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STUDY ON VIBRATION CHARACTERISTICS OF SINGLE CRYSTAL BLADE AND DIRECTIONALLY SOLIDIFIED BLADE

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ABSTRACT

In modern gas turbines, SC (Single Crystal) and DS (Directionally Solidified) nickel alloys are applied which, compared to CC (Conventionally Casting) alloys, hold a higher cyclic life and a significantly improved creep rupture strength. Because SC and DS alloys feature a significant directionally dependence of material properties, the vibration analysis of the SC and DS blade has to be carried out, taking account of the anisotropy of material properties. In the vibration analysis by FEA (Finite Element Analysis), the DS blade has to be modeled approximately as transverse orthotropic material, while the SC blade can be modeled exactly as orthotropic material in lattice directions. In order to design the SC and DS blade with high reliability, it is necessary to establish the analysis model and to clarify the influence of the anisotropy of the material properties on the vibration characteristics of the blade.

In this paper, first, the effect of the anisotropy of elastic constants on the vibration characteristics of the SC and DS blade is investigated. Second, the validity of the assumption of the transverse isotropy for the DS blade, which is applied in the current blade design, is examined by Monte Carlo simulation. Finally, the frequency deviation of the SC and DS blade is analyzed by the sensitivity analysis method, and is compared to that of the CC blade.

1. INTRODUCTION

Recently, DS (Directionally Solidified) and SC (Single Crystal) alloys have been widely applied for gas turbine blades instead of CC (Conventionally Casting) alloys to meet the requirement of the high temperature operation and to improve the thermal efficiency. Jet engine is well advanced to industrial engine in this field, and SC alloy has already been in service. Following this trend, SC and DS blades have been introduced to large industrial gas turbine [1][2]. The SC blade consists of one columnar grain, and therefore the creep rupture and thermal fatigue resistance can be improved by eliminating the grain boundary. Since the elastic constants of the SC blades are anisotropic, the vibration analysis of the SC blade has to be carried out, taking account of the anisotropy. On the other hand, the DS blade consists of several columnar grains where the growing direction of the columnar crystal is set to the direction of the centrifugal force to yield a large creep resistance by eliminating the grain boundary perpendicular to the loading direction. The study on the fatigue life of the DS blade with anisotropic material property has been carried out extensively [3][4][5]. The vibration analysis of the DS blade also has to be carried out, taking account of the anisotropy because the direction of the columnar crystal growth is parallel to the loading direction [6].

In the vibration analysis of the DS blade by FEA (Finite Element Analysis), it seems that all of the columnar grains included in the blade have to be modeled exactly [7]. In practice, however, it is difficult to know the exact growing direction and the geometry of the columnar grain. It is reported that if the volume and the rotation angle in the transverse plane of each columnar crystal are known, the vibration analysis of the DS blade can be carried out with the practical accuracy by use of the elastic constants averaged by the Reuss and Voigt method [8]. However, it is also difficult to get such data of the actual DS blade in advance. Therefore, in the actual mechanical design of the DS blade, the vibration analysis is carried out, assuming that the number of the columnar grains included in the blade is large, and the DS blade can be considered transverse isotropic [1].

In this study, taking account of the above-mentioned situation, first, the effect of the anisotropy of elastic constants on the vibration characteristics of the SC and DS blade is investigated. Second, the validity of the assumption of the transverse isotropy for the DS blade is examined, carrying out the Monte Carlo simulation. That is, in the actual DS blade, the number of the columnar grains included in the blade is 5 to 10 at most, and is not enough. Therefore, whether the assumption of the transverse isotropy can be applied to the vibration analysis of the actual DS blade is examined. In addition, the frequency deviation of the SC and DS blade caused by the deviation of the material property seems to become larger than that of the CC blade, because of the number of the independent elastic constants more than the CC blade, the deviation of the elastic constants due to the deviation of the crystal growing direction, and so on. And therefore, the frequency deviation of the SC and DS blade is analyzed by the sensitivity analysis method, and is compared to that of the CC blade.

2. ANALYSIS METHOD

2.1 Elastic stress-strain relationship of SC and DS blade

This paper employs two coordinate systems as shown in Fig.1 and Fig. 2. One is the structural coordinate expressed by the small letter x_1, x_2, and x_3, which defines the geometry of the blade. The other system is the alloy coordinate system, expressed by the capital letter X_1, X_2, and X_3 and fixed to each columnar grain. The alloy coordinate system are directed to the crystal axes of [100], [010], and [001]. The angle θ defines the rotation angle around [010] axis, while the angle ϕ defines the rotation angle around [001] axis. The angle γ defines the rotation angle around [100] axis, which is not written in Fig. 2 for lack of space. In the mechanical design of the SC and DS blade, the vibratory stress to be considered is sufficiently smaller than the yield strength of the material. Therefore, the vibration analysis of the SC and DS blade can be carried out in the elastic region as well as the CC blade.

Nickel base alloys are used as the material of the SC and DS blade of gas turbine. They are the face centered cubic and the elastic stress strain relationship in the single crystal alloy can be expressed by Eq.(1), taking the coordinate along the crystal axis as shown in Fig. 1 [1].

$$\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \gamma_{12} \\ \gamma_{23} \\ \gamma_{31} \end{Bmatrix} = \begin{bmatrix} S_{33} & S_{13} & S_{13} & 0 & 0 & 0 \\ S_{13} & S_{33} & S_{13} & 0 & 0 & 0 \\ S_{13} & S_{13} & S_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{44} \end{bmatrix} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \tau_{12} \\ \tau_{23} \\ \tau_{31} \end{Bmatrix} \quad (1)$$

Where, S_{ij} is the element of the compliance matrix [S] in the X_1 X_2 X_3 coordinate, and the independent constants are three of S_{33}, S_{13} and S_{44}, which can be expressed by Eq.(2), using the engineering elastic constants.

$$S_{33} = 1/E_0, \quad S_{13} = -\nu_0/E_0, \quad S_{44} = 1/G_0 \quad (2)$$

In Eq.(2), the subscript '0' denotes the quantity corresponding to the X_3 axis as shown in Fig. 1. If the material is isotropic, Eq.(3) is satisfied. Therefore, the independent material constants are two of E_0 and G_0.

$$G_0 = \frac{E_0}{2(1+\nu_0)} \quad (3)$$

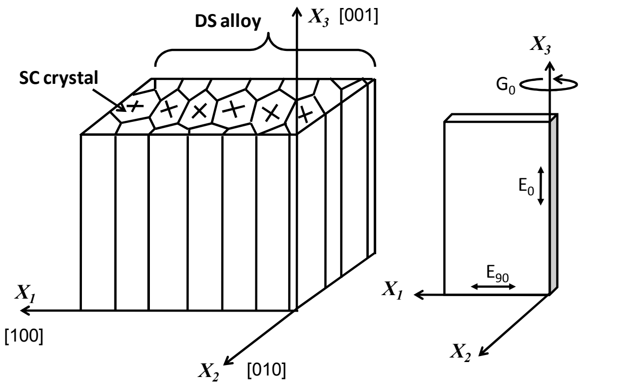


Fig. 1. Coordinate system and material property of SC and DS alloy

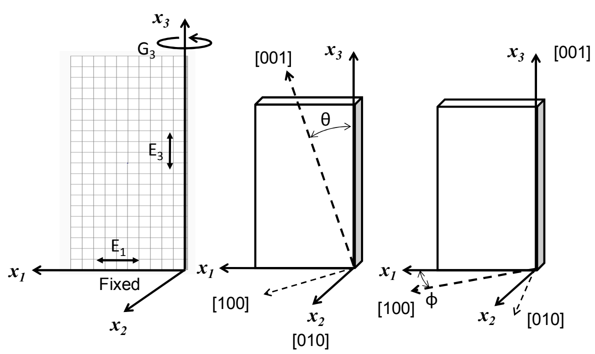


Fig. 2. Coordinate system of blade and FE model

The DS alloy is a bundle of single crystals having the same X_3 direction (the solidification direction) as shown in Fig. 1. In other words, although the solidification directions of all single crystals in the DS alloy are the same, the crystal direction in the X_1-X_2 plane is random. Therefore, if the number of the single crystals is large, the material property in this plane can be treated isotropically, and the stress strain relationship of the DS alloy can be expressed by Eq.(4).

$$\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \gamma_{12} \\ \gamma_{23} \\ \gamma_{31} \end{Bmatrix} = \begin{bmatrix} \overline{S_{11}} & \overline{S_{12}} & \overline{S_{13}} & 0 & 0 & 0 \\ \overline{S_{12}} & \overline{S_{11}} & \overline{S_{23}} & 0 & 0 & 0 \\ \overline{S_{13}} & \overline{S_{23}} & \overline{S_{33}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \overline{S_{44}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \overline{S_{55}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \overline{S_{66}} \end{bmatrix} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \tau_{12} \\ \tau_{23} \\ \tau_{31} \end{Bmatrix} \quad (4)$$

Where,

$$\overline{S_{44}} = 2\left(\overline{S_{11}} - \overline{S_{12}}\right) \quad (5)$$

Since the material constants in X_3 axis is the same as the single crystal, Eq.(6) is derived. In this case, independent material constants are five.

$$\begin{aligned} \overline{S_{33}} &= S_{33} \\ \overline{S_{55}} &= \overline{S_{66}} = S_{44} \\ \overline{S_{13}} &= \overline{S_{23}} = S_{13} \end{aligned} \quad (6)$$

Material constants of the DS alloy in the X_1-X_2 plane can be obtained, averaging the material constants of the SC alloy in the transverse plane under the assumption that the distribution of the crystal direction in the plane is random. The material constants of the DS alloy in the X_1-X_2 plane can be described by Eq.(7), if the Wells' method [1][9] is adopted in averaging.

$$\begin{aligned} \overline{S_{11}} &= \overline{S_{22}} = \frac{1}{2}\sqrt{S_{33}(2S_{33} + 2S_{13} + S_{44})} \\ \overline{S_{44}} &= \sqrt{2S_{44}(S_{33} - S_{13})} \end{aligned} \quad (7)$$

And Eq.(8) is also derived from Eq.(5).

$$\overline{S_{12}} = \overline{S_{11}} - \frac{\overline{S_{44}}}{2} \quad (8)$$

In short, if the number of the single crystal alloys is large enough to assume that the material is isotopic in the X_1-X_2 plane, the independent material constants become three. Therefore, at least 3 independent tests are necessary to determine the elastic constants of the DS alloy. Usually the tests to determine the three engineering elastic constants of E_0, E_{90}, and G_0 in Fig. 1 are adopted, because the test method and the deviation of the solidification direction have little effect on these engineering elastic constants. The relationship between these elastic constants and the elements of compliance matrix is described by Eq.(9).

$$\overline{S_{33}} = 1/E_0, \quad \overline{S_{11}} = 1/E_{90}, \quad \overline{S_{55}} = 1/G_0 \quad (9)$$

Where, the elastic constants of E_0 and E_{90} correspond to the Young's modulus in the direction of the blade height and blade chord, respectively, while G_0 corresponds to the shear rigidity around the X_3 axis as shown in Fig. 1.

Next, consider the case where the alloy axis makes a difference from the structural axis due to manufacturing error as shown in Fig.2. In this case, the compliance matrix $[S']$ in the x_1 x_2 x_3 coordinate can be obtained by Eq.(10).

$$[S'] = [\Psi]^T [S][\Psi] \qquad (10)$$

Where, $[\Psi]$ is stress transformation matrix from the X_1 X_2 X_3 coordinate to the x_1 x_2 x_3 coordinate, and superscript T indicates the transpose of the matrix. Substituting Eq.(2) and Eq.(4) into Eq.(10), elastic constants of the actual DS blade, in which the direction between the alloy axis and the blade axis is slightly different, can be calculated. In addition, carrying out vibration analysis based on FEA by use of the compliance matrix of Eq.(10), the vibration characteristics of the actual DS blade can be obtained.

2.2 Verification of material constants averaged by Wells' method

It is reported that if the volume and the rotation angle in the transverse plane of the columnar crystal are known, the vibration analysis of the DS blade can be carried out with the practical accuracy by use of the averaging elastic constants by the Reuss and Voigt average [8]. However, it is difficult to get such data of the actual DS blade in advance. Therefore, in the actual mechanical design of the DS blade, the vibration analysis is carried out, assuming that the number of the columnar grains included in the blade is large enough, and the DS blade can be considered transverse isotropic [1]. The Wells' method is one of the typical methods which can average the material constants over the transverse plane, assuming that the number of the columnar grains is infinite. Using Wells' method (Eq.(7) and Eq.(8)), the material constants are treated isotropically in the transverse plane. In this paper, one of the objectives is to examine the validity of the application of Wells' method for the actual DS blade, where the number of the columnar grains included in the blade is 5 to 10 at most.

In order to achieve this objective, first, the material constants obtained from the Wells' method are compared with those obtained by the Reuss method and the Voigt method, where the columnar grains of the finite number are averaged with respect to the volume. The Reuss average corresponds to the stress constant model, while the Voigt average the strain constant model. The compliances matrix by the Reuss and Voigt averages, $[S]_R$ and $[S]_V$, are equated as:

$$[S]_R = \sum_k V_k [\Psi]_k^T [S]_k [\Psi]_k \qquad (11)$$

$$[S]_V = \sum_k V_k \left([\Phi]_k^T [C]_k [\Phi]_k\right)^{-1} \qquad (12)$$

Where, V_k is the volume percent of each columnar grain. $[S]_k$ and $[C]_k$ are the elastic compliance and stiffness matrices of each columnar grain. $[\Psi]_k$ and $[\Phi]_k$ are the transformation matrices of stress and strain from the alloy coordinate to the structural coordinate.

Figure 3 shows the procedure for verifying the validity of the material constants obtained by the Wells' method, based on the Monte Carlo simulation. In this procedure, first, the number of the SC columnar grains included in the DS alloy is given. Next, after the rotation angle of each columnar grain in the transverse plane is selected at random, the DS alloy is assembled. The material constants of the DS alloy is calculated by the Reuss and Voigt methods. This process is repeated many times (one handed times), and the calculated results are treated statistically to estimate the distribution of the elastic constants. The results are compared with the material constants calculated by the Wells' method based on the assumption of the isotropy in the transverse plane.

In the second verification procedure, the DS blade is modeled as a plate including 5 columnar grains as shown in Fig. 4. FE models, where the rotation angle ϕ of each columnar grain is changed in many patterns, are made, and the vibration analysis is carried out directly by use of FE models. This model is called the direct FE model hereafter. In this study, the results calculated by the direct FE model are considered correct, and are used as reference, because the columnar grains included in the DS blade are modeled exactly in the direct FE model. The distribution of the natural frequencies calculated by the direct FE models are compared with those calculated by FEA, which uses the material constant calculated by the Wells' method. Comparing the natural frequencies by both methods, prediction error of the natural frequency of the Wells' method for each vibration modes is clarified.

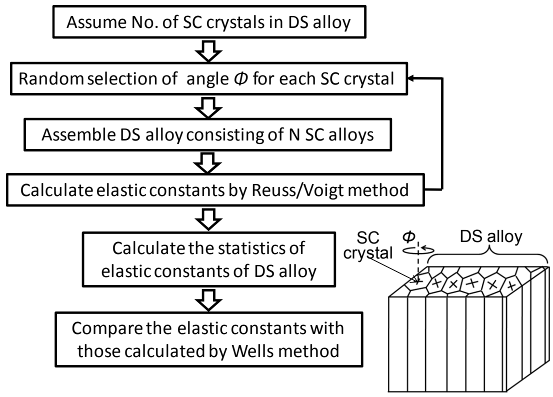


Fig. 3. Procedure of Monte Carlo simulation for elastic constants of DS alloy

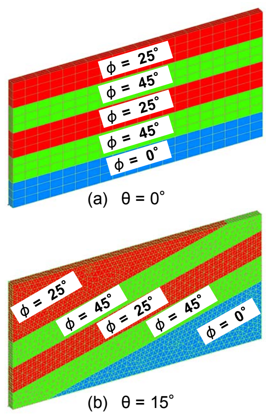


Fig. 4. Example of DS blade model consisting of 5 SC alloys

2.3 Analysis of frequency deviation due to deviation of material constant

In the DS blade used in gas turbine, the growing direction of the crystal is set to the direction of the centrifugal force to yield a large creep resistance by eliminating the grain boundary perpendicular to the loading direction. In the actual DS blade, however, the solidification direction of each blade shows a slight difference from the loading direction in casting process. The allowable deviation of the solidification direction is usually determined by the casting process adopted, the yield of acceptable product, and so on, and the rotation angle θ in Fig. 2 is prescribed to be within ±15 deg. As a result, although the factors of the frequency deviation in the isotropic blade are the deviation of two independent material constants （E_0 and G_0）, the deviation of the solidification direction should be considered as a factor in the SC and DS blade, in addition to three independent material constants.

In this study, the frequency deviation caused by the deviation of the crystal direction and the material constants is evaluated by the first order second moment method [10]. Based on the first order second moment method, when Y is defined by Eq.(13) as a function of probabilistic variables $Z_1, Z_2, ..., Z_n$, the expectation of the probabilistic variable Y, E[Y] and variance, σ_Y^2 are expressed by Eq.(14) and Eq.(15), respectively.

$$Y = g(Z_1, Z_2, \cdots, Z_n) \tag{13}$$

$$E[Y] = g(\overline{Z}_1, \overline{Z}_2, \cdots, \overline{Z}_n) \tag{14}$$

$$\sigma_Y^2 = \sum_{i=1}^{n}\sum_{j=1}^{n}\left(\frac{\partial g}{\partial Z_i}\right)_{\overline{Z}_i}\left(\frac{\partial g}{\partial Z_j}\right)_{\overline{Z}_j}\rho_{Z_iZ_j}\sigma_{Z_i}\sigma_{Z_j} \tag{15}$$

Where, $\overline{Z}_i$ is the expectation of Z_i, $\rho_{Z_iZ_j}$ is the correlation factor between Z_i and Z_j, and σ_{Z_i} and σ_{Z_j} are the standard deviation of Z_i and Z_j. Namely, if the probabilistic variable Y is considered the natural frequency of the blade, its variance, σ_Y^2 can be calculated from the correlation factor and standard deviation of Z_i, which is the independent variable of the probabilistic variable Y, including the material constants and the crystal direction.

3. ANALYSIS RESULT

3.1 Effect of deviation of the crystal direction on the natural frequency

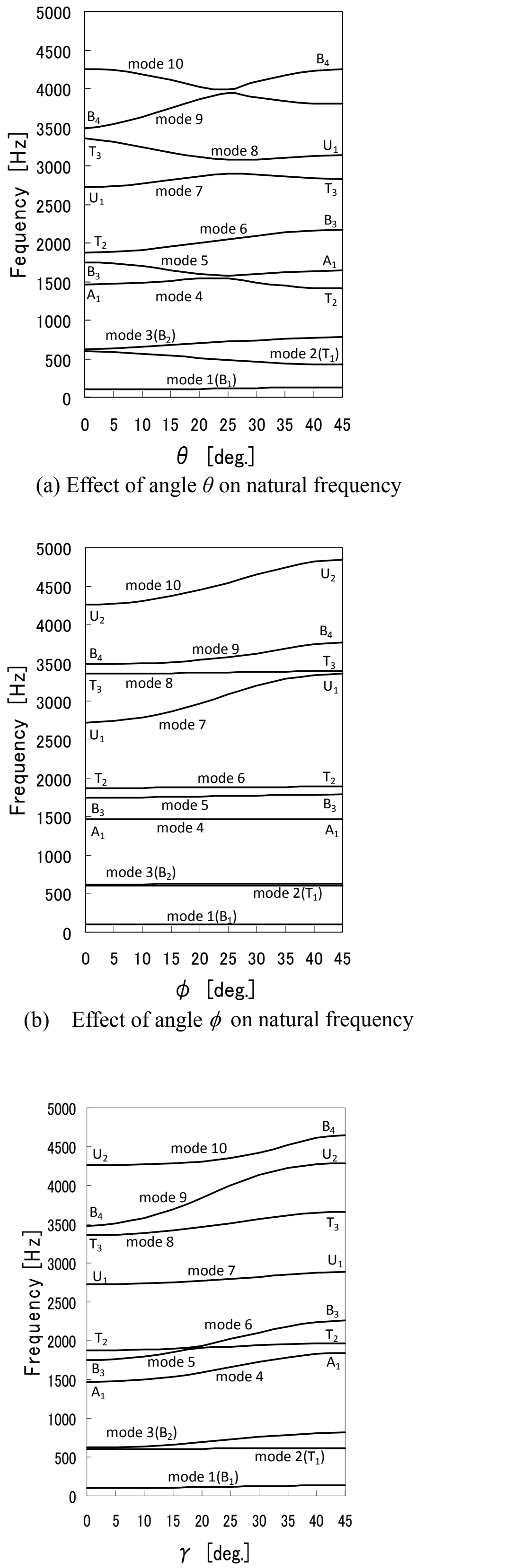
Since the DS alloy is a bundle of the SC alloys, it is important to examine the vibration characteristics of the SC blade for understanding the vibration characteristics of the DS blade. Therefore, the SC blade is modeled as a plate as shown in Fig. 2, and the effect of the crystal direction on the natural frequency was examined in detail. The SC blade has dimensions of 100×200×6 in millimeter. The value of the compliances of the nickel single crystal [7] (s_{11}=0.73×10^{-5}Mpa^{-1}, s_{12}=-0.27×10^{-5}Mpa^{-1}, s_{44}=0.80×10^{-5}Mpa^{-1}) were used in the vibration analysis done by the commercial FEA code NASTRAN.

Figure 5 shows relationship between the natural frequency of the SC blade and the crystal direction. Figure 5(a) shows the frequency change due to the deviation of the crystal direction θ, while Fig. 5(b) and Fig. 5(c) show the frequency change due to the deviations of the crystal directions ϕ and γ, respectively. The angels of θ, ϕ and γ are defined as the rotation angle around [010], [001] and [100] axis, respectively as shown in Fig. 2, and are changed in Fig. 5 from 0° to 45° considering the symmetry of the crystal. In Fig. 5, the B_1, B_2, B_3, and B_4 mode denote the bending modes (out-of-plane modes) along the blade height, while the A_1 mode denotes the in-plane bending mode. On the other hand, T_1, T_2, and T_3 mode are torsion modes, while U_1 and U_2 mode are the bending modes along the blade chord.

Figure 6 shows the vibration modes of the SC blade, in which the solid lines correspond to the nodal lines.

Figure 7 and Fig. 8 shows the relationship between the material elastic constants (E_1, E_3 and G_3) and the crystal directions (θ, ϕ and γ). The material elastic constants of E_1 and E_3 correspond to Young's modulus along the blade chord and the blade height, respectively, while G_3 is the shear rigidity around x_3 axis, as shown in Fig. 2.

As shown in Fig. 5(b), the effects of the angle ϕ on the lower



(a) Effect of angle θ on natural frequency

(b) Effect of angle ϕ on natural frequency

(c) Effect of angle γ on natural frequency

Fig. 5. Natural frequency and crystal angle of SC blade

natural frequencies are very small, and the lower frequencies up to 6th mode are hardly changed due to the change of the angle ϕ. Namely, it is clear that the effect of the angle ϕ is large only for the bending modes along the blade chord like the 7th U_1 mode and the 10th U_2 mode. On the other hand, as shown in Fig. 5(a), the effect of the deviation of the angle θ is observed from the lower vibration modes, and for the bending modes along the blade height (mode B_1, mode B_2, mode A_1, mode B_3), the natural frequencies increase with increase of the deviation of the angle θ. Inversely for the torsion modes (mode T_1 and mode T_2) the natural frequencies decrease with increase of the deviation of the angle θ. It is also shown from Fig. 5(c) that the effect of the angle γ is also observed from the lower vibration modes, and for the bending modes along the blade height (mode B_1, mode B_2, mode A_1, mode B_3) , the natural frequencies increase with increase of the deviation of the angle γ. That is, the effects of the angle θ and γ on the natural frequencies of the bending modes along the blade height are the nearly same. On the natural frequencies of torsion modes (mode T_1 and mode T_2), however, the effects of the angle θ and γ are clearly different. The natural frequencies of the torsion modes decrease with increase of the deviation of the angle θ, while they hardly change with increase of the deviation of the angle γ.

The effects of the crystal direction on the natural frequencies shown in Fig. 5 can be clearly explained, comparing the results in Fig. 5, Fig. 7 and Fig. 8. Namely, as shown in Fig. 7(a), although Young's modulus along the blade height, E_3 hardly changes due to the deviation of the crystal angle ϕ, E_3 increases with increase of the angle θ, and reaches its maximum at θ= 45°. On the other hand, as shown in Fig. 7(b), Young's modulus along the blade chord, E_1 increase with increase of the angle θ and ϕ. And as shown in Fig. 7(c), although the shear rigidity, G_3 around x_3 axis hardly changes with increase of the angle ϕ, G_3 decreases with increase of the angle θ and reaches its minimum at θ=45°. As a result, as shown in Fig. 5(a), with increase of the angle θ, the natural frequencies of the bending modes along the blade height increase, while the natural frequencies of the torsion modes decrease. And the effect of the deviation of the angle ϕ appears only for the bending modes along the blade chord (U_1 and U_2 modes), as shown in Fig. 5(b).

Similarly, the frequency change due to the deviation of the angle γ can be explained, comparing the results in Fig. 5(c) and Fig. 8. Namely, Young's modulus along the blade height, E_3 increases with increase of the angle γ as shown in Fig. 8(a), while Young's modulus along the blade chord, E_1 and the shear rigidity around x_3 axis, G_3 do not change

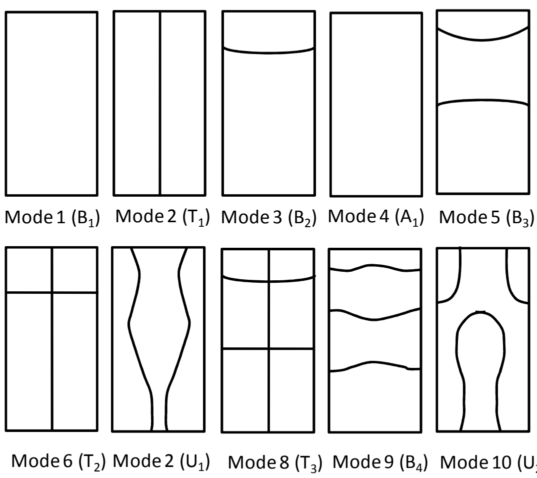
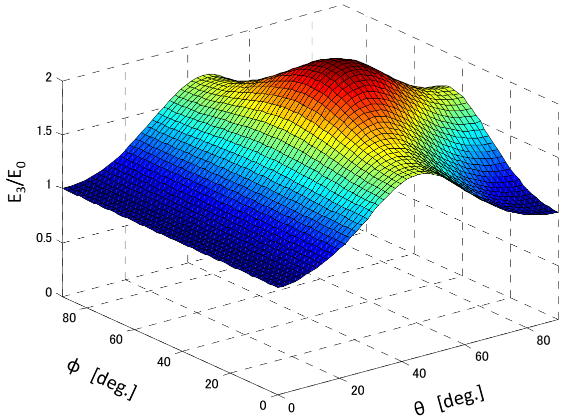
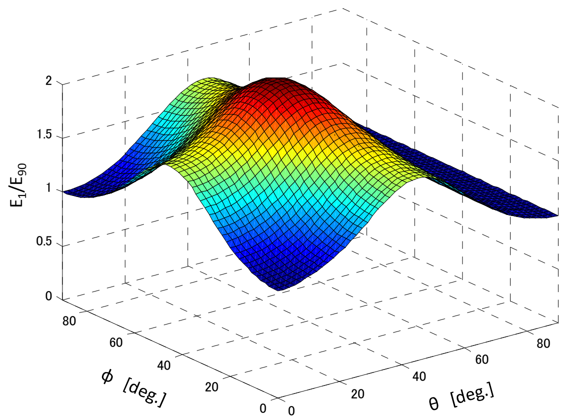


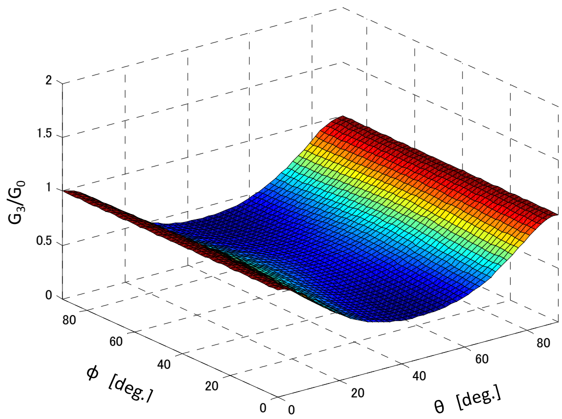
Fig. 6. Vibration modes of SC blade (θ = ϕ = γ = 0)



(a) Young's modulus E_3

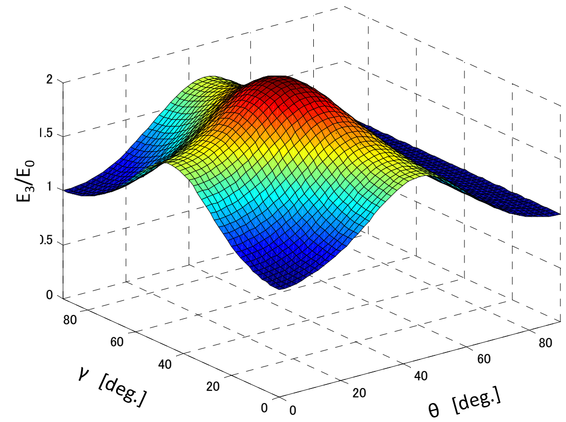


(b) Young's modulus E_1

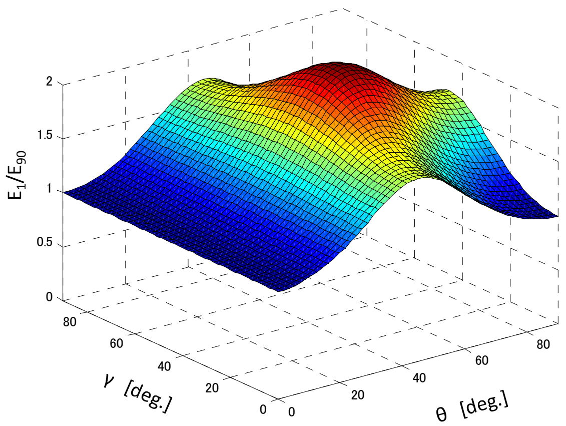


(c) Shear rigidity G_3

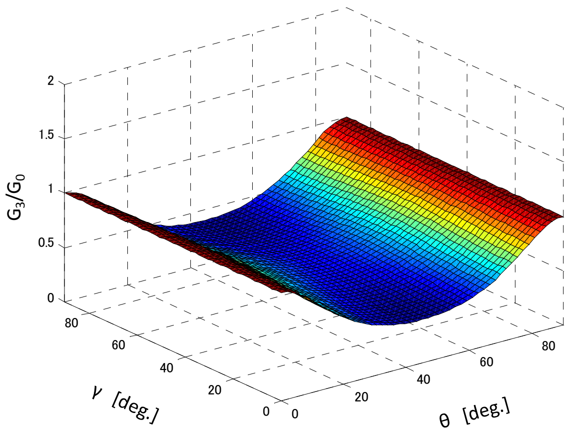
Fig. 7. Elastic constant and crystal angle θ , ϕ of SC blade



(a) Young's modulus E_3



(b) Young's modulus E_1



(c) Shear rigidity G_3

Fig. 8. Elastic constant and crystal angle θ , γ of SC blade

as shown in Fig. 8(b) and Fig. 8(c). As a result, only the bending modes along the blade height increase with increase of the angle γ, and the torsion modes and the bending modes along the blade chord hardly change as shown in Fig. 5(c).

3・2 Verification of assumption of the transverse isotropy of DS blade (Comparison of elastic constant)

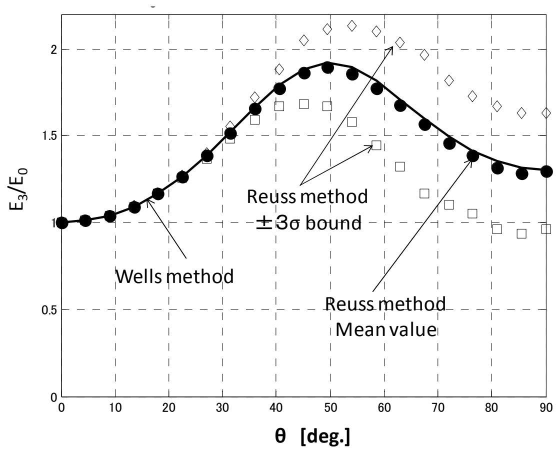
In order to examine whether the DS blade can be treated as a transversely isotropic material, the material constants averaged by the Wells' method are compared with those averaged by the Reuss method. In averaging the material constants by the Reuss method, a DS alloy is manufactured selecting the crystal angle ϕ of each columnar grain at random, and then the averaged material constants are obtained from Eq.(11). This process is repeated one hundred times, and the calculated results are treated statistically to estimate the distribution of the elastic constants (Monte Carlo simulation).

Figure 9 shows the comparison of the elastic constants averaged by the Wells' method and the Reuss method. As shown in Fig. 9, even if the number of the columnar grains is small (N=5), the mean values of the elastic constants averaged by the Wells' method shows good agreement with those averaged by the Reuss method. The deviations of Young's modulus along the blade height, E_3 and the shear rigidity around x_3 axis, G_3 due to the deviation of the angle ϕ hardly appear when the angle θ is less than 15° as shown in Fig. 9(a) and Fig. 9(c). This is because E_3 and G_3 of the single crystal alloy hardly change by the change of the angle ϕ when the angle θ is less than 15°, as shown in Fig. 7(a) and Fig. 7(c).

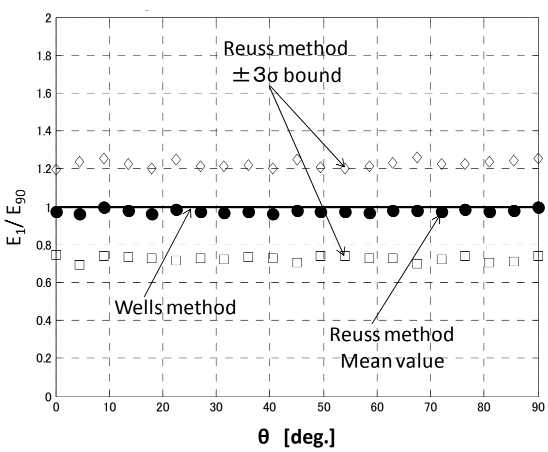
On the other hand, as for the Young's modulus along the blade chord, E_1, the deviation caused by the randomness of the angle ϕ becomes large as shown in Fig. 9(b), when the number of the columnar grains is small. In the actual mechanical design of the DS blade, however, the vibration modes, whose frequencies have to be adjusted for tuning, are lower bending and torsion modes, in which the effects of E_3 and G_3 are dominant. And from the viewpoint of the quality control, the angle θ is usually controlled within ±15°. Namely, the effect of the finite number of the columnar grains in the actual DS blade is limited to the higher vibration modes, whose natural frequencies are not managed in the mechanical design. Therefore, it is considered that even if the number of the columnar grains is small, the DS blade can be treated as a transversely isotropic material, and the vibration analysis can be carried out using the elastic constants averaged by the Wells' method.

Figure 10 shows the deviation of the elastic constants averaged by the Reuss method, where the number of the columnar grains included in the DS alloy is very large (N=100). Figure 11 shows the coefficient of variation of the elastic constants (the ratio of the standard deviation to the mean value) for the DS alloy where the solidification direction of each columnar grain is constant (θ=15°). As shown in these figures, the more the number of the columnar grains in the DS alloy, the less the deviation of the elastic constant is. It can be also said from Fig. 11 that in order to reduce the standard deviation of the Young's modulus along the blade chord, E_1 to less than 1 %, it is necessary to increase the number of columnar grains included in the DS alloy up to 200. In the actual blade design, however, only the natural frequencies of lower vibration modes, in which the effects of E_3 and G_3 are dominant, are adjusted. Therefore, it can be said that since the standard deviations of E_3 and G_3 are less than 1% even if the number of the columnar grains is small (around 5), the vibration analysis of the DS blade can be carried out under the assumption of the transverse isotropy.

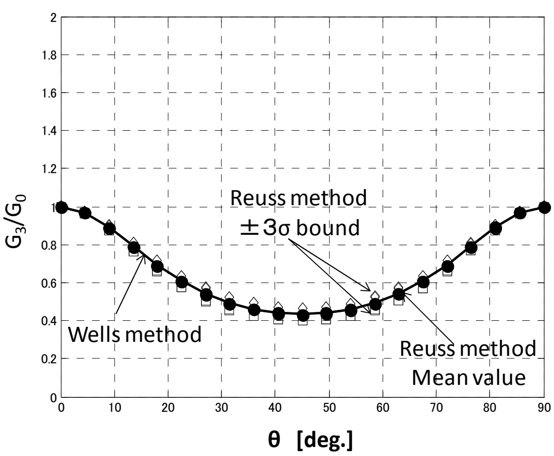
Figure 12 shows the comparison of the elastic constants averaged by the Wells' method and the Voigt method. In averaging the elastic constants by the Voigt method, a DS alloy is manufactured selecting the crystal angle ϕ of each columnar grain at random, and then the averaged elastic constants are obtained from Eq.(11). The distributions of the elastic constants obtained by the Reuss method and the Voigt



(a) Young's modulus E_3

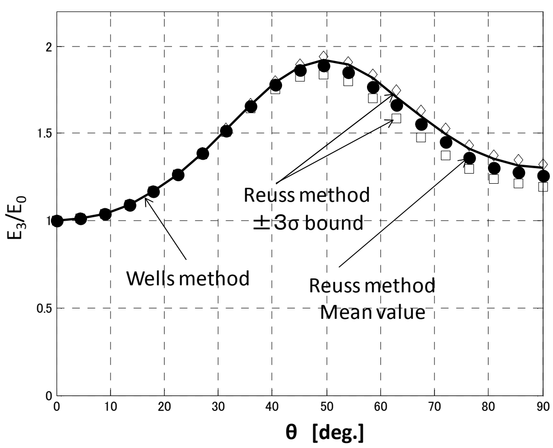


(b) Young's modulus E_1

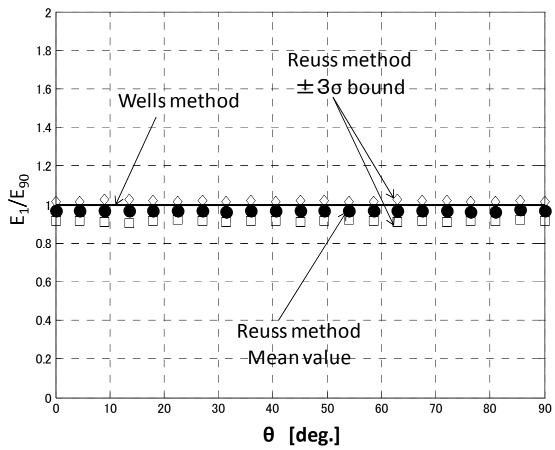


(c) Shear modulus G_3

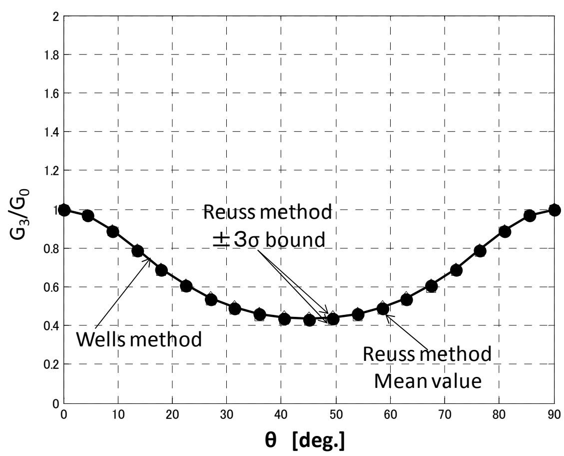
Fig. 9. Elastic constant of DS blade calculated by Wells and Reuss average method (N=5)



(a) Young's modulus E_3



(b) Young's modulus E_1



(c) Shear modulus G_3

Fig. 10. Elastic constant of DS blade calculated by Wells and Reuss average method (N=100)

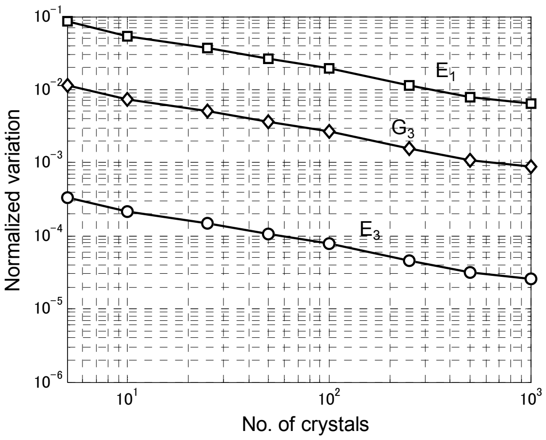
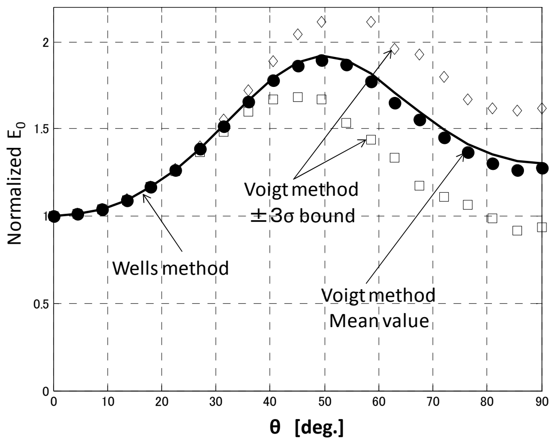
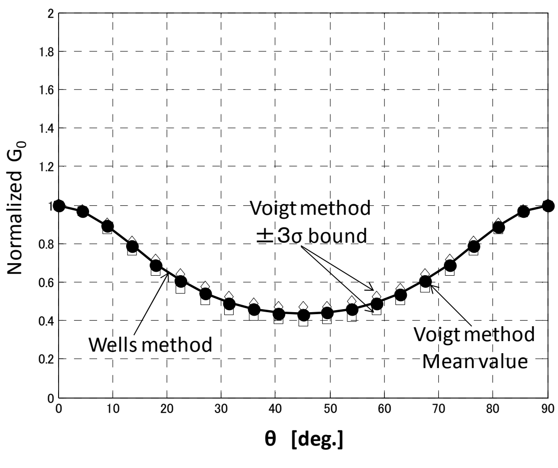


Fig. 11. No. of crystals and variation of elastic modulus by Reuss method (θ=15deg)



(a) Young's modulus E_3



(b) Shear modulus G_3

Fig. 12. Elastic constant of DS blade calculated by Wells and Voigt average method (N=5)

method are nearly the same as shown in Fig. 9 and Fig. 12, and there is almost no difference in the elastic constants between the Reuss and Voigt average.

3.3 Verification of assumption of the transverse isotropy of DS blade (Comparison of natural frequency)

Figure 13 shows the comparison of the natural frequencies of the DS blade with θ=0° in Fig. 4. The open circle denotes the natural frequency of the DS blade consisting of different combination of the columnar grains, where the five columnar grains are modeled exactly and the natural frequency is calculated by the direct FEA. The solid line corresponds to the natural frequency of the DS blade calculated by FEA, where the elastic constants averaged by the Wells' method are used as a material property of the DS blade with the transverse isotropy. In the vibration analysis, it is assumed that the DS blade consisting of five columnar grains has one columnar grain of ϕ=0°, two columnar grains of ϕ=25°, and two columnar grains of ϕ=45°, based on the reference [7]. Under this assumption, the number of all combinations of the columnar grains is sixteen, and therefore, the vibration analysis was carried out for all sixteen models as shown in Fig. 13.

Figure 14 shows the frequency error between the average of the direct FEA results of 16 models and the result by the Wells' method. In Fig. 14, the deviation of the natural frequencies of the 16 models, 3σ/μ (σ: standard deviation, μ: mean value) is also plotted. As shown in these figures, the natural frequencies of the lower bending and torsion modes calculated by both methods show good agreement, and the frequency error is less than 1%. In addition, the frequency deviation of the 16 models is negligible for the lower bending and torsion modes. On the other hand, it is shown that for the higher mode of U_1 (the bending mode along the blade chord), the frequency error and frequency deviation become large. However, in the mechanical design of the DS blade, the natural frequency of the higher mode is out of frequency control. Therefore, it can be said from these results that even if the number of the columnar grains is small, the DS blade can be treated as a transversely isotropic material, and the vibration analysis can be carried out using the elastic constants averaged by the Wells' method.

Figure 15 shows the comparison of the natural frequencies of the DS blade with θ= 15°. Figure 16 shows the frequency error between the average of the direct FEA results of 16 models （θ= 15°） and the result by the Wells' method. In Fig. 16, the deviation of the natural frequencies of the 16 models, 3σ/μ is also plotted. Even in case that the angle of θ becomes the allowable limit, the natural frequencies of the

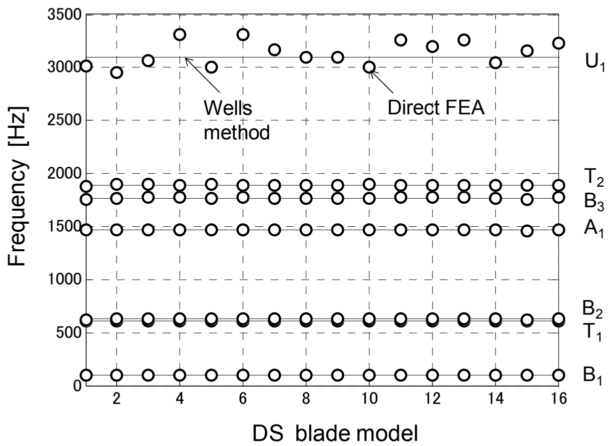


Fig. 13. Frequency of DS blade (θ=0 deg.)

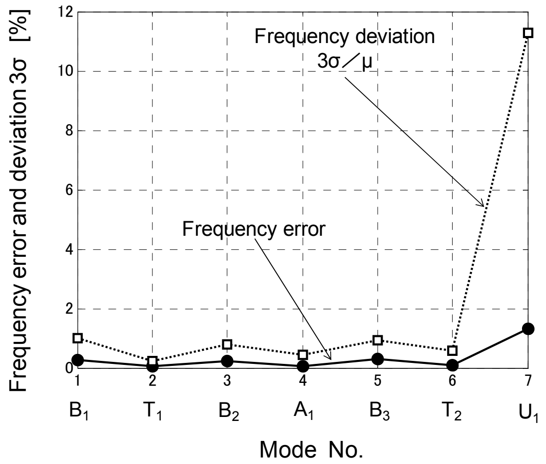


Fig. 14. Frequency error and deviation of DS blade (θ=0 deg.)

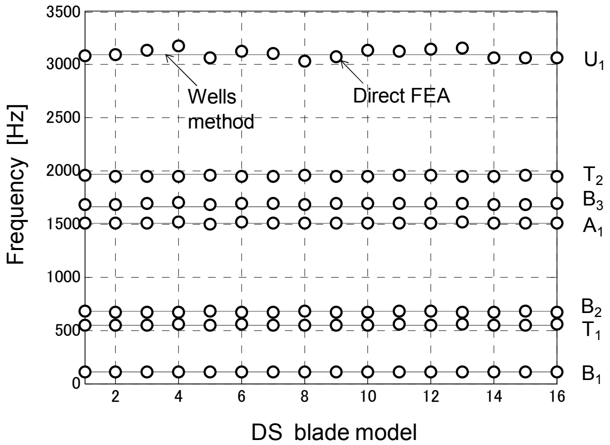


Fig.15. Frequency of DS blade (θ=15 deg.)

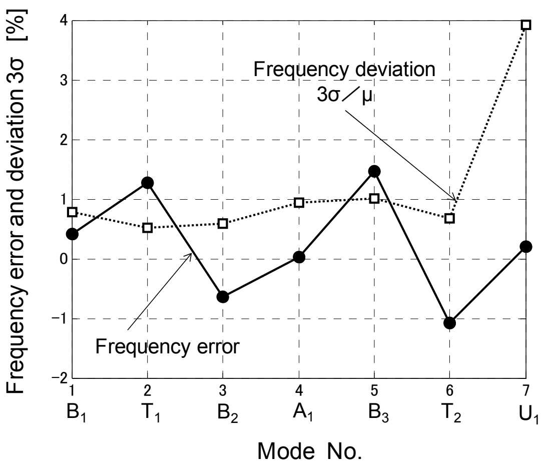


Fig.16. Frequency error and deviation of DS blade (θ=15 deg.)

lower bending and torsion modes calculated by both methods show good agreement, and the frequency error is less than 1%. The frequency deviation of the 16 models is also negligible for the lower bending and torsion modes.

3.4 Analysis result of frequency deviation

Figure 17 shows the standard deviation of the natural frequency of the CC, SC and DS blade, which is calculated by the first order second moment method. The elastic constant of the CC blade is assumed to be the same as the values used in reference [7] (E_0=2.49×10^5 Mpa, G_0=0.947×10^5 Mpa). The DS blade is assumed to be transversely isotropic, and the elastic constant of the DS blade is calculated by the Wells' method. As explained in previous section, the number of the independent elastic constants of the CC blade is two, while the number of the independent elastic constants of the SC and DS blade is three. In the analysis of the frequency deviation, the standard deviation of each independent elastic constant is assumed to be 5%. In addition, for the SC blade, the standard deviation of the crystal angle of θ and ϕ is assumed to be 10°, while the standard deviation of the angle θ is assumed to be 10° for the DS blade.

As shown in Fig. 17, the frequency deviation of the SC and DS blade caused by the deviation of the elastic constants is nearly twice larger than that of the CC blade.

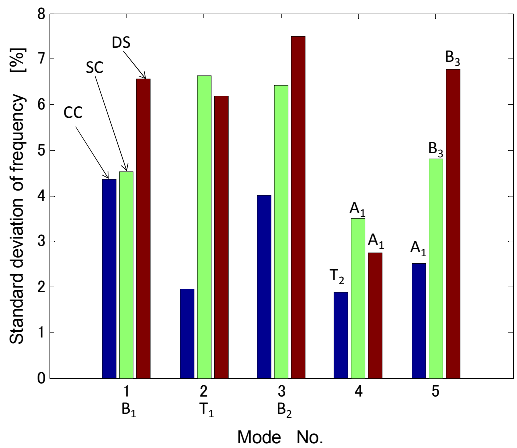


Fig. 17. Standard deviation of natural frequency due to variety of material constant

4. CONCLUSIONS

In this study, first, the effect of the anisotropy of elastic constants on the vibration characteristics of the SC and DS blade was investigated systematically. Second, the validity of the assumption of the transverse isotropy for the DS blade was examined, carrying out a Monte Carlo simulation on the elastic constants and FE analysis of the DS blade. Finally, the frequency deviation of the SC and DS blade was analyzed by the first order second moment method.

From these results, it is concluded that even if the number of the columnar grains included in the DS blade is small, the DS blade can be treated as a transversely isotropic material, and the vibration analysis can be carried out using the elastic constants averaged by the Wells' method. And it is clarified that the frequency deviation of the SC and DS blade caused by the deviation of the elastic constants is nearly twice larger than that of the CC blade.

REFERENCE

[1] H. Kaneko and T. Sakon, "A Study on Biaxial Thermal Fatigue Life of A Directionally Solidified Alloy for Gas Turbine Blade",

Proceedings of International Conference on Power Engineering-97, Vol.2(1997), pp.113-117.
[2] M. Manetti, et al., “The Dynamic Influence of Crystal Orientation on A Second Generation Single Crystal Material for Turbine Buckets”, ASME paper, GT2009-59091(2009), pp.1-9.
[3] T. Hasebe, M. Sakane and M. Ohnami, “High Temperature Low Cycle Fatigue and Cyclic Constitutive Relation of MAR-M247 Directionally Solidified Superalloy”, ASME, Journal of Engineering Material and Technology, Vol. 114(1992), pp162-167.
[4] H. Kaneko, T. Sakon and T. Kouji, “Study on Biaxial Thermal Fatigue Life of Directionally Solidified Alloy for Gas Turbine Blade”, Mitsubishi Technical Report, Vol. 34, No. 4(1997), pp.242-245.
[5] Roland Mücke and Piyawan Woratat, “A Cyclic Life Prediction Approach for Directionally Solidified Nickel Superalloys”, ASME paper, GT2009-59180(2009), pp.1-9.
[6] M. Mori et al., “Structural Analysis Technique of DS-Alloy Blade for Gas Turbine”, Mitsubishi Technical Report, Vol. 33, No. 1(1996), pp.42-45.
[7] T. Hasebe and M. Sakane, “Vibration of Nickel-Base Single Crystal and Directionally Solidified Superalloy Plates”, JSME International Journal, Series A, Vol. 37, No. 4(1994), pp.481-487.
[8] T. Hasebe, M. Sakane and M. Ohnami, “Elastic Anisotropy of Directionally Solidified Superalloy”, ASME, Journal of Engineering Material and Technology, Vol. 114(1992), pp141-146.
[9] Wells., C. H., “The Elastic Constants of a Directionally-Solidified, Nickel-Base Superalloy, Mar. M-200”, Trans. of the ASM, Vol. 60(1967), pp270-271.
[10] Y. Kaneko, et al., “Analysis of Variation of Natural Frequency and Resonant Stress of Blade”, ASME paper, GT2006-90176(2006), pp.1-7.