For simplicity, the blade is free standing and its hub is encastre. Neither material damping nor friction damping is included in either case, in order that the response peak is purely limited by aero damping. To produce the response characteristics, both frequency and speed sweeps have been carried out.

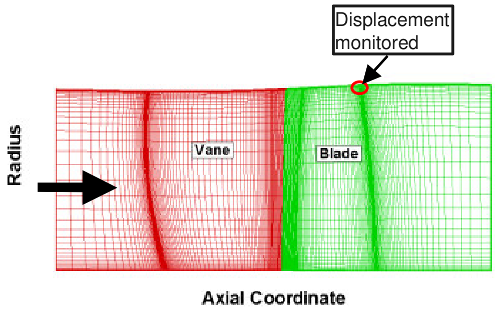


Figure 8. CFD Mesh and monitor point of the simulated turbine stage

Frequency Sweep

First, the rotating speed / excitation frequency (F) is kept constant and the natural frequency (Fn) is varied by adjusting the blade properties (Young's Modulus) to give a frequency ratio between 0.67 and 1.5 (Fig. 9). The tangential dynamic displacement at the blade tip trailing edge is shown for comparison. The peak response from the one-way coupling method is approximately 8% higher than that from the two-way coupling. The one-way coupling produces its peak response when the blade natural frequency matches the vane passing frequency. However two-way coupling produces only half of the response at that frequency compared to one-way coupling at the same forcing frequency. This is due to the fact that the two-way coupling demonstrates a frequency shift (FS), see section Frequency Shift), and produces its peak response at a reduced excitation frequency.

The FS is caused by the effect of aerodynamic coupling, which has been well documented by Moffatt and He [15]. It is produced by the phase lag between the unsteady damping force and the blade vibration velocity. Thus the aero-damping can be calculated from the imaginary part (out-of-phase) of the modal damping force. Similarly the FS can be calculated from the real part (in-phase) of the modal damping force. By applying this calculated FS, one-way coupling produces the peak response at the same excitation frequency as the two-way coupling method

(Fig. 9). The two response characteristics have very similar shapes despite their difference of up to 8% in absolute value.

During the process three points on the blade surface are monitored for both the one-way coupling and two-way coupling methods. It confirms that the mode shapes are consistent between the two approaches. The difference in peak response can probably be attributed to the nonlinearity, which is missing in the one-way coupling. This discrepancy due to nonlinearity is considered acceptable for practical application to HP/IP steam turbines where flow is normally subsonic, because the one-way coupling predicts a conservative higher response. However one should be cautious applying one-way coupling when a shock is present and strong nonlinearity is expected.

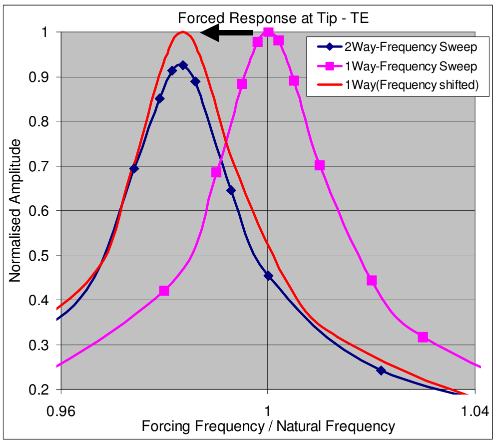


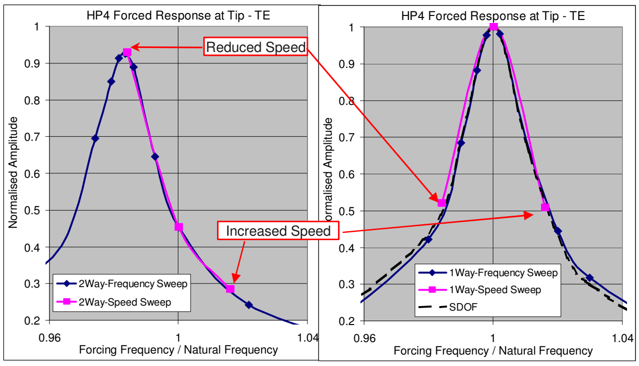
Figure 9. Comparison of response characteristics for frequency sweep

Speed Sweep

For the speed sweep, the blade natural frequency is kept constant and the rotating speed is varied. At a reduced speed, two-way coupling produces a very similar peak response to the frequency sweep, when its excitation frequency matches the resonant frequency after adjustment for aerodynamic coupling (Fig. 10a). Furthermore, both the frequency sweep and the speed sweep produce very similar response at increased speed. This demonstrates that two-way coupling implicitly includes the FS.

For one-way coupling, both the frequency sweep and the speed sweep also produce similar results (Fig. 10b). Furthermore, both of them agree perfectly with the classic single degree of freedom (DOF) response characteristics, especially at near resonant frequency. This means, in practice, only a single set of one-way coupling calculations are necessary, and the response characteristics can be generated from both the predicted damping and FS.

In this particular example, each set of one-way coupling calculations is approximately seven times faster than the two-way coupling calculations. One-way coupling needs only a single set of calculations to predict the response characteristics whereas two-way coupling needs a speed/frequency sweep to find the peak response. Therefore one-way coupling is judged a much more efficient and practical way to predict the response characteristics in a design cycle.



a. Two-way coupling b. One-way coupling

Figure 10. Comparison between frequency sweep and speed sweep

Frequency Shift

The forced response of each mode is modelled using Mode Based Dynamic Analysis in ABAQUS. It solves a single degree of freedom modal equation:

$$m \cdot \ddot{q} + 2\xi_M \cdot \omega_n \cdot m \cdot \dot{q} + k \cdot q = f + d(q) \tag{1}$$

Assuming harmonic motion at the excitation frequency under harmonic loading, and letting modal displacement $q = \tilde{q}e^{i\omega t}$, aero excitation force $f = \tilde{f}e^{i\omega t}$ and aero damping force $d = \tilde{d}e^{i\omega t}$, where $i^2=-1$, the harmonic force amplitudes acting on the blade surface are transformed into modal coordinates by taking the dot product between complex eigenvector $\{\Phi\}$ and the complex force vectors:

$$f = \sum_{j=1}^{n} \{\Phi_j \cdot (F^{\mathrm{Real}} + iF^{\mathrm{Imag}})\} \tag{2}$$

$$d = \sum_{j=1}^{n} \{\Phi_j \cdot (D^{\mathrm{Real}} + iD^{\mathrm{Imag}})\} \tag{3}$$

where n is the number of degrees of freedom, {F} is the complex excitation force vector which is predicted by CFD and interpolated to the FE mesh, and {D} is the complex damping force vector induced by the modal shape.

From Eqs. (2) and (3) the modal response can be re-written in the form

$$[\omega_n{}^2 - \omega^2(1+\frac{\tilde{d}_n{}^{\mathrm{Real}}}{m\omega^2}) + i\cdot 2\omega_n\omega(\xi_M - \frac{\tilde{d}_n{}^{\mathrm{Imag}}}{2m\omega_n\omega})]m\tilde{q} = \tilde{f} \quad (4)$$

Thus the imaginary part of the normalised modal damping force contributes to the equivalent aero damping,

$$\xi_a = -\frac{\tilde{d}_n{}^{\mathrm{Imag}}}{2m\omega_n\omega} \quad (5)$$

and the real part contributes to a mass change:

$$\frac{m'}{m} = 1 + \frac{\tilde{d}_n{}^{\mathrm{Real}}}{m\omega^2} \quad (6)$$

thus the resonance frequency ratio for a lightly damped blade is:

$$\frac{\omega'}{\omega} = 1/\sqrt{1+\frac{\tilde{d}_n{}^{\mathrm{Real}}}{m\omega^2}} \approx 1 - \frac{\tilde{d}_n{}^{\mathrm{Real}}}{2m\omega^2} \quad (7)$$

hence the FS can be expressed as:

$$FS \approx -\frac{\tilde{d}_n{}^{\mathrm{Real}}}{2m\omega^2} \quad (8)$$

Comparing Eq. (8) to Eq. (5), the FS has a very similar form to the aero damping at near resonant conditions, but the former is derived from the real part of the modal damping force and the latter from the imaginary part.

In two-way coupling, the FS is implicitly included. In one-way coupling however, it needs to be post-processed in a similar way to the aero-damping to produce the proper response characteristics. It should be noted that the FS can be either negative or positive depending on mode shape, flow condition, frequency and inter blade phase angle. In the current paper, the peak response frequency is reduced for the first mode. However it is reported the resonance frequency of the NASA67 fan blade is increased for the first mode but reduced for all the higher modes (Moffatt and He [15]).

Comparison with experimental data of both vibration stress and frequency is planned in the future.

FRICTION DAMPING AT CONTACT INTERFACES

Friction damping due to the relative sliding motion of assembled components during vibration can have a significant effect on the response levels close to a system natural frequency. One particular case of interest is for a pre-twisted T-root blade where the mode of vibration is predominantly tangential. In such a case, slip can occur between the rotor and the support neck of the blade, introducing significant friction damping into the system.

To model such behaviour, a simple two-mass model (Fig. 11, Eqs. 9-10) is derived to determine the effect of friction on the response amplitude of the blade:

$$F - K_1\cdot(X_1 - X_2) - C\cdot\dot{X}_1 = M_1\ddot{X}_1 \quad (9)$$

$$K_1\cdot(X_1 - X_2) - K_2\cdot X_2 - sign(\dot{X}_2)\cdot F_R = M_2\ddot{X}_2 \quad (10)$$

The equations of motion are solved in the time domain for a harmonic excitation F, such as that occurring from the wakes of upstream stationary blades. The tuning of the model relies on tuning the masses and springs to achieve the frequency separation between full sticking and frictionless sliding modes of the support neck, calculated from a more detailed FE analysis. It also requires an estimate of the moving mass of the blade aerofoil M_1 for the sticking mode, and the mass of the blade root M_2, which is moving for the sliding mode. The effect of friction in the blade root is applied to the single mass M2, which can exhibit a stick-slip behaviour under excitation. A friction force F_R is applied directly to the root mass and can be equated to a friction coefficient for a given support load on the neck. The two-mass model is very quick to run and can be used to identify the response of the system to varying excitation frequency.

As a verification of the simple representation, a more complex approach has been adopted using a FE model of a blade with a surface based contact simulation for the root (Fig. 12). The nonlinear effects of friction are modelled by commercial FE code ABAQUS/Explicit. An initial preload is applied to the structure in ABAQUS/Standard to generate a contact force between the blade root and a rigid surface, which represents the rotor contact face, as would be the case for a blade with pretwisted integral shroud. The model is then imported into Explicit to solve in the time domain for a harmonic excitation, with a friction coefficient applied to the contact faces. Constraints are applied at the blade tip to enforce an anti-phase mode similar to a high nodal diameter response of a coupled blade row. The calculation is run for a large number of vibration cycles until a converged solution is obtained, which is approximately 3 orders of magnitude more expensive than the simple two-mass model. The main damping comes from the friction but additionally a small amount of material damping C is included which equates to 0.2% of critical damping for the first mode. This will bound the solution when the root does not slide.

The results are presented in Fig. 13, which shows the effect on response for a friction coefficient μ of 0.1 and 0.3, with the models tuned to give the same sticking frequency. It shows the two-mass model is a sufficient representation and captures reasonably well the effect of the nonlinear friction forces on the peak response close to a natural frequency. The direct-solution FE calculations for the frictionless sliding and fully sticking cases are calculated in the frequency domain as linear perturbation procedures, and the response is limited only by the material damping. The Explicit calculation shows contact nodes sliding to a greater or lesser extent, including a mixture of some nodes sticking and some nodes sliding. The intermittent stick-slip in the root appears as a truncated sinusoidal response, and the same effect is seen in the two-mass model. The overall trend confirms a reduction of the peak response without significant change in resonance frequency. However, as the friction forces reduce further there will come a point where the damping becomes less significant and the response will start to increase at the frictionless sliding frequency.

The peak response calculated by the simple model with coefficient of friction of 0.3, which is typical for steel, is very close to that calculated by the FE Explicit analysis. Therefore we can see that the simple model is a practical design tool that can allow consideration of operating much closer to the nominal resonant frequencies than traditional practice would allow.

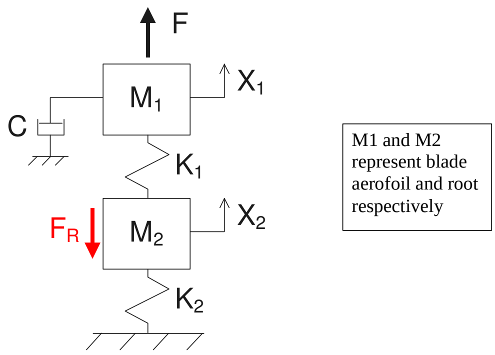


Figure 11. Two-mass model for friction damping

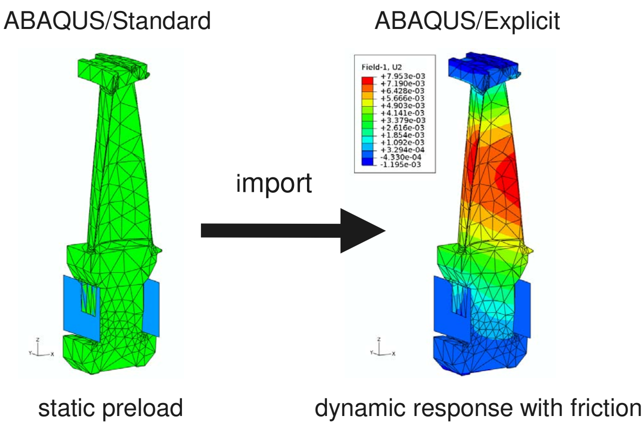


Figure 12. FE model for friction damping verification

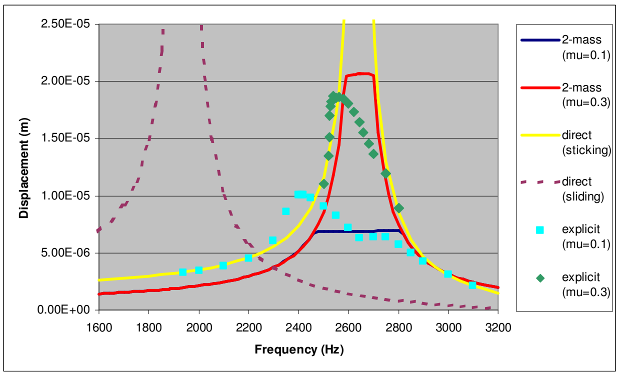


Figure 13. Effect of friction damping on blade response

CONCLUSIONS

A state of the art dynamic stress prediction methodology has been developed, enabling the design of narrower blades and smaller axial spacing in steam turbines whilst maintaining mechanical integrity.

It has been demonstrated on a free-standing subsonic turbine stage that one-way coupling predicts a similar peak response to two-way coupling. One-way coupling needs a single set of calculations to predict the response characteristics, so it is two orders of magnitude faster than the two-way coupling method. Thus the one-way coupling approach is more suitable for prediction of blade forced response in the routine design of order specific steam turbine cylinders comprising of perhaps 40 rows of blades. This is necessary to safely allow the use of smaller axial spacings and narrower blade chords so that more stages can be put into the axial extent of the design envelope.

The frequency shift due to aerodynamic coupling is implicitly included in two-way coupling. It is calculated in a similar way to aero damping and can thus be included in the one-way coupling calculations to properly predict the response characteristics.

The response characteristics of a blade exhibiting a frictional stick-slip damping mode can be predicted by one-way coupling when the response is further corrected by a simple two-mass response model. The two-mass model has been verified against ABAQUS/Explicit for a T-root blade with an integral shroud. It captures reasonably well the effect of the nonlinear friction forces on the peak response close to a natural frequency, as well as the changing stick-slip modeshape with reducing friction coefficient.

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