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CONSTRUCTION OF SHOCK RESPONSE MAP FOR A FLEXIBLE ROTOR-BEARING SYSTEM WITH MOUNT SYSTEM TO BASE EXCITATIONS USING THE FE TRANSIENT ANALYSIS METHOD

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ABSTRACT

Turbomachinery such as turbines, pumps and compressors, which are installed in transportation systems such as warships, submarines and space vehicles, etc., often perform crucial missions and are exposed to potential dangerous impact environments such as base-transferred shock forces. To protect the machines from such excessive shock forces, one may need to accurately analyze transient responses of rotors earlier on in their design stages, considering the dynamics of mount designs to be applied with.

In this study, utilizing the generalized FE transient response analysis method of a flexible rotor-bearing system with a mount system to base-transferred shock forces, constructions of the shock response and static deflection maps of turbine rotor-bearing and mount system are devised, introducing the mount mass, resilient support stiffness and damping ratios to the counterpart rotor mass, bearing stiffness and damping and the mount system natural frequency. For the given turbine rotor system design a best available mount system design, composed of a mount plate and resilient support, can be readily selected from the constructed maps to meet the rotor's shock response and mount's static design limits. The shock response maps also show that for the same shock the FE flexible rotor model used herein yield a more compact light-weighed mount system design than the conventional simple rigid rotor model. Therefore, the shock response map approach in conjunction with the more complicated FE flexible rotor transient response analysis method is justified.

1 INTRODUCTION

Turbomachinery such as turbines, pumps and compressors, which are installed in transportation systems such as warships, submarines and space vehicles, etc., frequently experience various sudden shock forces during their life cycles, depending on the operating conditions and external environments. These shock forces may be directly transferred through bases or foundations to the core rotor-bearing systems of turbomachinery, and may induce severe damage due to direct impact collisions between the rotors and bearings, seals and other stators or the rotors can generate dangerously high vibrations due to rubbing contact. Therefore, in cases of turbomachinery that are exposed to potential dangerous impact environments and perform critical missions, it is necessary to accurately predict the transient responses of rotor systems under base-transferred shock forces and estimate their safety in the early design stages. In particular, to protect turbomachinery from any excessive shock forces, an accurate analysis of transient responses of the rotors may be needed, considering the dynamics of the mount designs to be applied with.

Previous work on transient response analyses of rotor-bearing systems to base-transferred shock forces can be classified into the system modeling methods and types of excitation involved. Hori and Kato [1] investigated the stability of a rotor system supported by fluid film bearings when acted on by seismic loads, using a Jeffcott rotor model. Tessarzik et al. [2] analyzed transient responses of a simple rotor model, considering relative coordinate systems, to random excitations that were imposed at the base in the axial direction, and Soni and Srinivasan [3] investigated the responses of a rigid rotor model to seismic loads. Singh et al. [4], Suarez et al. [5] and Gaganis et al. [6] proposed FE rotor models with no mount consideration but by taking base motions into consideration and analyzed transient responses to seismic excitations. Generally speaking, in the past, mount systems had been applied to rotor-bearing systems to improve the stability and to minimize the response amplitudes and transferred forces, and so on. Utilizing

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a Jeffcott rotor model on a mount system modeled by an elastic spring and viscous damper, Kirk and Gunter [7, 8], Pilkey et al. [9] and Barrett et al. [10] discussed the optimal mount stiffness and damping, in an attempt to minimize rotor vibrations to unbalances and sudden losses and showed that a mount system can effectively improve the stability of a rotor-bearing system and greatly reduce its vibration amplitude. They also showed that the stability thresholds of rotor systems supported by fluid film bearings are improved. Ehrich [11] investigated mount stiffness and damping to minimize the force transmitted to the base due to a rotor unbalance. Utilizing rotor models that took the gyroscopic effects into an account, Ota et al. [12] focused on mount stiffness and damping to minimize rotor unbalance responses and Subbiah et al. [13] investigated the effects of mount stiffness and damping on transient rotor responses to base random excitations.

On the other hand, Kim and Lee [14] proposed the state-space Newmark method, based on the average velocity concept, for obtaining transient response solutions for general dynamic systems, and showed that the proposed method is unconditionally numerically-stable, regardless of a size of time step. Lee et al. [15] investigated transient responses of a flexible rotor-bearing system to base excitations, introducing the FE method but without considering any mount system and also attempted comparisons with experimental results. Further, Lee and Kim [16] proposed a generalized FE transient response analysis model to accurately predict the transient responses of a flexible rotor-bearing system with mount systems to base-transferred shock forces, utilizing the state-space Newmark method of a direct time integration scheme.

In this study, utilizing the recently-proposed generalized FE transient response analysis method of a flexible rotor-bearing system with a mount system to base-transferred shock forces, constructions of shock response and static mount deflection maps of a turbine rotor-bearing system are devised, introducing the mount mass, resilient support stiffness and damping ratios to the counterpart rotor mass, bearing stiffness and damping and the mount system natural frequency. The results by using the FE-based detailed flexible rotor model are compared to those by using the conventional simple model or Jeffcott rotor, which treats the rotor-bearing system as a concentrated lumped mass, equivalent spring and damper.

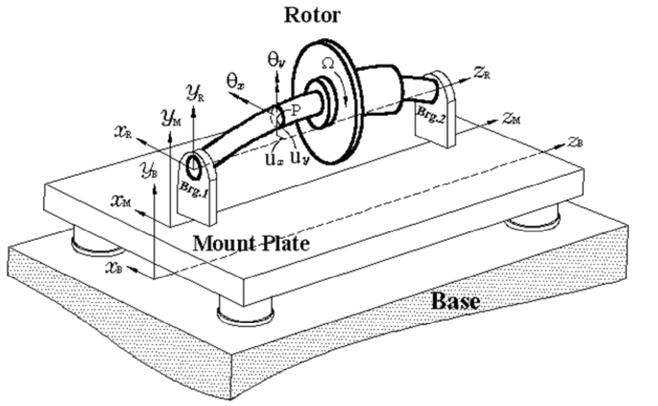
2. ANALYSIS MODELS, SOLUTION METHOD AND SOME TRANSIENT RESPONSES

2.1 FE Flexible Rotor Analysis Model:

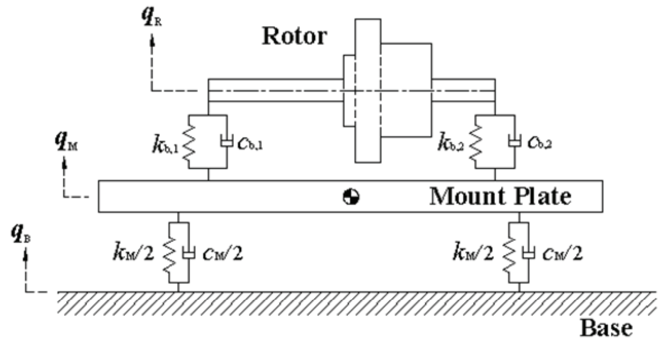
Figure 1 shows a flexible rotor-bearing system installed on a mount system and coordinate systems. Define the generalized relative displacement vectors of the rotor to the mount plate as $\{z_R\}$ and of the mount plate to the base as $\{z_M\}$,

$$\{z_R\} = \{q_R\} - \{q_M\} \tag{1}$$

$$\{z_M\} = \{q_M\} - \{q_B\} \tag{2}$$



(a) A 3D view



(b) A 2D front view

Figure 1. A flexible rotor-bearing system model on a mount system and coordinate systems.

where $\{q_R\}$, $\{q_M\}$ and $\{q_B\}$ are the generalized displacement vectors of the rotor, mount plate and base, respectively. $\{q_B\} = [w_x \quad w_y \quad \phi_x \quad \phi_y]^T$, and w_x and w_y represent the translational displacements and ϕ_x and ϕ_y the angular displacements. Detailed FE formulations of the equations of motion of a disk element, shaft element and rigid mount plate and associated nomenclatures of variables may be referred well to Lee and Kim [16]. Here, only the equations of motion for a disk element and rigid mount plate are introduced.

The equation of motion of a disk element is expressed by

$$\begin{bmatrix}[M_d] & [M_d]\end{bmatrix}\begin{Bmatrix}\{\ddot{z}_R\}\\ \{\ddot{z}_M\}\end{Bmatrix} + \begin{bmatrix}[G_d] & [G_d]\end{bmatrix}\begin{Bmatrix}\{\dot{z}_R\}\\ \{\dot{z}_M\}\end{Bmatrix} = \{F_d\} \tag{3}$$

where $[M_d] = \begin{bmatrix} m_d & 0 & 0 & 0 \\ 0 & m_d & 0 & 0 \\ 0 & 0 & I_d^t & 0 \\ 0 & 0 & 0 & I_d^t \end{bmatrix}$, $[G_d] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \Omega I_d^p \\ 0 & 0 & -\Omega I_d^p & 0 \end{bmatrix}$

and

$$\{F_d\} = \begin{Bmatrix} -m_d \ddot{w}_x \\ -m_d \ddot{w}_y \\ -I_d^t \ddot{\phi}_x - I_d^p \Omega \dot{\phi}_y \\ -I_d^t \ddot{\phi}_y + I_d^p \Omega \dot{\phi}_x \end{Bmatrix}$$

In Eq. (3) the equation of motion of the disk element is expressed by the relative motions of the rotor and mount plate. In addition, since the forcing vector contains terms due to the translational acceleration and the angular velocity and acceleration of the base, it can be seen that the motion of the base acts as the forcing.

Assuming the mount plate as a rigid body, the equation of motion of the mount plate is obtained by

$$[M_M]\{\ddot{z}_M\} + [C_M]\{\dot{z}_M\} + [K_M]\{z_M\} = \{F_M\} \tag{4}$$

where $[M_M] = \begin{bmatrix} m_M & 0 & 0 & 0 \\ 0 & m_M & 0 & 0 \\ 0 & 0 & I_M^x & 0 \\ 0 & 0 & 0 & I_M^y \end{bmatrix}$ and $\{F_M\} = \begin{Bmatrix} -m_M \ddot{w}_x \\ -m_M \ddot{w}_y \\ -I_M^x \ddot{\phi}_x \\ -I_M^y \ddot{\phi}_y \end{Bmatrix}$.

It can be seen from $\{F_M\}$ that the translational and angular accelerations of the base acts as the forcing to the mount system. On the other hand, in the case of treating the mount plate as a flexible body, the mount plate can be modeled readily by FEM.

2.2 Transient Solution Method:

Upon assembling the derived equations of motion of the disk elements, shaft elements and mount system along with the forces acting on the rotor and the mount plate by the bearings and mount support elements, the entire assembled equation of motion of a rotor-bearing system installed on a mount system can be expressed by

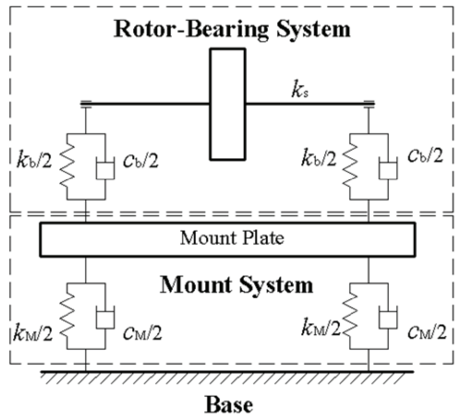
$$[M]\{\ddot{R}\} + (\Omega[G] + [C])\{\dot{R}\} + [K]\{R\} = \{Q\} \tag{5}$$

where $\{R\}$ and $\{Q\}$ are the relative displacement and forcing vectors of the entire system, and $[M]$, $[G]$, $[C]$ and $[K]$ are the inertia, gyroscopic, damping and stiffness matrices of the entire system, considering the mount system, too.

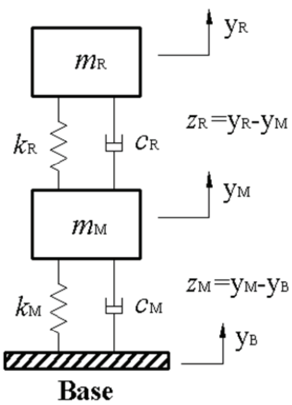
Introducing the state-space vector, Eq. (5) can be transformed into the first-order differential equation as given by

$$\{\dot{r}\} = [A]\{r\} + \{F\} \tag{6}$$

where $[A] = \begin{bmatrix} -[M]^{-1}[\Omega[G]+[C]] & -[M]^{-1}[K] \\ [I] & [0] \end{bmatrix}$,



(a) A simple Jeffcott rotor model on a mount system



(b) An equivalent 2 degrees of freedom model with base motion

Figure 2. A simple rotor model on a mount system.

$\{F\} = \begin{Bmatrix} [M]^{-1}\{Q(t)\} \\ \{0\} \end{Bmatrix}$ and $\{r\} = \{\{\dot{R}\} \quad \{R\}\}^T$

Finally, introducing the state-space Newmark method [14] for a transient response analysis, which assumes the average velocity over the time interval Δt, the response at time t_{n+1} is obtained by

$$\{r\}_{n+1} = ([I] - \frac{\Delta t}{2}[A])^{-1}(\{r\}_n + \frac{\Delta t}{2}\{\dot{r}\}_n + \frac{\Delta t}{2}\{F\}_{n+1}) \tag{7}$$

Upon calculating Eq. (7) with the known state values at time t_n and forcing $\{F\}_{n+1}$ at t_{n+1}, the state values or displacements and velocities at t_{n+1} are obtained.

2.3 Simple Rotor Analysis Model:

A simple Jeffcott rotor model on a mount system is shown in Fig. 2(a) and its equivalent 2 degrees of freedom model, considering the motion of the base, is shown in Fig. 2(b).

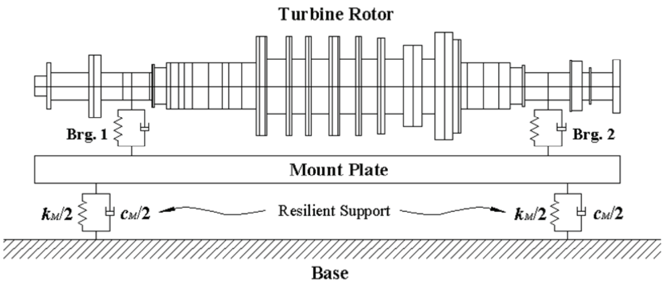


Figure 3. Schematic of a turbine rotor-bearing system on its mount system.

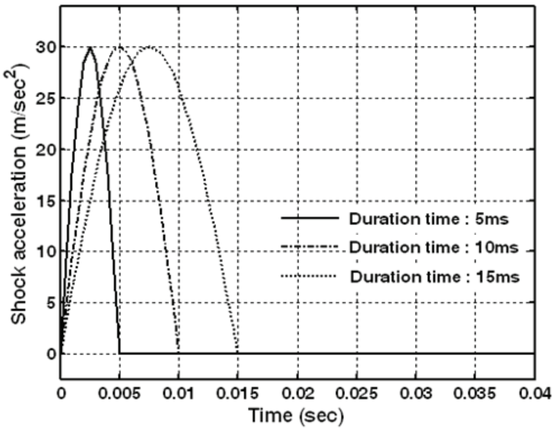


Figure 4. Series of ideal half-sine shock-waves imposed on the base.

Referring to Fig. 2(b) and introducing the relative coordinates, $z_R = y_R - y_M$ and $z_M = y_M - y_B$, the equations of motion of the rotor and mount plate in the vertical direction only are given by

$$\begin{bmatrix} m_R & m_R \\ 0 & m_M \end{bmatrix}\begin{Bmatrix} \ddot{z}_R \\ \ddot{z}_M \end{Bmatrix} + \begin{bmatrix} c_R & 0 \\ -c_R & c_M \end{bmatrix}\begin{Bmatrix} \dot{z}_R \\ \dot{z}_M \end{Bmatrix} + \begin{bmatrix} k_R & 0 \\ -k_R & k_M \end{bmatrix}\begin{Bmatrix} z_R \\ z_M \end{Bmatrix} = \begin{Bmatrix} -m_R\ddot{y}_B \\ -m_M\ddot{y}_B \end{Bmatrix} \tag{8}$$

where $k_R = k_s k_b/(k_s + k_b)$ is the equivalent support stiffness for the rotor, combining the bearing stiffness, k_b, and the shaft stiffness, k_s, in series, and $c_R(=c_b)$ is the bearing damping. From Eq. (8) expressed in the relative coordinates, it can be also seen that the acceleration of the base acts as the forcing.

2.4 Some Transient Response Results:

Figure. 3 shows a schematic of the turbine rotor-bearing system on a mount system used in the analysis. The rotor has a mass of 2,134 kg and axial length of 3.235 m. The rotor operates at the rated speed of 6,000 rpm with the first critical speed 3,469 rpm and the second critical speed 8,171 rpm. The mount system is composed of a mount plate and resilient

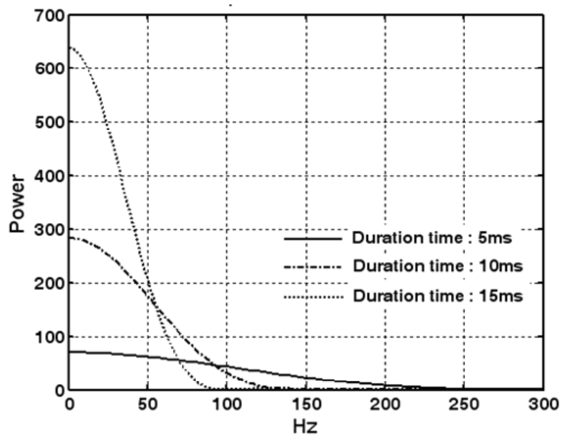


Figure 5. Power spectra of ideal half-sine shock-waves imposed on the base.

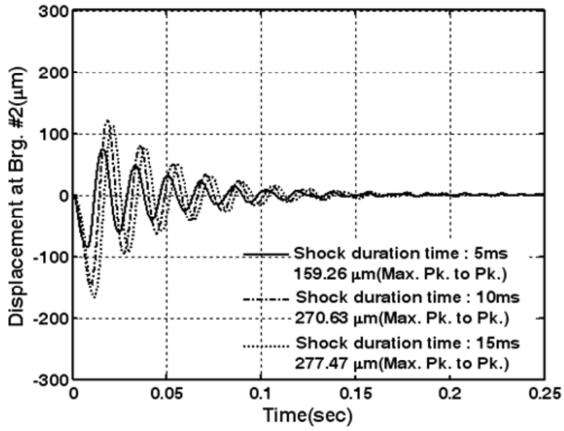


Figure 6. Transient rotor responses for the FE flexible rotor model with no mount system.

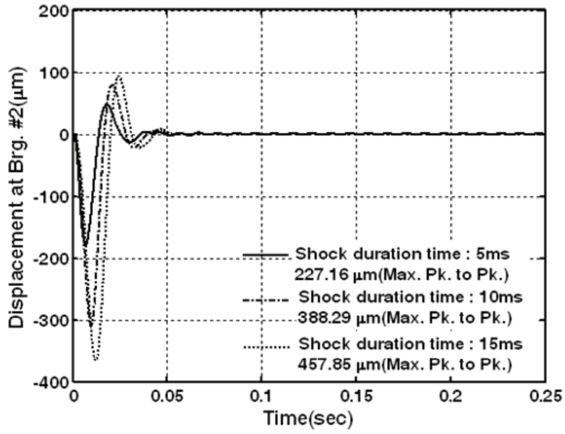


Figure 7. Transient rotor responses for the simple rotor model with no mount system.

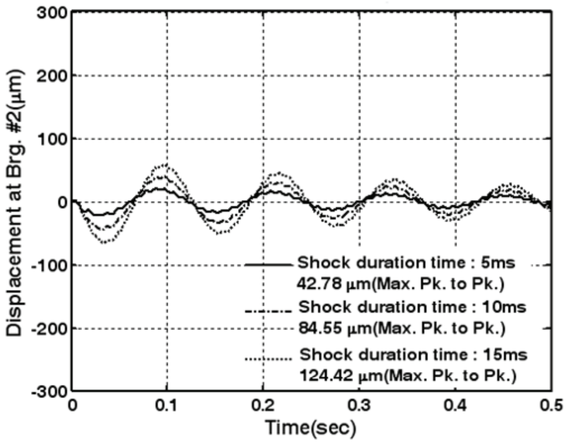


Figure 8. Transient rotor responses for the FE flexible rotor model with a mount system of natural frequency of 10 Hz.

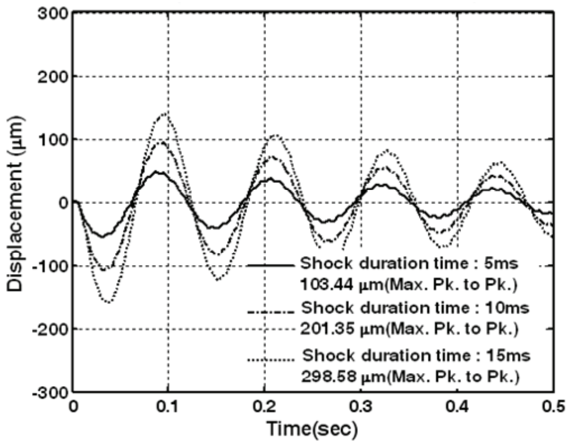


Figure 9. Transient rotor responses for the simple rotor model with a mount system of natural frequency of 10 Hz.

support. For a given mass, 6,402 *kg*, of the mount plate, the stiffness of the resilient support is selected to yield a natural frequency of the mount system to be 10 Hz. Further, a damping ratio, ζ=0.01, of the mount resilient support is considered.

Figure. 4 shows a series of ideal half-sine waves imposed in the vertical direction as shock excitations to the base. All the shock waves have a magnitude of 3*g* and their duration times are 5, 10 and 15 ms, respectively. Figure. 5 shows the power spectra of the half-sine waves, and it can be seen that there exist considerable exciting components up to 100 Hz.

By the way all transient rotor responses shown in the following represent the responses of the rotor at the position of bearing #2 relative to the mount plate.

Analysis with No Mount-System

In the analyses, the mount system was not taken into consideration. *For the FE flexible rotor model*, the transient responses of the rotor to the base shock excitations are shown in Fig. 6. For duration times of 5, 10 and 15 ms the maximum responses of the rotor are 159.3, 270.6 and 277.5 μm Pk. to Pk. *For the case of the simple rotor model*, the transient responses of the rotor to the base shock excitations are shown in Fig. 7. For duration times of 5, 10 and 15 ms the maximum responses of the rotor are 227.2, 388.3 and 457.9 μm Pk. to Pk.

Analysis with Mount-System

In the analyses, the mount system was taken into consideration and its natural frequency was 10 Hz. *For the case of the FE flexible rotor model*, the transient responses of the rotor to the base shock excitations are shown in Fig. 8. For duration times of 5, 10 and 15 ms the maximum responses of the rotor are 42.8, 84.6 and 124.4 μm Pk. to Pk. *For the case of the simple rotor model*, the transient responses of the rotor to the base shock excitations are shown in Fig. 9. For duration times of 5, 10 and 15 ms the maximum responses of the rotor are 103.4, 201.4 and 298.6 μm Pk. to Pk.

From the above, it is clear that the transient rotor response increases as the duration time increases and that particularly the FE flexible rotor model yields the rotor responses that are more reduced, compared to the simple rotor model. The reason for the later is as follows: while in the simple rotor model the shock energy transferred to the rotor is concentrated on the equivalent support element, the FE flexible rotor model treats the rotor as a series of discrete inertia and elastic systems along the shaft and as a result some of the transferred shock energy is stored in the rotor shaft as well. Thereby, for the FE flexible rotor model the vibrations of the rotor are more reduced. Consequently and more importantly, in order to obtain the same level of transient response of the rotor the FE flexible rotor model can adopt a mount system with a higher natural frequency than the simple rotor model. On the other hand, as expected, it can also be seen that the rotor with a mount system always has less transient vibration than that without one.

Synthetically speaking, to protect a rotor-bearing system from excessive shocks imposed on the base of a rotating machine, installing an appropriate mount system between the rotating machine and base structure can be a practical solution, and further, to permit a rotor-bearing system design with a more compact light-weighted mount system the FE flexible rotor analysis model here shall be recommended.

3. CONSTRUCTION OF SHOCK RESPONSE MAP

For a given or fixed design of rotor-bearing system the effect of mount system design on the transient rotor response to the base-transferred shock excitation is explored as a shock response map. In order to construct the shock response map the design variable ratios of mount system to the rotor-bearing system such as the mount mass ratio, α, stiffness ratio, β, damping ratio, γ, and the natural frequency of mount system, ω_M, are defined as follows, referring to Fig. 2:

$$\alpha = \frac{m_M}{m_R}\ , \quad \beta = \frac{k_M}{k_R}\ , \quad \gamma = \frac{c_M}{c_R}\ , \quad \omega_M = \sqrt{\frac{k_M}{m_M}} \qquad (9)$$

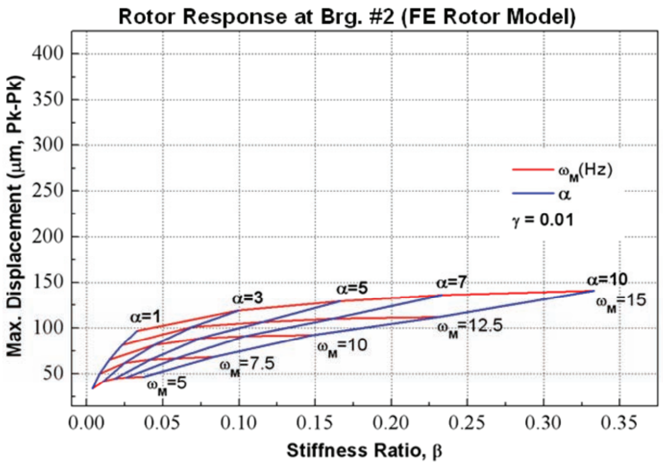


Figure 10. The shock response map of the rotor at $\gamma = 0.01$ for the FE flexible rotor model.

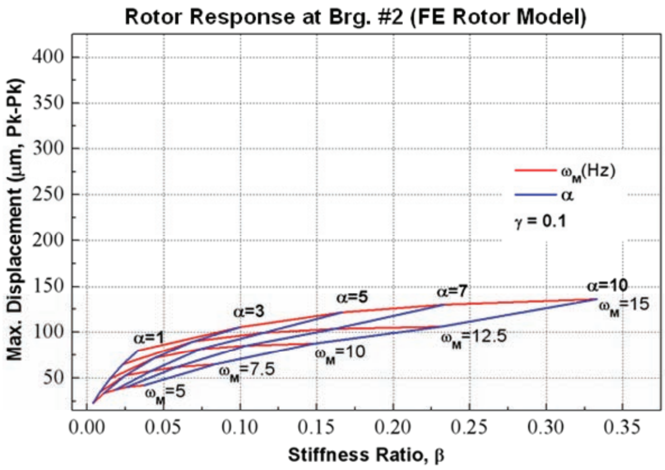


Figure 11. The shock response map of the rotor at $\gamma = 0.1$ for the FE flexible rotor model.

The base shock excitation considered here is an ideal half-sine wave with a magnitude of 3g and duration time of 10 ms in the vertical direction. *For the case of the FE flexible rotor model*, the maximum transient responses of the rotor at the position of bearing #2 relative to the mount plate are plotted in Figs. 10 to 13 as the shock response maps of the rotor as the functions of α, β and ω_M for each γ value of 0.01, 0.1, 0.5 and 1, respectively. From the figures it can be seen that as the γ increases from 0.01 to 1, the rotor responses decrease in overall. Also, for a given value of γ, as α, β and ω_M all increase, the rotor responses increase in overall. *For the case of the simple rotor model*, the maximum transient responses of the rotor are shown in Fig. 14 as the shock response map for $\gamma = 0.1$, only for a reference purpose, since the maximum responses for the simple rotor model are all substantially greater than the maximum responses for the FE flexible rotor

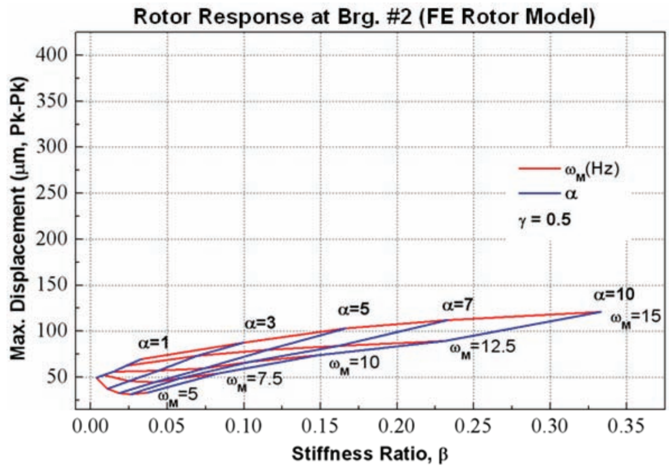


Figure 12. The shock response map of the rotor at $\gamma = 0.5$ for the FE flexible rotor model.

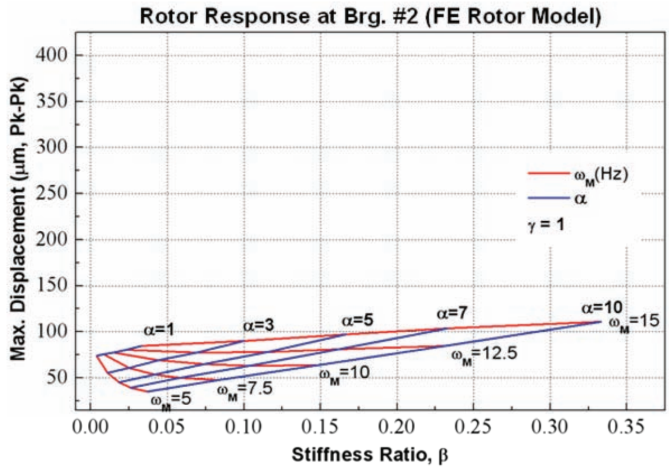


Figure 13. The shock response map of the rotor at $\gamma = 1$ for the FE flexible rotor model.

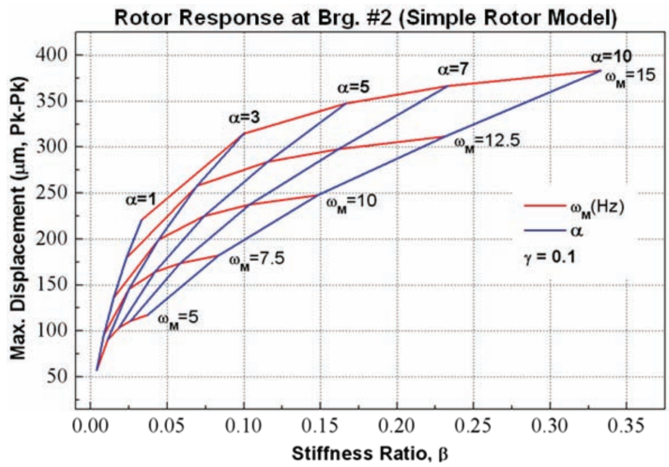


Figure 14. The shock response map of the rotor at $\gamma = 0.1$ for the simple rotor model.

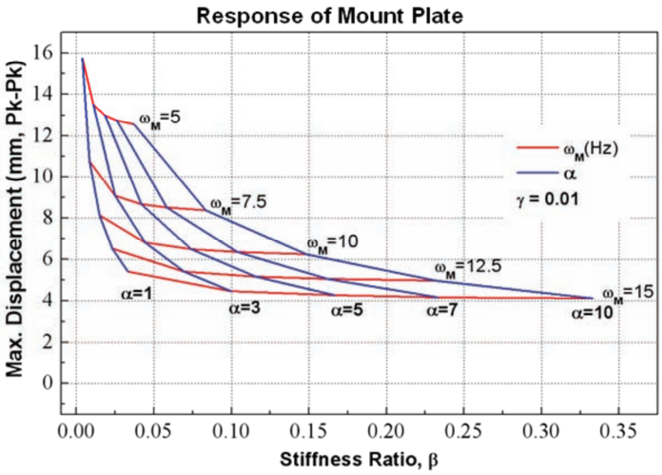


Figure 15. The shock response map of the mount plate at $\gamma = 0.01$.

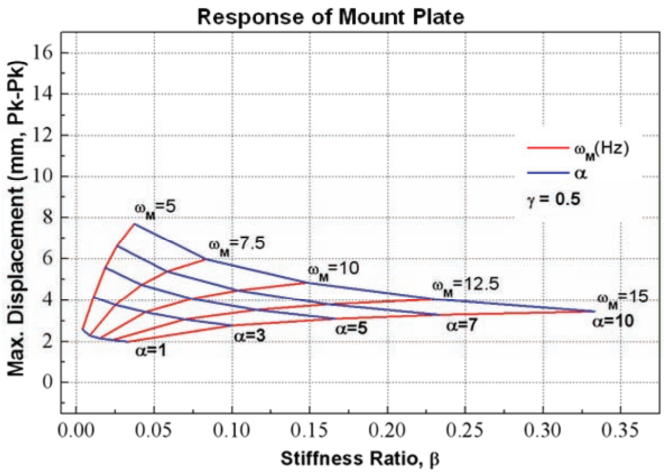


Figure 17. The shock response map of the mount plate at $\gamma = 0.5$.

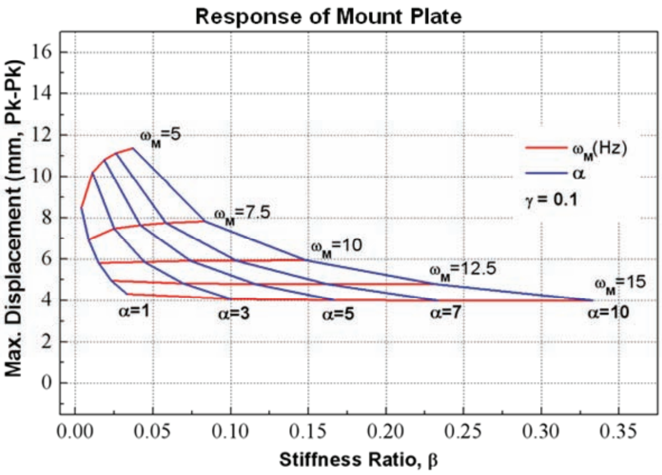


Figure 16. The shock response map of the mount plate at $\gamma = 0.1$.

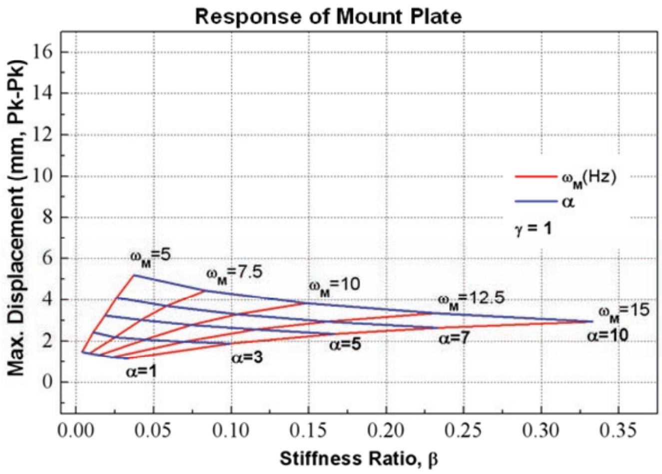


Figure 18. The shock response map of the mount plate at $\gamma = 1$.

model and also the trends of the shock response maps for the simple rotor model, with respect to of γ, α, β and ω_M, are the same as those for the FE flexible rotor model, as expected.

The maximum transient responses of the mount plate relative to the base are plotted in Figs. 15 to 18 as the shock response maps of the mount plate as the functions of α, β and ω_M for each γ value of 0.01, 0.1, 0.5 and 1, respectively. From the figures it can be seen that as the γ increases from 0.01 to 1, the responses of the mount plate decrease in general. But, depending on the values of γ and β, the effects of α and ω_M on the shock response of the mount plate can change.

In addition, the static deflection map of the mount plate, which is also an important design consideration, is plotted in Fig. 19 as the functions of α, β and ω_M. As the ω_M increases, the static deflections decrease in general. And along a given line of equal ω_M, as α and β increase the static deflections decrease.

Now, utilizing the shock response maps of the rotor-bearing system and mount system and the static deflection map of the mount plate, plotted above, and also considering any design limitations or restrictions imposed on the mount system, a best compromised design of the mount system may be obtained to effectively control the response of the rotor-bearing system to the base-transferred shock excitation.

4. CONCLUSIVE REMARKS

In order to protect turbomachinery such as turbines, pumps and compressors, which are installed in transportation systems such as warships, submarines and space vehicles, etc., from excessive base-transferred shock forces, a designer may need to accurately analyze transient responses of machine rotors earlier

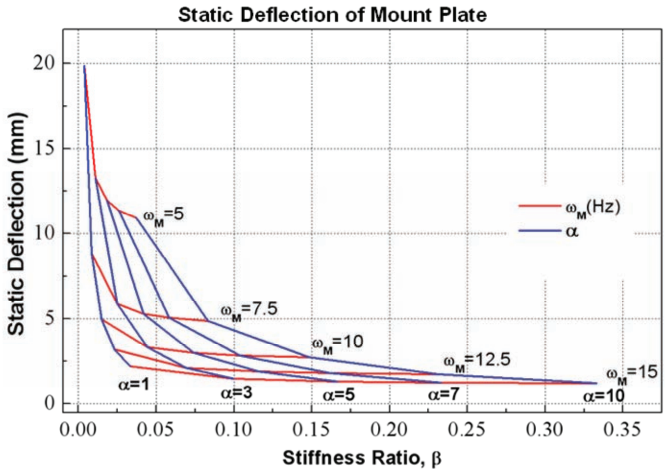


Figure 19. The static deflection map of the mount plate.

on in their design stages, considering the dynamics of mount designs to be applied with.

In this study, utilizing the generalized FE transient response analysis method of a flexible rotor-bearing system with a mount system to base-transferred shock forces, constructions of the shock response maps of rotor-bearing system and mount system and the static deflection map of mount plate have been devised, introducing the mount mass, resilient support stiffness and damping ratios to the counterpart rotor mass, bearing stiffness and damping and the mount system natural frequency as well. For the given turbine rotor system design a best available mount system design, composed of a mount plate and resilient support, can be readily selected from the constructed maps to meet any design limits of the rotor's shock response and static mount deflection. The shock response maps of the rotor also have shown that for the same shock the FE flexible rotor model used herein would yield a more compact light-weighed mount system design than the conventional simple rigid rotor model. Therefore, the shock response map approach in conjunction with the more complicated FE flexible rotor transient response analysis method is justified.

5. REFERENCES

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