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PROBABILISTIC ANALYSIS OF GAS TURBINE DISK MULTI-CRACK PROPAGATION

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ABSTRACT

An approach of predicting probabilistic life of multi-source surface crack growth problem is described in the paper. Multi-source circumferential surface cracks occur occasionally on aircraft engine turbine disk. A serious of semi-elliptical cracks which located in a certain radius but different circumferential position on turbine disk was caused by low-cycle fatigue. The residual life should be predicted according to the damage tolerance design theory. A typical turbine disk was studied as a numerical example by using finite element method. The existing experimental result in literature shows that the surface crack propagates faster on circumferential direction than on axial direction, and the surface cracks are going to merge on circumferential direction, which made the multi-crack a closed ring crack. The experiment shows that the multi-cracks would merge into a 2/3 circle crack after certain number cycles which agrees the numerical simulation. Then the crack model was reasonable simplified to axial direction 2-D crack problem in meridian plane of turbine disk. This 2-D axisymmetric simplification significantly reduces the probabilistic analysis computational time. The randomness of materials, load and geometric imperfection is considered in sensitivity analysis by using response surface method. Then the probabilistic life is predicted by considering the major random variables of initial crack length and engine speed. The probabilistic life analysis result is also compared with the existing experimental results.

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NOMENCLATURE

J J-integral.
W Strain energy density.
T Stress vector
p_i Stress components
F_i Body force
N Number of cycle
K Stress intensity factor
C,n Material constants of Paris equation
a Crack length
a_0 Initial crack length
a_{cr} Critical crack length
L Crack propagation life
ω Engine rotating speed
E Elastic modulus
ρ Density
T_0 Temperature of disk flange
μ Mean value of samples
σ Standard Deviation
COV Coefficients of variation

INTRODUCTION

Modern aero-engine structure is designed based on damage tolerance theory. Crack propagation life prediction is a crucial process in structural integrity assessment. Structures intrinsically involve randomness and uncertainty, for example in the material properties, boundary conditions, external loads. In recent years, probabilistic fracture mechanics (PFM) has been studied by many scholars [1] [2] [3] [4]. Probabilistic crack propagation analysis is also used in the turbine disk life prediction. In numerical simulation, finite element method is widely used in computational structure field. FEM represents a suitable approach for the study of fracture mechanics. This paper uses FEM in fracture mechanics calculation.

The crack caused by low cycle fatigue usually located on high stress region of aeroengine turbine disk. Bolt hole and fir tree groove are the most common stress concentration region where fatigue crack initiates [5] [6]. These locations are non-axisymmetric structures in geometry and the cracks are single-source. Stress concentration also occurs in some axisymmetric turbine disk structures where radius changes rapidly in axial direction. Stress concentration may lead to multi-source crack initiation. In this paper, a realistic multi-source crack problem of a areoengine turbine disk is studied. A serious of semi-elliptical cracks located in a certain radius but different circumferential position on labyrinth gas seals chamfer region. In order predict the probabilistic life of turbine disk, a serious of simplified and calculation method is developed in this paper.

Theory

J-integral Method in Turbine Disk Calculation

The J-integral parameter proposed by Rice [7] is extensively used in assessing fracture integrity of cracked engineering structures. J-integral method allows large plastic deformation at the crack tip. For a linear or nonlinear elastic material, in the absence of body forces, thermal strains, Eq. 1 shows the form of J-integral. W is strain energy density, Γ is integral contour.

$$J = \int_{\Gamma} (W dy - T \frac{\partial u}{\partial x} ds) \tag{1}$$

Turbine disk works at high rotation speed and the centrifugal force can not be ignored. Thus, J-integral is modified as Eq, 2 shows.

$$J = \int_{\Gamma} (W dx_2 - p_i \frac{\partial u_i}{\partial x_1} ds) - \int_A \frac{\partial}{\partial x_3} dA - \int_A (F_i \frac{\partial u_i}{\partial x_1}) dA \tag{2}$$

The first term is original form of Eq. 2, the second term is the affect of three-dimensional stress and the last term is the affect of body force. A refers to the area of the integral contour Γ.

Paris Equation in Crack Propagation Life calculation

In stable propagation stage, Eq. 3 shows the Paris equation [8] which describes the relationship between crack growth rate $\Delta a/\Delta N$ and increment of stress intensity factor ΔK.

$$da/dN = C(\Delta K)^n \tag{3}$$

Crack propagation life is calculated from the integral given by Eq. 4.

$$L = \int_{a_0}^{a_{cr}} \frac{1}{C(\Delta K)^n} da \tag{4}$$

In Eq. 3, 4, the coefficients C and n are obtained from material test. Initial crack length a_0 is measured by nondestructive testing. Critical crack length a_{cr} is calculate based on the fracture toughness and requirement of structure residual strength.

Probabilistic Fracture Mechanics Analysis

Two-dimensional Simplification

The multi-source cracks located on labyrinth gas seals chamfer region of turbine disk, as FIGURE1 shows. The experimental result in literature [9] shows that the surface cracks would converge into 2/3 or full circle crack when cracks propagates at the beginning stage of propagation. The crack propagation problem can be simplified into a two-dimensional problem.

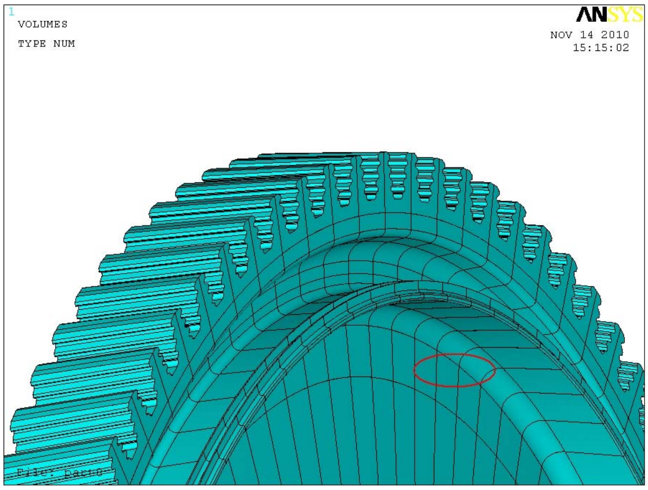

FIGURE 1 Failure Crack Location

Process of Probabilistic Fracture Mechanics Analysis

According to II, the multi-crack problem can be reduced to two-dimensional axisymmetric problem in order to reduce the probability computational cost. The determinate crack propagation life is studied firstly. Then initial crack length and engine speed are randomized for the probabilistic life calculation.

FIGURE2 shows the process of predicting probabilistic life by using axisymmetric model. There are three steps.

(a) Calculate the stress intensity factor K on propagation path by finite element method. The randomness of material and load is introduced into the results of K.

(b) Calculate the relationship between stress intensity factor and propagation path location. $K = f(a)$ is obtained by polynomial fitting.

(c) Integral Paris equation to obtain the determinate crack propagation life. In probabilistic life calculation, a_0 is introduced into the integral as the lower limit.

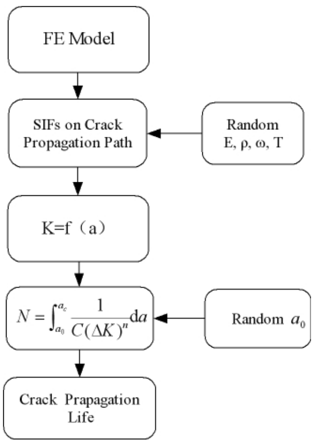

FIGURE 2 Process of Probabilistic Life Analysis

Crack Modeling Technique FEM and other numerical method in which the discrete method depend on mesh need to rebuild the FEM model when crack grows. In probabilistic fracture mechanics problem, a large number of samples is calculated, therefore parametric model should be established and updated automatically during calculation loop.

The two dimensional disk model is established in commercial FEM code ANSYS:

(1) The model is established from base to top: point - line - area - mesh.

(2) In order to keep the quality of mesh in crack tip during automatically remodeling loop, the meridian mesh of the disk is divide into two area as FIGURE 3 shows: the crack tip area within red contour, other disk area outside the red contour.

(3) The inner region is meshed firstly.

(4) Then the outside region is meshed.

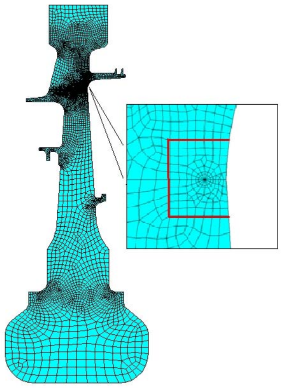

FIGURE 3 Two-dimensional turbine disk FE model

(5) Fracture parameters is calculated. SIF is obtained by J-integral method. The crack model is remeshed when calculating stress intensity factor in next crack length. FIGURE4 shows the cracktip model of different length.

(6) After calculation, the objects are deleted from top to base: mesh - area - line - point.

(7) New calculation loop.

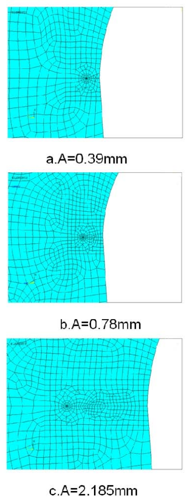

FIGURE 4 Crack Tip FE Model of Different Length

Fracture Mechanics Parameter Calculation by FEM The crack growth path is defined as a strait line in axial direction of engine. The crack model is remeshed when calculating stress intensity factor in next crack length. Eq.5 is the relationship between stress intensity factor K and crack length a. FIGURE 5 shows the curve of K=f(a), which is plotted by polynomial fitting.

$$K(a) = 3.7185a^3 - 27.7207a^2 + 79.9787a + 31.9178 \quad (5)$$

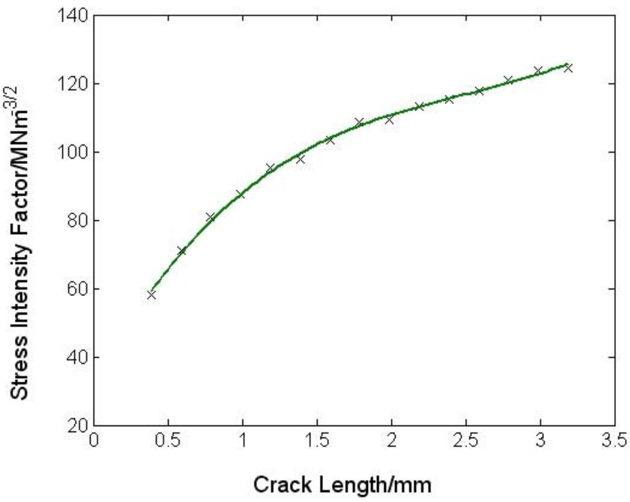

FIGURE 5 Relationship between Crack Length a and Stress Intensity Factor K

Determinate Crack Propagation Life Calculation As the basis of the probabilistic calculation, determinate fracture mechanics calculation is studied in this paper at first.

Critical crack length a_{cr} is calculate based on the fracture toughness and requirement of structure residual strength. The fracture toughness of turbine disk material is $128 MN.m^{-3/2}$. At the maximum engine speed 1393rad/s, the corresponding critical crack length a_{cr} is 3.2mm.

In literature [9], the same model of turbine disk which had multi-source surface cracks was studied by a serious of crack propagation experiments. In experiment, the crack propagated from 0.39mm to 1.8mm. The crack propagation life of the two experimental disk is 380 and 500.

In order to compare simulation and experiment results, the simulation also calculate the crack propagation from 0.39mm to

TABLE 1: Comparison of experimental and simulation results

Results	Test Disk NO.1	Test Disk NO.2	Simulation Result
Crack Propagation Life/cycle	380	500	475

1.8mm. The load of calculation and experiment is "0-Maximum-0". The increment of the stress intensity factor is $\Delta K = K_{max} - K_{min} = K_{max} - 0 = K_{max}$. The life integral use the mean value of the engine speed ω 1392.8 rad/s and initial crack length a_0 0.39mm. Determinate life is calculated in Eq.6.

$$L_1 = \int_{0.39}^{1.8} \frac{1}{C(\Delta K)^n} da = 475 \quad (6)$$

Table 1 shows the results of simulation and experiments. 380 and 500 cycles from experiments shows that turbine disks crack propagation life have large dispersion and probabilistic analysis is necessary.

475 cycles from simulation is close to the experiment results. Previous assumptions and two-dimensional reduction is reasonable.

Random Variable Distribution

In this paper, five random factors are concerned: density of disk material ρ, Young's modulus of material E, temperature of disk flange T_0, engine rotate speed ω, initial crack length a_0.

The initial crack initiates in low cycle fatigue of turbine disk. Low cycle fatigue life also has significant stochastic character, in other words, a group of turbines in the same production batch usually have very different life after certain service time. The distribution of elliptical crack length is exponential type [10].

The distribution of the engine speed is usually considered as normal form. The normal distribution of variables in the probability of the region $[\mu - 3\sigma, \mu + 3\sigma]$ account for about 99.74 % overall. This paper assumes this region instead of the entire range. The mean value of the random variable is considered as the mean value of the range of the engine speed fluctuation. The upper and lower limit of the speed fluctuation is regarded as the $\mu \pm 3\sigma$ value of the random variables. The engine main load spectrum is "0-maximum-0". The maximum speed range is 98%-105%. The minimum speed is constant 0. The maximum speed

TABLE 2: Random Variables Distribution

Random variable	μ	COV	σ	Distribution
$\rho/Kg.m^{-3}$	8320	0.01	83.2	Normal
E/GPa	205	0.01	2.05	Normal
$\omega/rad.s^{-1}$	1393	0.01	13.93	Normal
T_0	520	0.02	10.4	Normal
a_0/mm	0.39	1	0.39	Exponential

is considered as the random variable of normal distribution, and the mean value of the maximum speed is given by Eq.7,

$$\mu = \frac{105\% - 98\%}{2} = 101.5\% \tag{7}$$

The standard deviation σ is given by Eq.8,

$$\sigma = \frac{105\% - 101.5\%}{3} = 1.17\% \approx 0.01 \tag{8}$$

The non-dimensionless parameter coefficient of variation is given by Eq.9 to describe the dispersion of random variable.

$$COV = \frac{\sigma}{\mu} \tag{9}$$

The COV of the engine speed is given by Eq.10,

$$COV = \frac{\sigma_{max}}{\mu_{max}} = \frac{105\% - 101.5\%}{101.5\%} \approx 0.01 \tag{10}$$

Table 2 shows the means, standard deviations, and probability distributions of random parameters.

Probabilistic Sensitivity Analysis

The sensitivities are used to determine the relative importance of the variables on the damage tolerance estimates. In this study, sensitivity analysis is made by using response surface method in ANSYS Probabilistic Design System (PDS).

In crack propagation problem, the sensitivities should be studied when the crack length is a specific value. The randomness of a_0 is shown in the

The random data in table 2 is input to calculate sensitivities of all random variables to SIF. In ANSYS PDS, 80 times Monte Carlo simulation is made to fit the response surface. Then the SIF is calculated 100,000 times through response surface. Models of different crack length is calculated: 0.79mm, 1.59mm, 2.39mm. The calculations show similar results. One of the typical results(crack length is 0.79mm) is plotted in the bar and pie chart as FIGURE6 shows. In FIGURE6, the red area which represents the sensitivity of ω accounts over 77% of the entire pie chart. SIF is most sensitive to the engine rotate speed ω. Relatively, other variables affect SIF insignificantly.

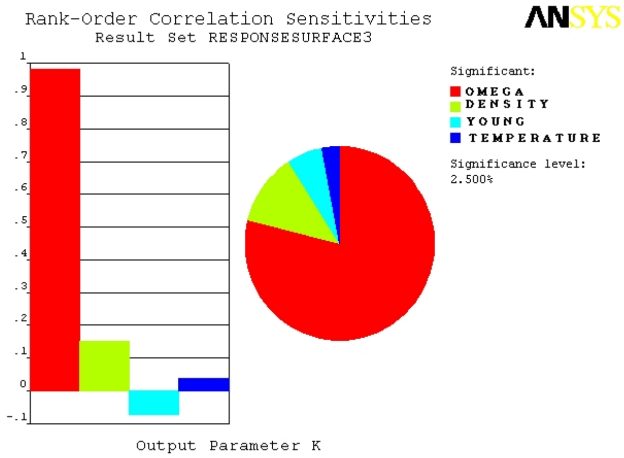

FIGURE 6 Sensitivity of SIF (a=0.79mm)

Probabilistic Life Prediction

According to the results of sensitivity analysis, ω is the major random variable to SIF. The effects of other random variables are insignificant relatively. The probabilistic life is predicted by considering the randomness of a_0 and ω only. The life calculation is made by using Monte Carlo simulation in Matlab.

The effect of random initial crack length Considering the effect of random initial crack length separately, the randomness of a_0 is introduced into Eq. 4 as the lower limit. There is no need to calculate finite element model in Monte Carlo simulation.

Use the data in Table 2, mean initial crack length a_0=0.39, COV=1.0, after 5000 times Monte Carlo simulation, crack propagation from 0.39 to 1.8mm, the distribution of probabilistic life is shown in FIGURE 7.

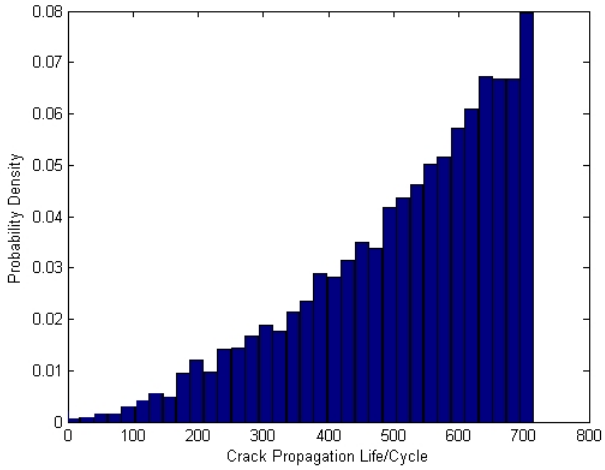

FIGURE 7 Lifetime Histogram by effect of random a_0 (crack length 0.39-1.8mm)

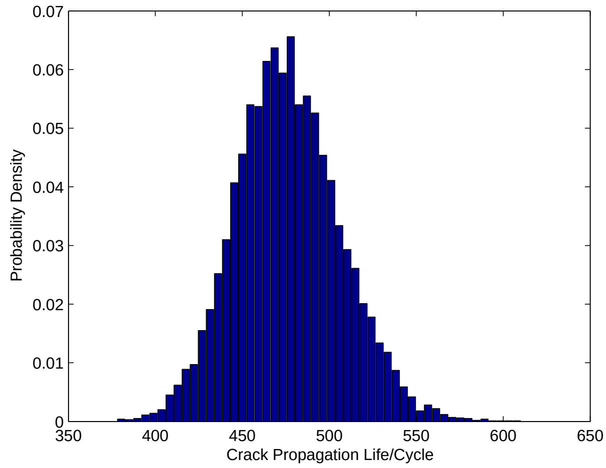

FIGURE 8 Lifetime histogram by effect of random ω (crack length 0.39-1.8mm)

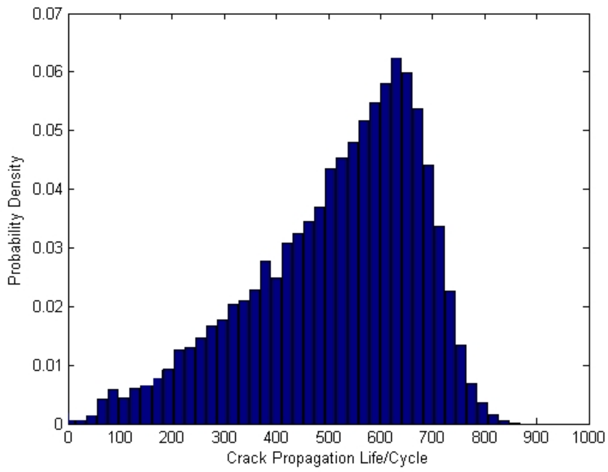

FIGURE 9 Lifetime histogram by effect of random ω and a_0 (crack length 0.39-1.8mm)

The effect of of random engine speed Considering the effect of random engine speed separately, finite element model is calculated in Monte Carlo simulation.

Use the data in Table 2, mean engine speed ω=1392.8rad/s, COV=0.01, after 5000 times Monte Carlo simulation, crack propagation from 0.39 to 1.8mm, the distribution of probabilistic life is shown in FIGURE 8.

The effect of of two random factors Considering the effect of above two factors, use the data in Table 2, mean engine speed is 1392.8rad/s, COV=0.01, initial crack length is 0.39mm, COV=0.1, after 5000 times Monte Carlo simulation, crack propagation from 0.39 to 1.8mm, the distribution of probabilistic life is shown in FIGURE 9.

Furthermore, the probabilistic life of was calculated when crack propagated from 0.39mm to 3.2mm. FIGURE10 shows the distribution of probabilistic life.

Result Discussion

Table 3 shows the calculation results of probability life.

(1)Considering separately for the random initial crack length a_0 in exponential distribution, the skewness of the life samples is -1.07. Figure 7 shows that maximum value of life is 715 cycle which corresponding to the minimum value of a_0.

(2) Considering separately for the engine speed ω in normal distribution, the skewness of the life samples is 0.235. Figure 9 shows that the life samples basically meet normal distribution.

(3) Considering both the randomness of initial crack length a_0 and engine speed ω, the skewness of the life samples is -0.952. Figure 9 shows that the life samples basically meet weibull distribution.

The mean value of life sample is 504 cycles. The life of 99% reliability is 316 cycle and life of 1% reliability is 827 cycle. In literature [2], two pieces of crack propagation life test results are 380 and 500 cycles, which are both in the range of simulation

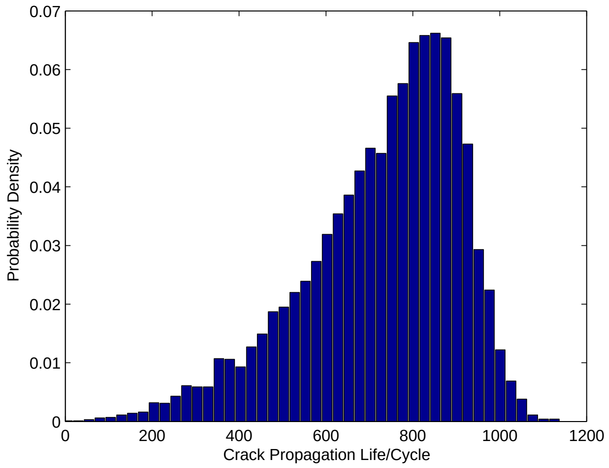

FIGURE 10 Lifetime histogram by effect of two random parameters (crack length 0.39-3.2mm)

TABLE 3: Result of Probability Life

Random Variable	a_0 to a_{cr} /mm	Mean Life /*Cycle*	Skewness	Variance
a_0	0.39-1.8	503	-1.07	2.972e4
ω	0.39-1.8	476	0.235	9.085e2
a_0, a_{cr}	0.39-1.8	504	-0.952	3.124e4
a_0, a_{cr}	0.39-3.2	729	-0.840	3.093e4

result of probabilistic lifetime. Therefore, the probability of life prediction method is reasonable.

(4) By using this method, taking into account the randomness of initial crack length and engine speed, crack propagation life from 0.39mm to 3.2mm is predicted, as shown in figure 10. Life samples meet normal distribution, and the mean value of life is 703 cycles. The life of 99% reliability is 560 cycle and life of 1% reliability is 1077 cycle. Therefore, the prediction is 560 cycles when the failure probability is 1%.

Conclusion

A realistic multi-source surface crack propagation problem of turbine disk is studied in this paper. We can conclude:

(1) In this paper, an approach of predicting propagation life of circumferential surface crack which located on gas turbine disk is established. A realistic turbine disk failure is studied. Compared with the experiment results, the prediction of probabilistic life is reasonable, which proved that the two-dimensional simplification is appropriate in specific multi-source crack failure.

(2) From sensitivity analysis, we can conclude that SIF is most sensitive to engine speed ω in this problem. Other variables such as properties of material is insignificant. In probabilistic fracture mechanics study, sensitivity analysis is necessary.

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