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PROBABILISTIC LIFE ASSESSMENT OF TURBINE VANES

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ABSTRACT

Life of a gas turbine vane generally depends on different factors such as scatter of material properties, load variation and manufacturing tolerances. However, deterministic finite element (FE) life analysis gives just a discrete value typically based on the nominal or worst case conditions. It precludes considering sensitivity to the input parameters and obtaining the expected life range. To consider the possible variations of the input parameters from their nominal values, a probabilistic approach has been applied to compute the LCF (Low Cyclic Fatigue) and creep life distributions for the uncooled vane.

The deterministic 3D FE life assessment of the gas turbine components is based on the input data such as physical and mechanical properties of the base material and coating at operating temperatures, nominal geometry of the component, thermal and mechanical loadings.

Each of the above mentioned inputs has its own scatter band characterized either by average and minimum values of mechanical properties (tensile strength, LCF, creep) or by variations of manufacturing tolerances; thermal boundary conditions and gas pressure distribution.

The probabilistic life analysis has been performed in order to assess individual impact of each input on vane's life scatter. LCF and creep life distributions as well as variation of the base metal oxidation layer thickness have been obtained for each scatter factor and for their overall contribution.

It is seen from results that LCF and creep lives of the analyzed vane have been influenced mainly by material properties and secondarily by OTDF (hot gas temperature variation in the circumferential direction) and uncertainties of thermal boundary conditions, which depended on the operation conditions of the engine. Manufacturing tolerances and alternation of ambient air temperature in the compressor intake have the lowest impact.

The derived model is useful for the risk analysis or maintenance planning. For instance, it has been shown how probability of small fatigue crack indication in one vane can be extended onto the overall probability for the failure detection of n vanes at the stator stage during regular inspection. The probability of micro crack growth due to creep after the determined amount of operating hours for the single vane may be also redefined into the overall stage probability for the detection of n such vanes.

To perform validation, normalized field data have been used for comparison with the analytical predictions. Good correlations between the field data and analytical predictions have been shown.

NOMENCLATURE

FE	Finite element
LCF	Low Cycle Fatigue
OTDF	Overall Temperature Deformation Factor – circumferential hot gas temperature variation in a row
LE	Leading edge
FM	Crack growth estimated by Fracture Mechanics
BC's	Boundary conditions
N	Number of LCF cycles
t	Creep rupture time
d	Oxidation thickness
T	Temperature
S	Standard deviation

P	Probability
M	Number of vanes in a row
K	Number of vanes with micro-cracks
Z'	Quantile of normal distribution with 95% confidence level
C	Number of vane combination
OH	Operating hours

Subscripts

LCF	Low Cycle Fatigue to crack initiation
FM	Crack growth estimated by Fracture Mechanics
MEAN	Mean value
MIN	Minimum value
MAT	Material properties
OTDF	Overall Temperature Deformation Factor
UNC	Uncertainties with thermal BC's
AMB	Ambient condition
W	Wall thickness
COMP	Composition
CI	Crack initiation

Superscripts

j	Variable number of vanes

INTRODUCTION

Life prediction for heavy duty gas turbine parts is based on both analysis of the new design and field experience of the prototype. Interaction between analysis and field experience is important in order to develop life estimation.

The most realistic (consolidated) life statement is obtained using probabilistic approach taking into account scatter bands characterized by variation of mechanical properties, manufacturing tolerances, thermal boundary conditions, gas pressure distribution etc.

As opposed to the deterministic discrete approach probabilistic and statistic methodology demonstrates the risk involved in operating components more clearly. Distributions are given instead of discrete minimum numbers. This has a potential to reduce calendar time of design process as the probabilistic life distribution can meet requirements whereas the deterministic ones do not meet these without some design iteration(s).

There are a lot of publications on this topic showing a wide range of probabilistic techniques application and benefits of probabilistic lifing for critical rotating engine components in gas turbines. For example, our company has used probabilistic approaches developed by R. Mücke *et al* [1], [2], L.A. Houck III *et al* [3] for life analysis of turbine blades.

R. Mücke *et al* [1-2] address their probabilistic approach to life prediction of cooled gas turbine blades where variations of load and material parameters are taken into account by a combination of a direct Monte Carlo Simulation and a Response Surface Method. In the paper [2] probabilistic design methods are applied to single crystal turbine blade where variations in three to single crystal orientations are also taken into account.

L.A. Houck III *et al* [3] presents a Bayesian updating technique to incorporate the analytical prediction with field data in order to improve the prediction. The field data are interpreted in terms of the probability of having defective hardware, and then the likelihood function is generated from the binomial distribution.

Apart from turbine blading, probabilistic life assessment techniques may be also applied for rotors. M. Gorelik *et al* [4] discuss the main elements of probabilistic lifing system development for a new alloy, including material characterization requirements, selection of appropriate modeling techniques and validation plans. R. Corran *et al* [5] propose probabilistic approach for induced anomalies along machined hole surfaces in engine rotors.

Probabilistic methodology may be also applied to assess fretting fatigue of aircraft engine disks. M.P. Enright *et al* [6] presents probabilistic model of the fretting process. This model calibrated by using available field data is applied to estimate risk reduction associated with non-destructive inspection of aircraft engine components subjected to fretting fatigue. K.S. Chan *et al* [7] identify steps required to develop a probabilistic crack growth methodology for treating high-cycle fretting fatigue in military engine disks, i.e. incorporation of high-frequency vibratory stress cycles into analysis.

Probabilistic approach makes it also possible to take small changes in the applied stress history of aircraft engine – see M.P. Enright *et al* [8] where a process is given for classifying and incorporating stress values taken from engine flight data recorders for use in probabilistic crack growth assessments.

One of the key random variables is a probability of detection representing quantified non-destructive evaluation capability, M. Gorelik *et al* [9] discuss how this variable may be used in probabilistic design and life management process. The paper focuses on non-destructive evaluation applications in detecting either pre-existing or fatigue induced component anomalies.

The aim of the current investigation is to introduce probabilistic life assessment methodology, calculate life range resulting from input data scatter and compare it with obtained field data statistics. An uncooled hollow turbine vane (Figure 1) was used as an example for checking the link between predicted life range (caused by variations and uncertainties of source data) with statistics from field experience. Source data sensitivity has been reviewed and factors having fundamental influence on life range limits have been identified.

Preliminary performed 3D FE steady state and transient LCF calculations as well as creep and oxidation calculations have indicated the most critical location at vane LE fillet (Figure 1). So only this location has been taken into account for probabilistic assessment. Comparison of calculated steady state and transient LCF lives has shown difference less than 5% for LE fillet that allows to use steady state LCF analysis for further deterministic calculations.

Life scatter has been evaluated as follows:

1. Deterministic calculation of the scatter sources influence on LCF (to crack initiation + FM) / creep life and oxidation layer thickness. The worst and the mean cases have been considered separately for each of the scatter factors (material data, thermal boundary conditions and manufacturing tolerances).
2. Corresponding normal distributions for LCF / FM / creep life and oxidation thickness obtained for each of the mentioned scatter factors using appropriate percentiles or three sigma rule.
3. Generation of probability distributions' composition for LCF / FM / creep life and oxidation thickness by adding standard deviations assuming that the scatter factors are uncorrelated random variables
4. Generation of probability distribution for total cyclic life using Monte-Carlo simulation of LCF and FM life
5. Conversion of crack initiation / growth probability in one vane to probability of at least K vanes with leading edge crack detection during an inspection (using binomial probability law).

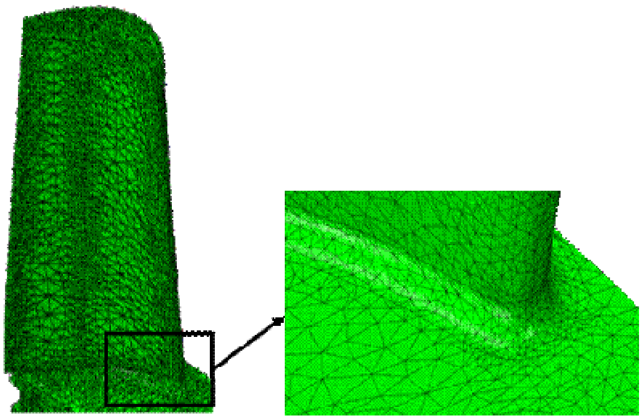

Figure 1: Uncooled turbine vane used for the investigation and its LE fillet

Variations of the following factors have been considered:

1. Material data scatter:
   - LCF material data scatter;
   - Crack growth material data scatter;
   - Creep material data scatter.
2. Thermal boundary conditions scatter:
   - Hot gas temperature variation in circumferential direction: OTDF in a row up to 5%;
   - Hot gas temperature increase of 5% to take into account uncertainties with thermal boundary conditions (BC's);
   - Ambient temperature range: -20 … +50^{0}C.
3. Manufacturing tolerances scatter:
   - Vane's wall thickness variation due to core position change.

DETERMINISTIC CALCULATIONS

The series of deterministic 3D FE LCF / creep life and oxidation layer thickness calculations has been done for analyzed vane to calculate input for probabilistic analysis and evaluation of life distribution parameters:

- N_{MEAN} (LCF and FM), or t_{MEAN}, or d_{MEAN} – mathematical expectation of LCF or creep life, or oxidation layer thickness for a vane with mean material properties, w/o OTDF, w/o uncertainties in thermal BC's, at standard ambient conditions, with nominal wall thickness.
- $N_{MIN\ MAT}$, or $t_{MIN\ MAT}$, or $d_{MiN\ MAT}$ – LCF or creep life, or oxidation thickness for a vane with minimum material properties; other parameters are nominal (w/o OTDF, w/o uncertainties in thermal BC's, standard ambient conditions, nominal wall thickness)
- $N_{MIN\ OTDF}$, or $t_{MIN\ OTDF}$, or $d_{MIN\ OTDF}$ – value for a vane with maximum OTDF (+5%); other parameters are nominal
- $N_{MIN\ UNC}$, or $t_{MIN\ UNC}$, or $d_{MIN\ UNC}$ – value for a vane with maximum uncertainties in thermal BC's (+5%); other parameters are nominal
- $N_{MIN\ AMB}$, or $t_{MIN\ AMB}$, or $d_{MIN\ AMB}$ – value for a vane on cold day -20ºC (the worst condition); other parameters are nominal
- $N_{MIN\ W}$, or $t_{MIN\ W}$, or $d_{MIN\ W}$ – value for a vane with min wall thickness near LE (separate FE model was created for this case); other parameters are nominal

Relative deterministic cyclic lives *N*, creep lives *t* and oxidation thicknesses *d* are shown in Table 1.

Calculation cases	N_{LCF}	N_{FM}	t	d
MEAN	1.00	0.44	1.00	1.00
MIN MAT	0.21	0.04	0.04	-
MIN OTDF	0.31	1.27	0.23	1.68
MIN UNC	0.31	1.27	0.23	1.68
MIN AMB	0.71	0.36	0.85	1.1
MIN W	0.37	0.25	0.28	1.05

Table 1: Relative deterministic cyclic lives (to crack initiation and crack growth), creep lives and oxidation thicknesses

It can be seen from the results that life of the analyzed vane has been influenced mainly by material properties and secondarily by OTDF and uncertainties with thermal boundary conditions, which depend on the operation conditions of the engine. Manufacturing tolerances and variation of ambient air temperature at the compressor intake have the lowest impact.

CYCLIC LIFE DISTRIBUTION

Both a number of engine cycles to crack initiation and a number of engine cycles for stable crack growth up to its critical length are assessed below. The algorithm is given for LCF assessment, for crack growth life it is analogous.

For the sake of simplicity the normal distribution has been used as probability distribution law for all scatter factors although, for example, material properties are better fitted by the Weibull distribution.

Material properties scatter

The materials properties can influence life assessment results by two ways.

First of all, static material properties are used to determine strain range and maximum temperature (used for elastic thermal stress analysis). It should be noted that the influence of variation of static material properties (elastic modulus, Poisson ratio) on steady state results is the minimal one; hence, it has not been considered herein.

Secondly, low cyclic fatigue material's data have direct influence on LCF life.

Currently their minimum properties with hold time under either compression or tension are used in practice to consider creep influence.

The difference between minimum properties and mean ones lies in probabilistic variation of materials properties. Minimum values correspond to material properties with 1% fail of the number of tested specimens (99% probability of survival with 95% confidence). It is also considered that mean values correspond to material properties with 50% fail of the number of tested specimens (i.e. to the nominal value of life).

Log-normal distribution has been used as probability distribution law of LCF material properties and corresponding life cycles: logarithm of cyclic life log(N) is assumed to be dominated by normal distribution.

The mean value of LCF life distribution caused by uncertainties in LCF material properties corresponds to N_{MEAN} cycles; the minimum value $N_{MIN\ MAT}$ cycles corresponds to minimum material properties, e.g. to the life with probability 1%. Therefore, standard deviation S_{MAT} for normal distribution of log(N) caused by material properties variations is evaluated using equation

$$S_{MAT} = \left(\lg\left(N_{MEAN}\right) - \lg\left(N_{MIN\ \ MAT}\right)\right) \Big/ Z'\left(1\%\right) \qquad (1)$$

where $Z'\left(1\%\right)$ – 1% quantile of normal distribution with 95% confidence level.

Hot gas temperature variation in circumferential direction

Hot gas temperature variation of 5% has been considered for OTDF to take into account uncertainties in it.

The mean value N_{MEAN} remains the same, whereas the minimum value corresponding to hot gas temperature increase of 5% is $N_{MIN\ \ OTDF}$ cycles. So, standard deviation is as follows

$$S_{OTDF} = \left(\lg\left(N_{MEAN}\right) - \lg\left(N_{MIN\ \ OTDF}\right)\right) \Big/ 3, \qquad (2)$$

i.e. 3 Sigma rule is applied (instead of 1% quantile because distribution for hot gas temperature is less known than LCF life distribution).

Uncertainties with thermal BC's

Standard deviation S_{UNC} corresponding to 5% uncertainties in hot gas temperature is the same, i.e. $S_{UNC} = S_{OTDF}$.

Ambient temperature scatter

Ambient temperature scatter has been chosen as the following range: -20 … +48^{0}C. A hot day gives increase of LCF life due to thermal strain range decreasing and a cold day gives its decrease. Therefore, a cold day is the worst case of ambient temperature scatter.

The same procedure as for hot gas temperature variation has been applied here, and in order to find the standard deviation S_{AMB} the 3 Sigma rule has been applied as well.

Manufacturing tolerances scatter

To evaluate influence of Vane's wall thickness the worst possible core position according to design tolerances has been considered. Only wall thickness variation has been taken into account because LE fillet radius scatter lead to reduction of LCF life by 15% whereas wall thickness variation lead to LCF life reduction by 2.7 times.

The same procedure as that for ambient temperature scatter has been applied and standard deviation S_W has been found.

Probability distribution of cyclic life due to factors' composition

Mathematical expectation remains the same and is equal to N_{MEAN}, whereas composed standard deviation was evaluated using the following equation [10]

$$S_{COMP} = \\ = \sqrt{\left(S_{MAT}\right)^2 + \left(S_{OTDF}\right)^2 + \left(S_{UNC}\right)^2 + \left(S_{Tamb}\right)^2 + \left(S_W\right)^2} \qquad (3)$$

This expression assumes that the input factors are uncorrelated, normally distributed random variables.

Relative standard deviations of log-normal probability distribution of life cycles to crack initiation are given in Table 2.

Calculation cases	S
MAT	0.24
OTDF	0.17
UNC	0.17
AMB	0.05
W	0.15
COMP	0.37

Table 2: Relative standard deviations of log-normal probability distribution of life cycles to crack initiation

Total cyclic life is the sum of the number of cycles to crack initiation and the number of cycles of stable crack growth until its critical length defined from Paris law.

Probability distribution of FM life considering all the mentioned scatter factors has been found using the same approach as distribution of life to crack initiation.

Combination of these two distributions has been found using Monte-Carlo simulation with assumptions that both events – crack initiation and crack growth up to critical value are independent.

Probability distributions themselves are shown in Figure 2.

Probability distribution of cracked vanes detection

During a regular inspection one row consisting of M vanes is checked, therefore, determination of probability of detection of one or even more damaged vanes may be of interest.

By means of binomial law of probability distribution, probability of at least K cracked vanes detection versus life cycles may be obtained.

$$P_K = \sum_{j=K}^{M} C_M^j \cdot P_{CI}^j \cdot (1 - P_{CI})^{M-j}, \qquad (4)$$

where P_{CI} – probability of crack initiation in one vane.

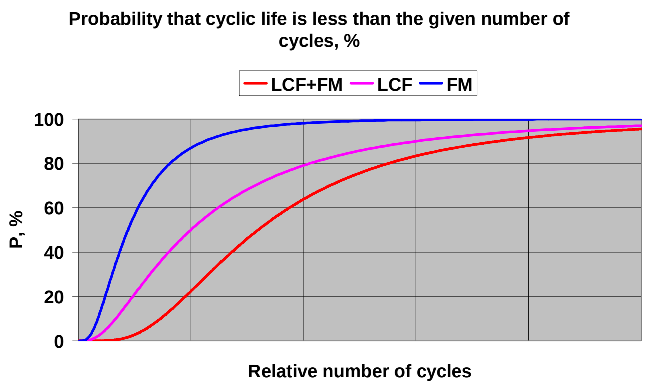

Figure 2: Relative cyclic life distribution

The obtained results are given in Table 3. So if probability of crack initiation is 0.5%, probability of detection at least one vane with a crack at LE during inspection of the engine is 28.2%. At least two cracked vanes in the same case will be found with probability of 4.34%.

Probability	Relative number of cycles of engine operation						
	0.07	0.11	0.14	0.24	0.33	0.56	1.00
Event 1	0.1	0.5	1	5	10	25	50
Event 2	6.39	28.2	48.5	96.6	99.9	100	100
Event 3	0.2	4.34	14.1	84.9	99.2	99.9	100

Table 3: Probability (%) of micro crack initiation in at least K vanes out of M in a row (Event 1 – Crack initiation in one vane; Event 2 – Detection of at least one vane with micro crack in one engine; Event 3 – Detection of at least two vanes with micro crack in one engine).

CREEP LIFE DISTRIBUTION

Probability distribution of creep rupture in one vane

In order to obtain creep life range same scatter factors as above have been considered. Relative deterministic creep lives are shown in Table 1. It has been assumed that creep life t (in OH) characterized by LE damage in the form of creep pores and/or micro cracks is distributed according to log-normal law for each of the factors.

Results for every factor have been obtained using the same procedures as described in previous Section for cyclic life. Certainly, a cold day is the worst case of ambient temperature scatter for a creep. Standard deviation has been obtained according to (3).

Creep life distribution is given on Figure 3.

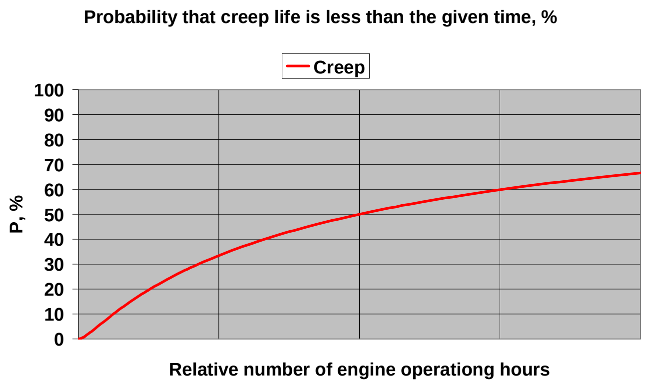

Figure 3: Relative creep life distribution

Probability distribution of detection of several vanes with creep pores

Same procedure as in previous Section for cyclic life has been applied to obtain probability of detection of one or even more damaged vanes out of M in one row. Obtained results are given in Table 4.

Probability	Relative number of engine operating hours						
	0.01	0.02	0.03	0.07	0.13	0.33	1.00
Event 1	0.1	0.5	1	5	10	25	50
Event 2	6.39	28.2	48.5	96.6	99.9	100	100
Event 3	0.2	4.34	14.1	84.9	99.2	99.9	100

Table 4: Probability (%) of creep pores in at least K vanes out of M in a row (Event 1 – Creep pores in one vane; Event 2 – Detection of at least one vane with creep pores in one engine; Event 3 – Detection of at least two vanes with creep pores in one engine).

BASE METAL OXIDATION THICKNESS DISTRIBUTION

In order to determine the range of base metal oxidation thickness values the same procedure as above has been applied. Relative deterministic oxidation thicknesses are shown in Table 1.

It has been assumed that logarithm of a base metal oxidation thickness is governed by the normal probability law.

Corresponding probability distribution is shown in Figure 4.

It should be mentioned that oxidation is an additional cracking cause. Cavities on the critical locations lead to additional significant worsening of LCF and creep material properties.

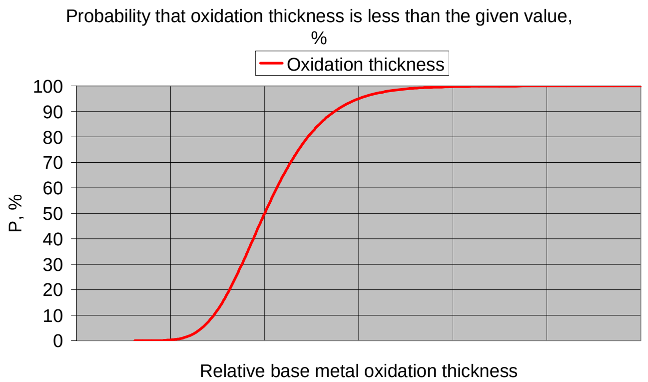

Figure 4: Base metal oxidation thickness distribution.

FIELD EXPERIENCE ANALYSIS AND COMPARISON WITH PROBABILISTIC RESULTS

Totally 48 vane rows (a row is a set of M Vanes installed in an engine at a time) installed in different engines have been analyzed during this research. The focus of the performed field experience analysis lies on the LE fillet.

Comparison between field experience and probabilistic results has been performed for vane rows as a whole and regarding probability of damage in at least one vane in a row only. Field experience data together with calculated probability of LE damage (cyclic / creep) in at least one vane in a row are shown in Figure 5 in the diagram with starts / operating hours (OH) axes where one start of operation corresponds to one calculated at least one LCF cycle in a vane row.

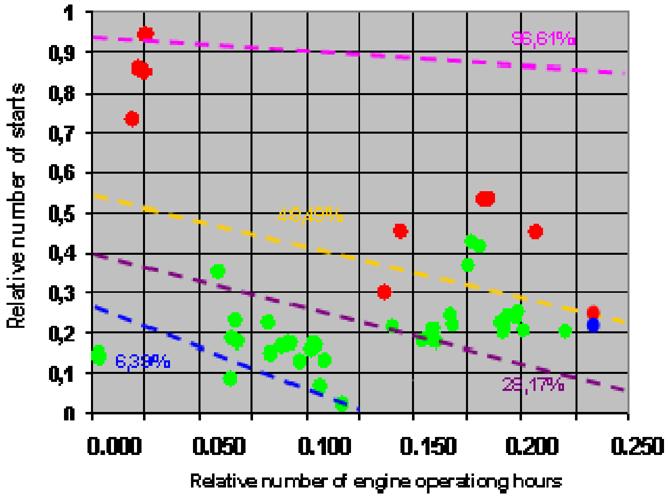

Figure 5: Damage diagram vs. probability of damage in at least one vane in a row. Green circles correspond to crack absence, red ones to LCF micro crack presence and blue ones to creep pores (micro crack)

Dashed lines in Figure 5 indicate linear accumulation of LCF and creep damage at different probabilities based on probabilistic analysis.

Calculated relative probabilistic cycles to crack initiation in at least one vane in a row are in a good agreement with available field experience data. Comparison between relative field statistics and results of probabilistic analysis is shown in Figure 6 for vane rows having at least one vane with micro crack.

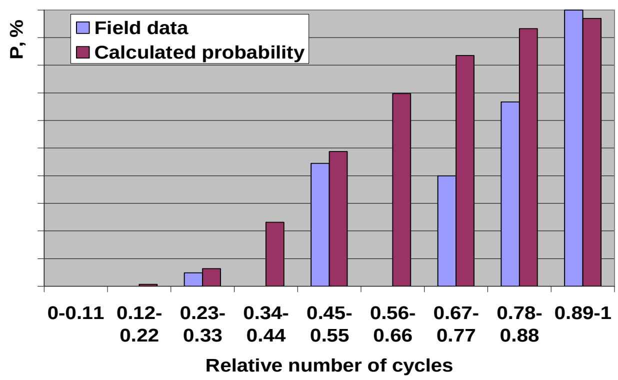

Figure 6: Comparison between field statistics and results of probabilistic analysis for vane rows having at least one vane with micro crack depending on the relative number of starts.

As for probabilistic time to creep pore (micro crack) in at least one vane in a row, it corresponds well to available field experience for relatively high number of operating hours – see Figure 7.

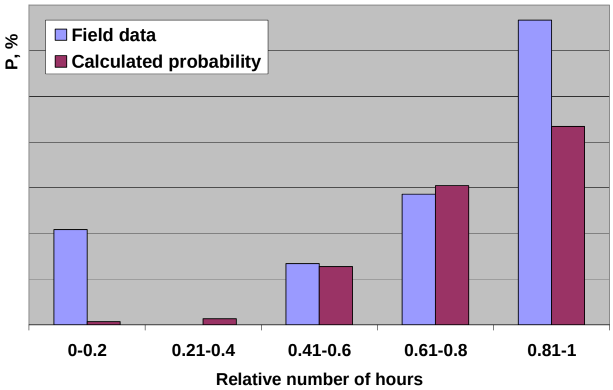

Figure 7: Comparison between field statistics and results of probabilistic analysis for vane rows having at least one vane with micro crack depending on the relative number of operating hours.

However, creep alone doesn't explain LE damage at relatively low number of operating hours, so, creep is not the only damage cause.

Although limited oxidation thickness measurements after operation are available that does not allowing comparison of probabilistic calculation with field statistics however measured maximum oxidation thickness corresponds well to thickness calculated for probability about 90%.

Thus combined mechanism of LCF, creep and oxidation damage was confirmed by Alstom metallurgical reports where LE micro cracks have shown triple effect of thermo-mechanical fatigue + creep + oxidation mode on micro crack propagation.

Finally it can be concluded that obtained results for cyclic / creep life distributions do correspond to available field experience.

CONCLUSIONS

1. Probabilistic approach has been applied for cyclic and creep life and oxidation thickness assessment of vane depending on material properties scatter, applied loads variation and manufacturing tolerances.
2. Cyclic / creep life distributions as well as base metal oxidation thickness distribution have been obtained for a leading edge fillet of an uncooled hollow vane.
3. Probability of cyclic / creep damage after a certain amount of cycles / operating hours have been calculated for one vane and then converted to probability of detection of at least K vanes with defects during an inspection.
4. The following source data sensitivity for life has been observed:
    - Material properties have the most pronounced effect on cyclic / creep life; the next two factors are OTDF and uncertainties in hot gas temperature and manufacturing tolerance; the least important factor is ambient temperature.
    - OTDF and uncertainties in hot gas temperature have the most profound influence on base metal oxidation thickness.
5. Obtained results have been compared with field experience for vane rows.
    - Probability of micro crack initiation in at least one vane in a row corresponds well to available field experience data for the whole range of starts.
    - Probabilistic time to creep pore formation in at least one vane in a row corresponds well to available field experience data for relatively high number of operating hours.
    - Creep alone doesn't explain LE damage at a relatively low number of operating hours where creep rupture is unlikely, therefore, creep is not only damage cause. This has been confirmed by metallurgical reports where LE micro cracks have shown triple effect of thermo-mechanical fatigue + creep + oxidation mode micro crack propagation.
6. The derived model is useful for the risk analysis or maintenance planning. Probability of small fatigue crack or micro crack growth due to creep detection after the determined amount of operating hours in one vane can be extended onto the overall probability for the failure detection of K vanes at the stator stage during regular inspection.

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