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NUMERICAL AND EXPERIMENTAL INVESTIGATION ON THE CONTROLLING FOR ROTOR-TO-STATIONARY PART RUBBING IN ROTATING MACHINERY

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ABSTRACT

Rotor may physically contacts with stationary elements of a rotating machine, and the subsequent rubbing at the contact area is a serious malfunction in rotating machinery that may lead to the machine's catastrophic failure. Usually, it is deemed as a secondary phenomenon resulting from a primary cause which perturbs the machine during normal operating conditions. Generally, there are two types of rubs, i.e., radial rub and axial rub. In this paper, the dynamic response of a rotor system with two types of rubs and unbalances is investigated numerically. Then, characteristics of dynamic behavior for both types of rubs could be achieved. It indicates that symptoms of axial rub are similar with that of unbalance, where 1X vibration is the main component in FFT results. While, radial rub will result 0.5X and 1X vibration in FFT result. Combing a troubleshooting process of a steam turbine in an ammonia plant and field test data, the numerical results are confirmed furthermore although there are some differences in vibration characteristics between numerical results and field test results. Under axial rub impact, the fault force emerges even at low speed. Its spectrum characteristics are more like those of radial rub impact at low speed and more like those of unbalance at high speed. On these bases, methods of preventing rub-impact faults as the machine operating are presented and investigated theoretically focusing on how to exert external forces to counteract those forces resulting from rubbing. Experimental investigations are conducted and their results indicate that the method presented in this paper is useful and feasible.

INTRODUCTION

Rub is an undesired contact between a rotating and stationary part. It would cause direct damage to the contacting parts. The damage may be mild under light rub or complete destruction of the machine under severe rub [1].

Rub is always a secondary effect that is caused by some other malfunctions that produce a combination of average and dynamic shaft centerline position that uses up the available clearance between the rotor and the stator. Extreme average shaft centerline position can be caused by excessive radial loads, looseness, and external or internal parts. Extreme dynamic shaft position can be caused by high vibration, due to excessive unbalance, rotor bow, or instability. Even when vibration is not extreme, it produces dynamic motion about the average shaft centerline position. Rub will occur whenever the instantaneous position of the rotor uses up the available clearance [2].

Rotor contact can occur in the radial direction or in the axial direction, or in combination. Axial rub can result from a mismatch in the thermal growth rates between the rotors expand faster than the more massive casing. For this reason, large steam turbines are usually heat soaked several times during the startup process to allow the casing catch up to the thermal growth of the rotor. A related problem can occur when a sideway on the machine casing hangs up and prevents free movement of the casing during startup or shutdown. The constrained machine casing can deform, displace internal parts, and cause an internal rub. Surge in compressors or in the compressor stage of gas turbines can cause highly damaging axial rubs on blades and impellers, and thrust bearing failure can produce catastrophic axial rub [1, 3].

The nonlinear dynamics and stability of rotating systems have been the subject of many studies over the past decades. The models presented in these studies may be classified into two groups, in the first of which, the rotor system is modeled as a continuous system and the second consists in lumped mass (Jeffcott rotor) models to study the nonlinear dynamics of rotating systems. Sakata et al. were among the first to study the vibration of a flexible blade-disk-shaft system using the Finite Element Method [4]. A comprehensive studies of stability and bifurcation analysis of the rotating shaft area presented in [5-7]. Sinha [8] studies the transient response of a beam with its tip deforming due to the rub in the rotating beam. Several researchers' [9-11] studies emphasize the stability analysis and the effect of bearing characteristics on nonlinear dynamics of the rotating shaft using the continuous model. Muszynska and Goldman [12] present analytical and experimental simulations of loose pedestal dynamic effects on a rotating machine vibration response, and they also found abundant non-periodic dynamic responses. Khanlo [13] investigates the effects of centrifugal and Coriolis forces on the incidence of the rub-impact. All these results can be useful in identifying the undesirable behaviors in machine.

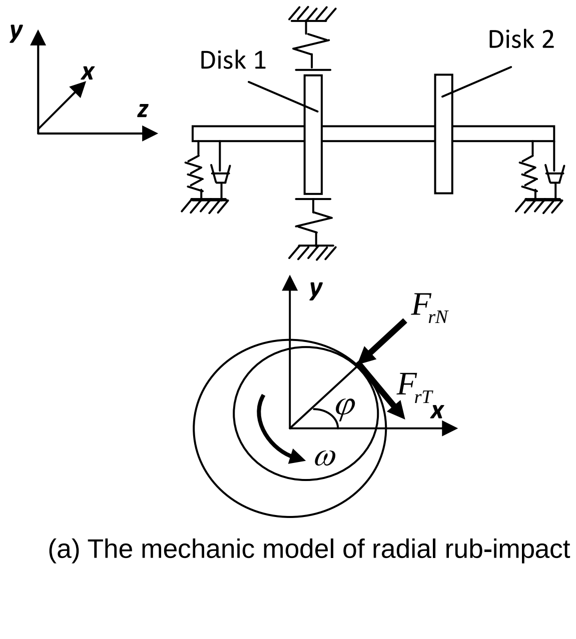
Although there are so many focuses on dynamic behaviors of rotor bearing system under rub impact, there is little attention paid on axial rub impact which is deemed as a more pertinacity fault than radial rub [3, 14]. Furthermore, there are few solutions for operating rotating machine in field when rub impact is emerged. In many conditions, only we can do is permitting this fault for a long time until the rotating or stationary parts are worn. So the clearance between them is so large that they can't contact continuously and rub impact is eliminated. It is well know that this will reduce the efficiency and reliability of machines. However, there aren't better methods in field practices. Furthermore, if the trouble is axial rub impact, it will maintain a longer time because contact stress is much lower than that at radial rub.

In this paper, a continuous flexible shaft and rigid disk model are considered to study the combining axial-radial rub-impact between the rotor and the stator firstly. Shaft-center orbit, poincaré section map, time wave plot and FFT results at the left disk are used to analyze the vibration behaviors of the system. Then an industry case study is introduced to illustrate the symptoms of axial rub impact, also rotordynamic analysis about the steam turbine is presented. Finally, experimental study is conducted to find a feasible way to prevent the rub impact problems as the machine operating.

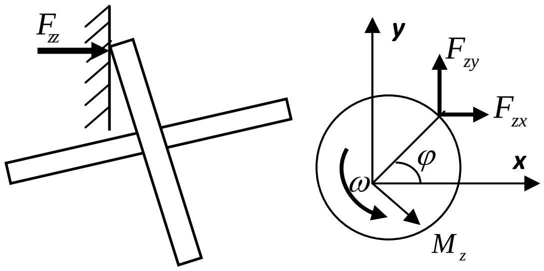
THE ANALYSIS MODEL OF ROTOR SYSTEM WITH RUB IMPACT

The Mechanic Model of Rotor System

Consider a double-disk isotropic rotor-bearing system with rub-impact taking place only on the left disk as shown in Fig.1. The two disks have the same eccentric magnitude 0.1mm, diameter D=300mm and thickness 50 mm. The length and diameter of the shaft are 1000mm and 50mm respectively. Both radial and axial rub-impact are included in this model. Their mechanic models are indicated in Fig.1. Here, F_{rx}, F_{ry}, F_{zx}, F_{zy} and M_z are the forces and moment caused by rub impact, respectively. The rotational speed is denoted by w. The action of the supporting bearing to rotor is simplified by means of linearized stiffness and damping coefficients as functions of rotating speed, which obtained from numerical simulation by DyRoBeS BePerf®.



(a) The mechanic model of radial rub-impact



(b) Sketch of axial rub-impact

Figure 1. The mechanic models of rub-impact of rotor system.

The Calculation of Radial Rub-Impact Forces

Without considering the thermal effect due to friction and based on Coulomb friction law, the radial rub-impact forces could be calculated as follows

$$\begin{Bmatrix} F_{rx} \\ F_{ry} \end{Bmatrix} = -\frac{k_r(e-\delta_r)}{e}\begin{bmatrix} 1 & -f \\ f & 1 \end{bmatrix}\begin{Bmatrix} x_{o1} \\ y_{o1} \end{Bmatrix} \quad (e \geq \delta_r) \qquad (1)$$

$$F_{rx} = F_{ry} = 0 \qquad (e < \delta_r)$$

$$M_{rz} = k_r fD(e-\delta_r)/2 \qquad (2)$$

where $e = \sqrt{x_{o1}^{\ 2} + y_{o1}^{\ 2}}$, x_{o1} and y_{o1} are the displacement components of the geometric center of the disk 1 in x- and y- directions, and f is the friction coefficient between rotor and stator. k_r is the radial stiffness of stator and δ_r is the radial clearance between rotor and stator. F_{rx} and F_{ry} denote the components of the radial rub-impact force in x- and y- directions. M_{rz} is the rub-impact moment around axis *z*-.

The Calculation of Axial Rub-Impact Forces

Assuming that the disk1 has a pitching angle θ, and θ_x and θ_y denote the rotation angles of the cross-section around axes x and y, respectively. According to the theoretical mechanics, the velocity components at rub position in x- and y- directions, v_x and v_y, are as follows

$$\begin{cases} v_x = \dot{x}_{o1} + D(\omega+\theta_z)\theta_x/2\theta \\ v_y = \dot{y}_{o1} + D(\omega+\theta_z)\theta_y/2\theta \end{cases} \qquad (3)$$

where θ_z is the rotation angle of the cross-section around axis z. Thus the forces and moments caused by axial rub-impact are stated as

$$\begin{cases} F_{zx} = fk_z\delta_\theta sign(v_x)\left|v_x/\sqrt{v_x^{\ 2}+v_y^{\ 2}}\right| \\ F_{zy} = fk_z\delta_\theta sign(v_y)\left|v_y/\sqrt{v_x^{\ 2}+v_y^{\ 2}}\right| \\ F_{zz} = k_z\delta_\theta \\ M_{zx} = -k_z\delta_\theta D\theta_x/2\theta \\ M_{zy} = -k_z\delta_\theta D\theta_y/2\theta \\ M_{zz} = F_{zx}D\theta_x/2\theta + F_{zy}D\theta_y/2\theta \end{cases} \qquad (4)$$

where $\delta_\theta = (\delta_z + z_{o1}) - D\theta/2$, z_{o1} is the displacement component of the geometric center of the disk 1 in z- direction and k_z is the axial stiffness of stator. In addition to the above rub-impact forces, the other external forces acting on the rotor are unavoidable unbalanced forces and gravity.

The FEM Model of Rotor System with Rub Impact

By use of a beam element taking into account the shear effect and gyroscopic effect, the finite element model of a double-disk rotor-bearing system with rub-impact has been made, as shown in Fig.2. The model is composed of 22 elements and 23 nodes. Every node has 6 degrees of freedom-three translational and three rotational: x, y and z are the translation freedoms in x-, y- and z- directions and θ_x; θ_y and θ_z denote the rotations of the cross-section around axes x-, y- and z-, respectively. The corresponding non-linear equations of this system are integrated in the time domain using the Newmark's implicit method.

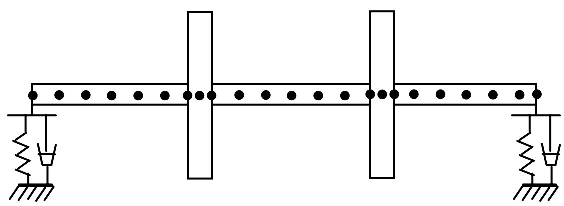


Figure 2. The finite element model of rotor system.

Numerical Calculation and Discussion

The relative clearances between rotor and stator are $\delta_r = 0.2$mm for radial rub-impact and $\delta_z = 1\mu m$ for axial rub-impact. $k_z = k_r = 45$e6 N/m and $f = 0.1$ (acquired by experience). For given parameters, the numerical solutions at different rotation speeds could be obtained when initial eccentric phase difference between two disks is zero ($\Delta\phi = 0$). Figure 3 is a bifurcation diagram of *x*-displacement of the left disk, which is characterized by various bifurcations. It can be seen that the response of the rotor system with rub-impact shows abundant nonlinear phenomena. The response of the rotor system may be a synchronous periodic motion or a pseudoperiodic motion, even a chaos motion, which depends on the parameters of the system. For $w = 600$ rad/s, Fig.4 shows that the shaft-center trajectory at the left disk, poincaré section map, time wave plot and amplitude spectrum, respectively. The corresponding numerical solutions at $w = 800$ rad/s are shown in Fig.5. When $w = 600$ rad/s, the response of the rotor system is synchronous periodic motion, where pseudoperiodic motion with fractional component when $w = 800$ rad/s. In Fig.4, the amplitude of translation vibration is 0.13mm, so radial rub-impact has not been taken place. However the maximum amplitude reaches to 0.21mm and both radial and axial rub-impact have been taken place in Fig.5. Also, it is indicates that vibration characteristics are similar with unbalance fault's if only axial rub takes place in Fig.4. However, 0.5X vibration is prominent when radial rub is also taken place. So diagnosing the axial rub correctly is difficult. From Fig.4 (d) and Fig.5 (d), it can be seen that only low frequency order is prominent and has the necessary to study.

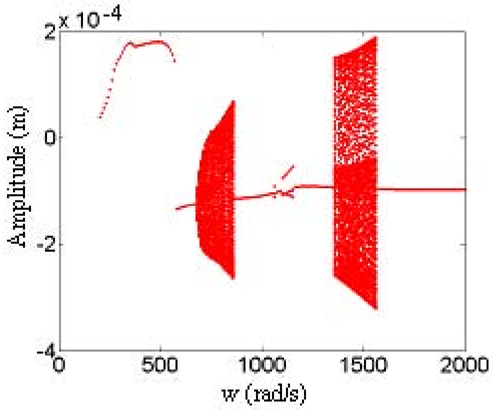
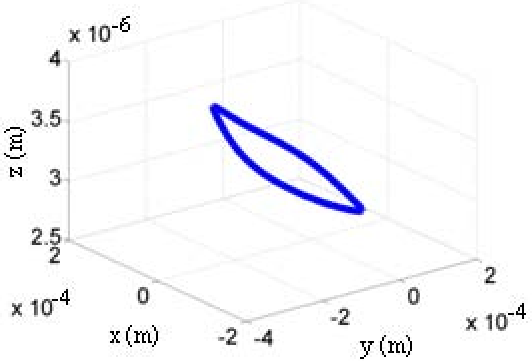
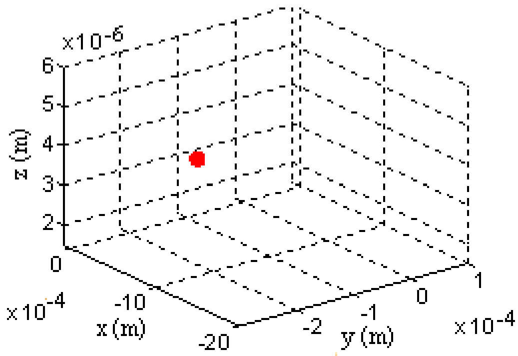


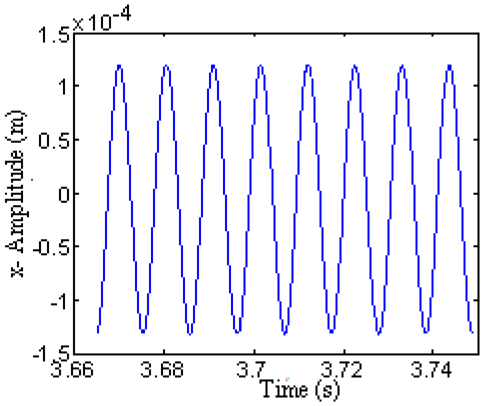
Figure 3. Bifurcation diagrams of x-displacement.



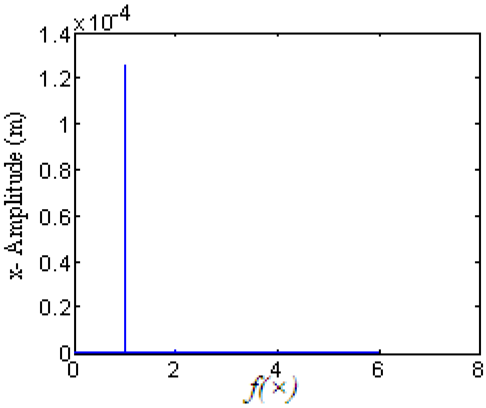
(a) Shaft-center trajectory



(b) Poincaré section map

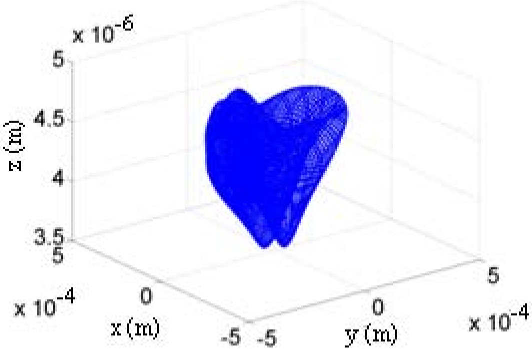


(c) Time wave plot

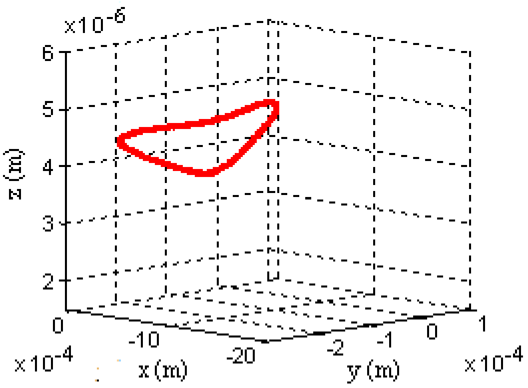


(d) Amplitude spectrum

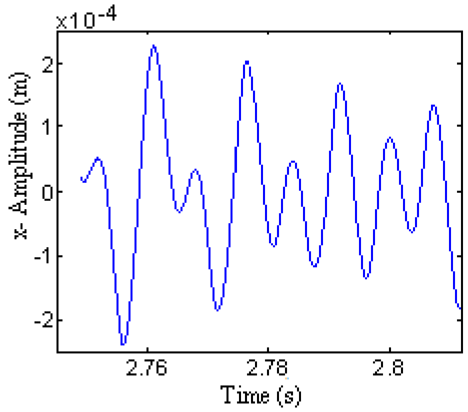
Figure 4. The dynamic behavior of rotor system (w=600 rad/s).



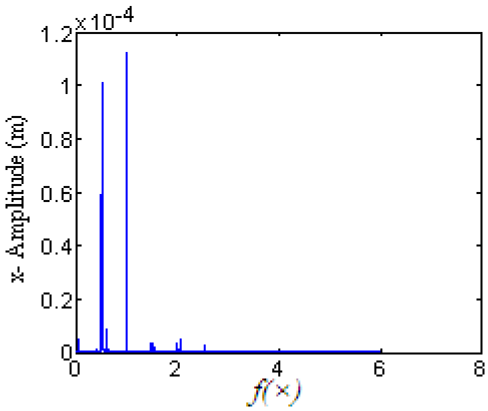
(a)Shaft-center trajectory



(b) Poincaré section map



(c) Time wave plot



(d) Amplitude spectrum

Figure 5. The dynamic behavior of rotor system (w=800 rad/s).

It could be seen that the rub-impact fault has significant influence on the dynamic behavior of the rotor-bearing system, and all of the different response characteristics could be helpful to diagnose the rub-impact fault.

CASE STUDY

Overview of Forces Acting Upon Rotor

There are many types of forces that act on a rotor-bearing system. These forces can be classified into three categories: (1) Casing and foundation forces, (2) forces generated by rotor motion, and (3) forces applied to a rotor.

Forces transmitted to casing and foundations can be due to foundation instability, other nearby unbalanced machinery, piping strains, rotation in gravitational or magnetic fields, or excitation of casing or foundation natural frequencies. Operation on reciprocating machinery in the same area can cause foundation forces and unduly excite the rotor of a turbomachine.

Forces generated by rotor motion can be classified into two categories: (1) forces due to mechanical and material properties, (2) forces caused by various loadings of the system. These forces from mechanical and material properties are unbalanced and are caused by a lack of homogeneity in materials, rotor bow, and elastic hysteresis of the rotor. Those forces caused by loading of the system are various. Hydrodynamic and blade loading forces in the rotor-bearing system may vary in the operational range of the unit.

Forces applied to a rotor can be due to drive torques, couplings gears, misalignment, and axial forces from piston and thrust unbalance. They can be destructive, and they often result in the total destruction of a machine.

For most of these forces, their symptoms are unique. The task of fault diagnosis and prognosis is to confirm the types of forces and the conditions generating these forces. However, the most difficult work is eliminating these forces so as to depress vibration. Especially, the development of large scale industry needs techniques to solve the problems without interrupts manufacturing process. It is the aim as well as the dream of every engineer dedicated in this realm.

Description and analysis of the steam turbine

Many process centrifugal compressors are driven by industry steam turbine. Figure 6 is the layout of one steam turbine driving centrifugal compressor systems used in a large scale ammonia synthesis plant. Both high pressure and low pressure casing are tested in shop and their respective vibrations are qualified. After these two machines are installed in field and standup, both of the rotors' vibrations exceed the limit. Figure 7 shows the sketch of high pressure turbine rotor and Fig.8 shows the low pressure turbine rotor sketch.

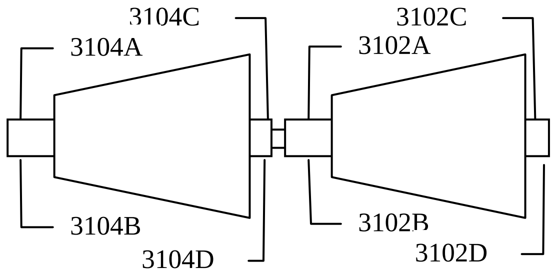


Figure 6. Layout of centrifugal compressor systems.

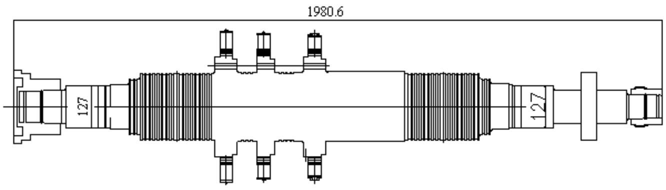


Figure 7. Sketch of rotor in high pressure casing.

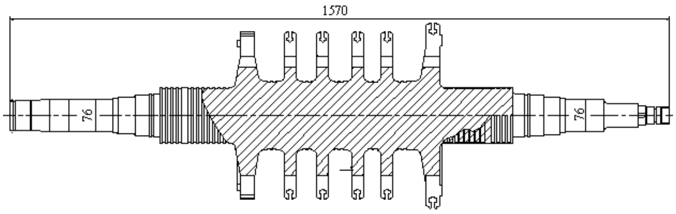


Figure 8. Sketch of rotor in low pressure casing.

By field test and analysis using OROS38 portable instrument, the results are as follows. Figure 9 shows the spectrum analysis result at the largest vibration test point (3012C). Its total vibration amplitude is 24.9 μm. Its 1X and 2X vibration are 14 μm and 4 μm, respectively. Just from symptom in this figure, we can infer that unbalance and slight misalignment may contribute to the vibration. Figure10 and Fig.11 are center orbits of rotor at left and right vibration testing plane. From these two figures, it can be concluded that the vibration isn't caused by instability. From the changing of vibration with time, it can be seen that both amplitude and phase are independent with time.

Unbalance mass can exert a rotating force resulting 1X vibration with constant amplitude and phase. In this condition, it's seemed that the vibration caused by this type of fault force can be counteracted by field balancing so as to depress vibration.

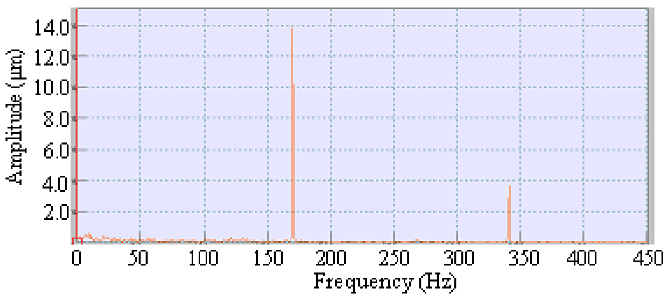


Figure 9. Result of spectrum analysis at 3012c.

Field Balancing

Field balancing is a convenient means to resolve the vibration problems caused by unbalancing force. Traditional affect coefficients based methods need run up machine three times at least. While, run up a machine is very expensive and time consuming. So many methods are presented to reduce the number of run up. In all of these methods, no trial weight method is most prominent. The author has used this method to resolve vibration problems [15]. Its basic process is calculating the affect coefficients by numerical method but by field test. Its effectiveness depends on the accuracy of simulation. About this, the FEA method provides the possibility. In this method, modeling of rotor-bearing system is set up firstly and rectified through run up or run down test result. The dynamic characteristics of the rotor-bearing system are simulated by DyRoBeS®.

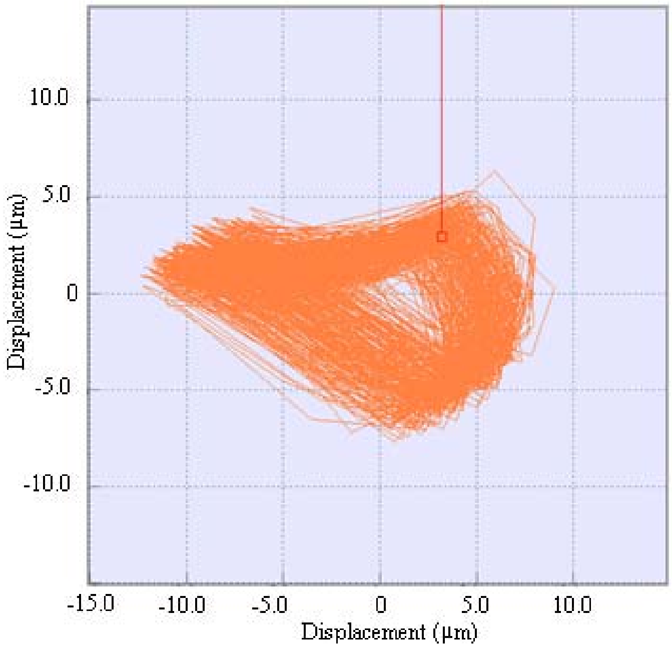


Figure 10. Center orbit of rotor at left test plane (3012A-3012B).

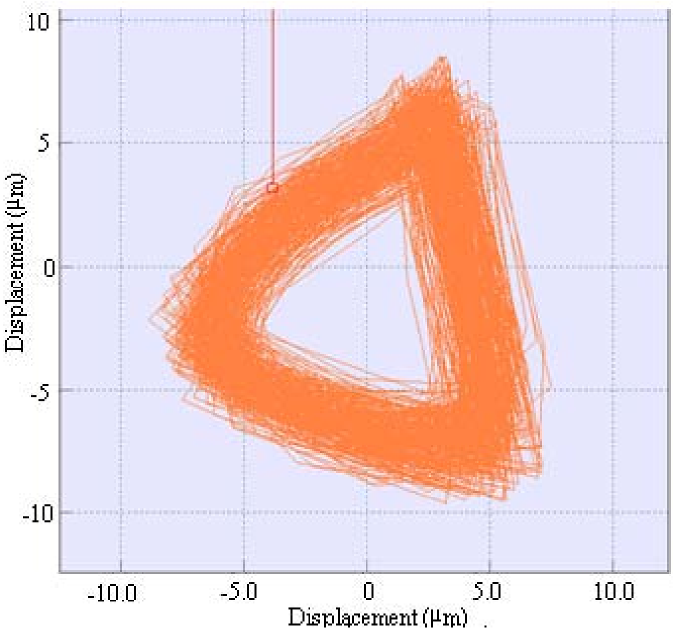


Figure 11. Center orbit of rotor at right test plane (3012C-3012D).

High pressure steam turbine rotor model is shown in Fig. 12. In this model, journal diameter is 127mm. Radial clearance of bearing is 0.12mm and there is 2425N radial load exerted on it. Inlet oil temperature is 40℃ and pads offset is 50%. Its pressure profile at working speed (10400r/min) is shown in Fig.13. Its maximum pressure is 1078.35kPa. Figure 14 indicates the rotor's critical speed map under different bearing stiffness. Here, k_{xx} and k_{yy} are the direct stiffness coefficients of bearing along *x*- and *y*- direction, respectively. The corresponding results for low pressure steam turbine are shown in Fig.15, Fig.16 and Fig.17. For low pressure steam turbine, journal diameter is 76 mm. Radial clearance of bearing is 0.08mm and there is 1600N radial load exerted on it. Inlet oil temperature is 40℃ and pads offset is 50%. Its pressure profile at working speed (10400r/min) is shown in Fig.16. Its maximum pressure is 2699.69kPa.

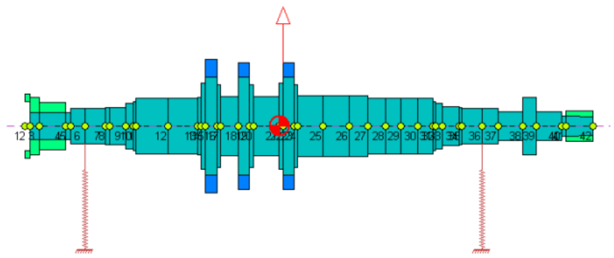


Figure 12. FEA model of high pressure turbine rotor.

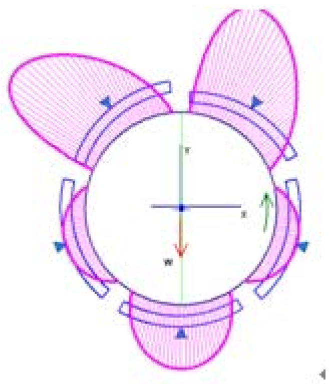


Figure 13. Pressure profile of one journal bearing for high pressure turbine rotor at operating speed.

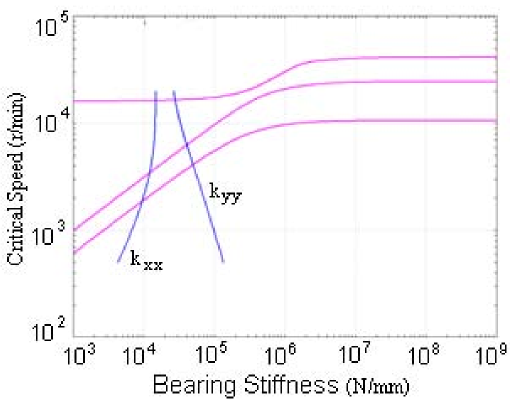


Figure 14. Critical speed map of high pressure turbine rotor.

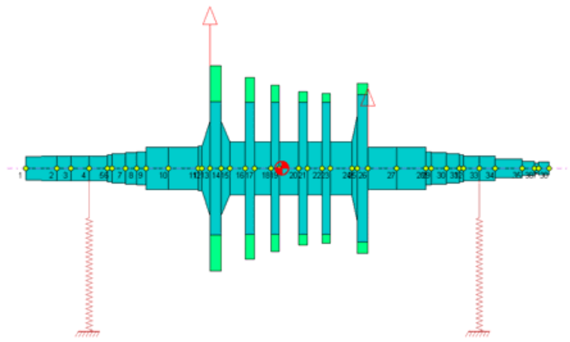


Figure 15. FEA model of low pressure turbine rotor.

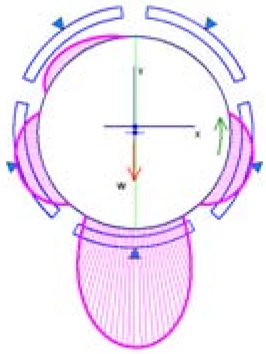


Figure 16. Pressure profile of one journal bearing for low pressure turbine rotor at operating speed.

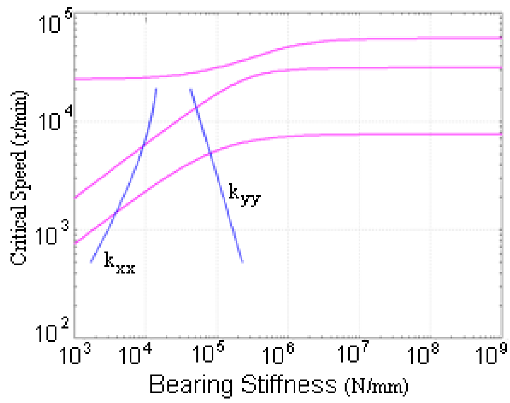
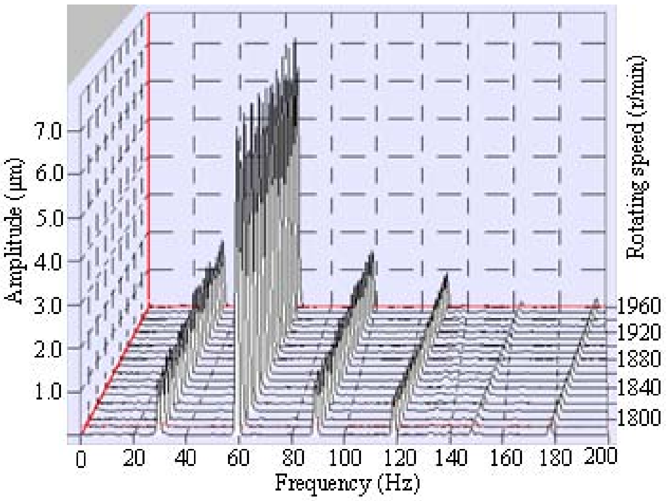


Figure 17. Critical speed map of low pressure turbine rotor.

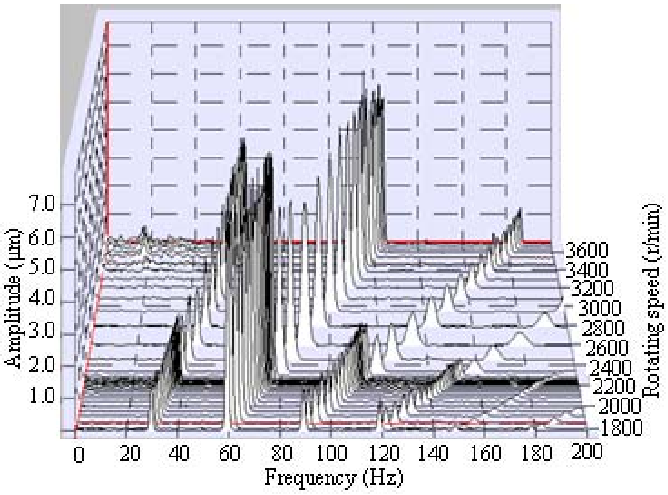
According to no trial weight method [15] and rotor dynamic analysis presented, 17g weight at 10.2° is need to depress vibration at right end of the high pressure steam rotor. While, after two run up and run down tests, it is failed. Maybe other causes but unbalance contributes to this type of serious vibration.

Vibration Analysis and Fault Diagnosis

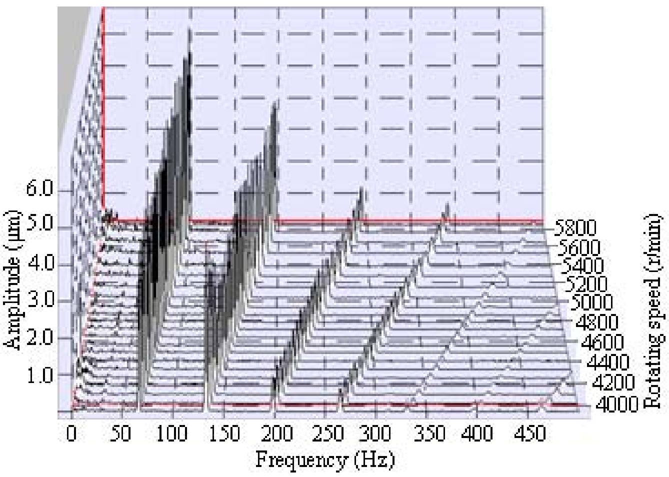
Spectrum analysis is a useful way to determine the cause of fault force. By waterfall, the change of every vibration component with rotating speed can be viewed. Figure 18 shows the vibration at 3102C for different speed range. Figure 18(a) shows the spectrum of vibration between 1000 r/min to 2000 r/min. In this range the components of vibration are abundant and 2X vibration is several times of 1X vibration. 3X and 4X vibration are also apparent. From the vibration characteristics in this range, it's seemed that the cause of fault is radial rub impact. From Fig.18 (b) and Fig.18(c), it can be seen that 1X vibration increases and 2X vibration decreases. From Fig.18 (d), 1X vibration increase continuously and 2X becomes constant. This type of symptom is more like that of unbalance plus slight misalignment.



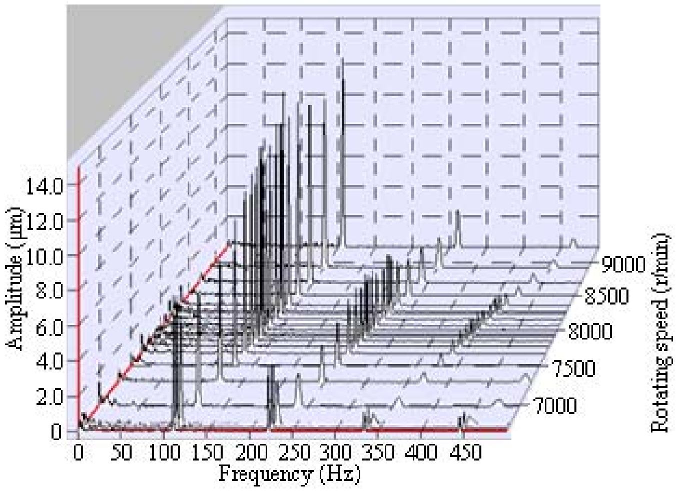
(a) 1000~2000r/min



(b) 2000~4000 r/min



(c) 4000~6000 r/min



(d) 6000~1050 r/min

Figure 18. Waterfall plot for vibration at 3102C.

Figure 19 shows the labyrinth seal after disassembled. It can be seen that the high tooth is worn down while the lower tooth is well relatively. It is the typical result of axial rub impact. From the characteristic acquired in the paper and many other industry case studies, it can be concluded that under axial rub impact, the fault force will emerge even at low speed. In this stage, the unbalance force is small and the symptom of spectrum analysis is more like that of radial rub impact. With the speed increasing and unbalance becoming large, the effect of impact also enlarges the unbalance response of rotor system. In this stage, the symptoms of spectrum analysis are more like that of unbalances. Although its amplitude and phase are constant, eliminating it by field balancing is difficult because affect coefficient can't correctly acquired.

Figure 19. Labyrinth seal after overhaul.

For radial rub impact, it is mainly caused by large radial vibration and usually seemed as secondary fault. So the best way to prevent it or eliminate it is to control the rotors vibration as it is working. In this paper, electromagnetic bearing is used as an actuator to control vibration. Three control strategies are presented and investigated experimentally to find which of them can decrease rotor's vibration in the shortest time. They are "searching in whole circle" strategy, "fast optimizing control" strategy and "None mistaking control" strategy respectively. Details of these strategies are already published in reference [16].

VIBRATION CONTROL INVESTIGATION

Test rig Description

The test rig used to verify the self-recovery control strategy is shown in Fig.20. One meter long rotor, which has a diameter of 50mm, is derived by a 20KW motor. The rotor is supported by two five pads tilting pad bearings. The span between two bearing pedestals is 490mm. The disk fixed on rotor has a diameter of 270mm. There are sixteen evenly distributed screwed holes used to add unbalance on the disk. A keyway is milled on coupling connected to motor. AME, which can provide multi-harmonic force, is taken as the actuator of vibration fault self-recovery system. Four Bentley 3300 proximity probes are used to measure the vibration near bearing. Data acquisition and analysis instrument OROS38 is used to monitor and evaluate the vibration of rotor as well as the efficiency of controlling.

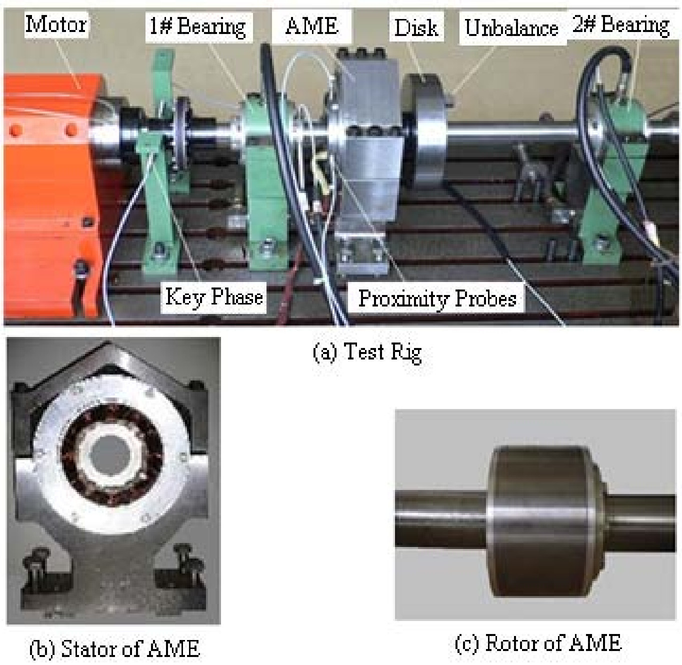
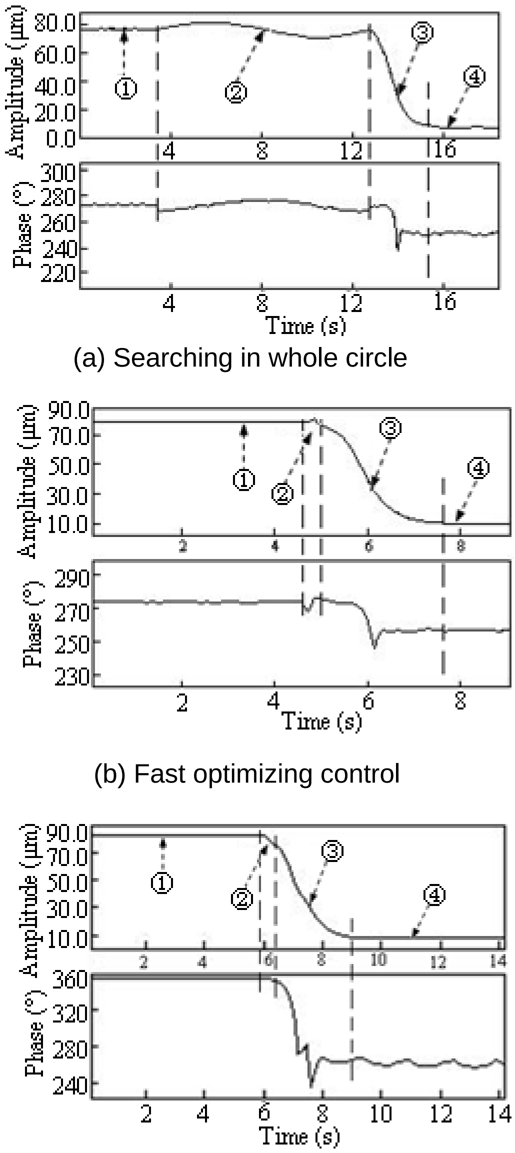


Figure 20. Test Rig.

Experimental Result and Analysis

To verify the effectiveness of three synchronous vibration control strategies, experimental investigation has been conducted at several rotating speed 2100r/min, 2700r/min, and 3000r/min, 3300r/min. At every speed, the results indicate that all of the three strategies can minimize the vibration though at different efficiency. Here take results at 3000 r/min as example as Fig.21 shows. In it, the whole controlling stage can be divided into four stages. The first stage ① is original vibration monitoring. The second stage ② is phase optimizing. The third stage ③ is amplitude optimizing and the fourth stage ④ is final state. The "searching in whole circle" strategy needs 20s to decrease vibration amplitude from 78 μm to 8 μm. While the "fast optimizing control" strategy needs only 4s and the "None mistaking control" strategy needs 2.5s only to realize the same objective. Even more, in the whole process of the third control strategy, the vibration is not exceed its originate value.



(a) Searching in whole circle

(b) Fast optimizing control

(c) None mistaking control

Figure 21. Effectiveness of vibration control.

CONCLUSION

In this paper, the dynamic response of a double-disk rotor system with two types of rubs and unbalances is investigated numerically. Vibration characteristics for both rub-impacting faults are achieved. It is indicated that vibration characteristics are similar with that of unbalance if only axial rub-impacting has been taken place. In an engineering case study, vibration fault of a steam turbine is deemed as unbalance by mistake. Field balancing work is carried out although there are little effects. Combining FFT result at different speed range and overhaul result, the causes of vibration are discovered. Also theoretical results are confirmed. On these bases, method of preventing radial rub-impacting fault as the machine operating is presented and investigated experimentally focusing on how to exert exciting force to counteract those forces resulting rubbing. Experimental results indicate that the method presented in this paper is useful and feasible.

In future work, thermal effect caused by rub impact will be included to finely simulate the dynamic behaviors of rotor-bearing system to fulfill the gap between numerical result and field test result. Also, new control strategy should be developed to solve the problem after rub taking place. Furthermore, techniques localizing the rub impact are needed to give an accurate solution.

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