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FREQUENCY CLUSTERING IN CYCLIC SYMMETRIC ROTORS CONSIDERING GEOMETRY NONLINEARITY

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ABSTRACT

Motivated by finite element results of a bladed disk model, a discrete model and numerical analysis are developed and presented in this paper with view toward understanding effect of large geometric design factors on the vibrations of a cyclic symmetric integrally bladed rotor (IBR). It is found that nonlinear eigenvalue-loci crossing for split frequency doublet modes and eigenvalue-loci veering for repeated frequency modes having the same wavenumber content can occur when large blade span distribution and blade to disk inertia ratio are considered in the discrete IBR model. Clustering of eigen-loci is found and characterized in blade span parameter study when blade's inertia is significant large. These three findings are new to the literature and have engineering implications on structure design and vibration control of engine components in turbine and automobile applications.

INSTRUCTION

Localized vibrations are commonly observed in rotationally periodic structures or cyclic symmetric structures such as bladed disk assemblies in jet engines. Such localized vibrations potentially contribute to large dynamic responses in the bladed disks under certain engine order excitations and rotating speed, which can be very harmful to the rotating systems and engine's health. From rich literature, the localized vibrations were reported to be caused by mistuning effect among substructures (the blades) due to manufacturing and assembling imperfections in stacked bladed disk assemblies. Limited by computation efficiency and the fact that turbine rotor disks are much stiffer than blades and blades are inserted into rotors, spring-mass models, in which mass elements are used to represent blade's inertia and spring elements are used to approximate flexibility between adjacent blades, are commonly employed as the first-order approximation for bladed disk vibrations. To capture the localized vibratory characteristics and to improve computation efficiency, reduced order method [1-3] and probability model [4] have been successfully developed to predict airfoil responses.

For integrally bladed rotors or integrally blisk systems, no insertion of blades is needed as blades and rotor systems are integrated during manufacturing process. Reduced order method is commonly employed for the case when rotor stiffness is much higher than blade stiffness. When stiffness and inertia of blades and rotor disk are comparable, the reduced order method and probability model are not sufficient to describe their complex structure dynamics. Construction of a system finite element model [5] is the common way for such case. Although it can certainly reveal significant amount of design information, it is obviously penalized by significant cost investment in computation.

On the other hand, studies on vibrations of axis-symmetric or slightly asymmetric continuous systems [6-14] provide an alternative in revealing the complex vibration modes in integrally bladed rotors. When slight inertia or stiffness

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elements such as blades in a turbine rotor or bolts in a disk drive spindle are added to the underlying axis-symmetric structure, the rotor disk., structure axis-symmetry is destroyed and becomes asymmetric or cyclic symmetric. The key method used in the literatures is to employ perturbation method. For the case of a bladed disk, the perturbation model assumes blades to be "point" masses to turbine rotor while in disk drive applications, fasteners or disk clamp bolts are assumed to be point stiffness to the magnetic annular disks [14]. The major findings from the aforementioned perturbation approach verified by calibrated experiments are spatial modulation for repeated frequency modes [10-13], frequency splitting for certain repeated modes whose number of base wavenumber follows certain relationship with respect to number of blades [7] and finally dispersive traveling wave components in forced responses [9, 11-13].

Motivated by the perturbation work [11], in this paper, nonlinear migration of eigenvalues will be presented and discussed when an IBR's geometry is relatively large. Supported by finite element model and experimental modal test results that will be presented in the following section, a discrete model will be developed and demonstration of its capability will be presented in the context of this paper through parametric studies.

IBR MODE STRUCTURE

When blades or fasteners are included in an axis-symmetry structure, certain repeated frequency (m,n) modes having "n" nodal diameters and "m" nodal circles in its base axis-symmetric case split into two distinct frequencies denoted as split doublet $P(m,n)SC$ following rule (split when $n= jNF$, j=1,2,3, …) developed in [10-13], where S and C stands for sine and cosine components, respectively. As an example in Fig. 1, natural frequencies or eigenvalues of the (0,5) mode at Δr=0 split into two distinct frequency P(0,5)S and P(0,5)C modes in a NF=5 bladed disk when $\Delta r \neq 0$. Here NF stands for number of features or blades in the IBR and P indicates that the periodicity occurs in the structure. Other repeated frequency doublet modes remain repeated and are denoted by P(m,n). In the illustrated example as shown in Fig. 1, steel bladed disks of uniform thickness of 0.05 cm with r_o=12.7 cm clamped at r_i=5.95 cm were used in experimental modal analysis as well as in finite element models.

The phenomena of crossing and veering of eigenvalue loci are of particular interest in bladed disk design for vibration control [15, 16]. If modes can be placed closely together, the structure has less chance to be at resonant as wider frequency range freedom is provided for designing engine order excitations. In the prior articles [11, 12] in which perturbation method was employed to model blades, crossing points of split doublet's eigenvalue-loci are predicted at

$$0 \le \alpha = j\frac{\pi}{n} \le \frac{2\pi}{NF}, \qquad (1)$$

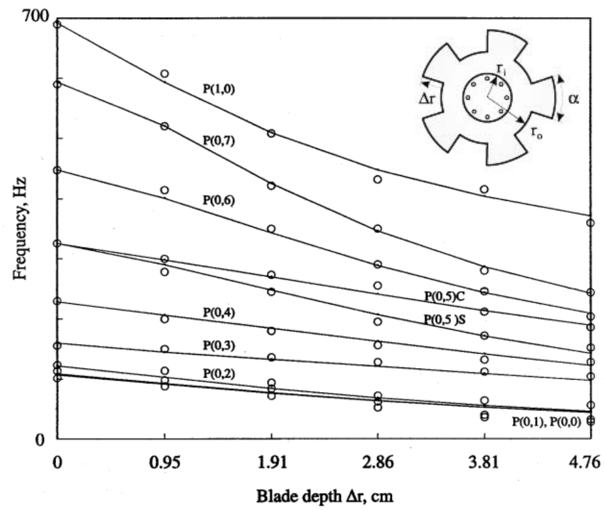

Fig.1 Spectrum of measured, ○, and predicted, –, natural frequencies of several modes of a NF=5 bladed IBR as function of blade depth, α= 36^{o}.

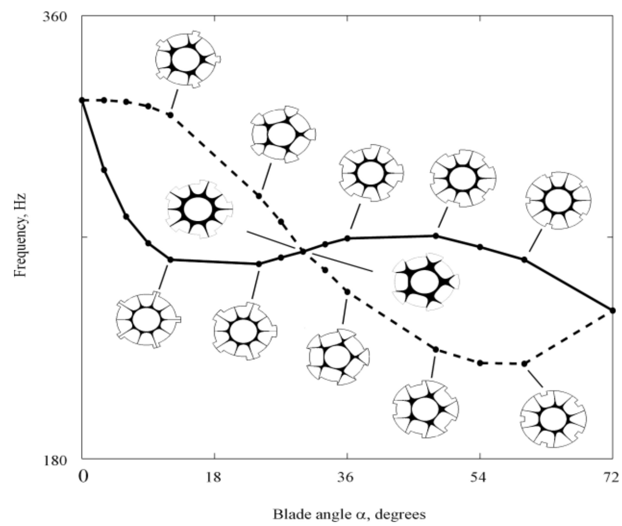

Fig.2 Predicted natural frequency loci of P(0,5)S (–•–) and P(0,5)C (--•--) modes as function of blade span angle, , Δr= 1.91 cm. The two loci cross at α=30^{o}.

where j=1, 2, 3, …Based on this rule, for a NF=5 IBR, split doublet P(0,5)SC modes cross their eigen-values when blade span angle is designed at α=36 degree. However, when blades deviate from perturbed domain, this prediction fails. An example of such deficiency is demonstrated by using finite element simulation as shown in Fig. 2 in which the crossing point for the NF=5 IBR's P(0,5)SC modes is at 30 degree. Therefore, instead of building a full finite element model for an IBR for the case in which stiffness and inertia of blades and rotors are comparable, the focus of this paper is aimed at developing a discrete model to study the nonlinear eigenvalue crossing and veering in IBR structures when geometry such as blade span deviates from linear case.

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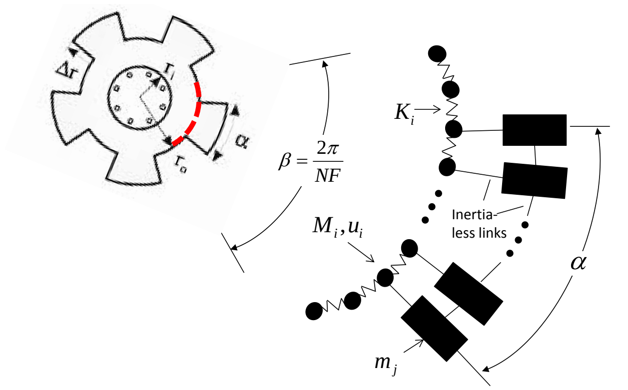

Fig.3 Modeling the IBR as a discrete spring-mass system.

DISCRETE MODEL

To consider the blades-rotor system as a whole, blades which are periodically placed on the rotor's outer rim can be assumed as additional mass elements to the axis-symmetric disk. As opposed to commonly used "perturbed point mass" approach in the literature, continuous inertia sectors forming as a square wave are used to model the blades in the discrete model. A discrete spring-mass model as illustrated in Fig. 3 is developed as the companion model for the continuous IBR system. The free vibration problem of the model discrete system is written as

$$M\ddot{U} + KU = 0, \tag{2}$$

where M is the system mass matrix, K is the system stiffness matrix and U is the displacement vector. The displacement vector contains nodal displacement components u_i for disk elements having inertia M_i connected by identical springs K_i, which represent IBR's disk subsystem's bending stiffness. The dots in equation (2) represent differentiation with respect to time. The M and K matrixes are positive definite and skew symmetric in which K is a $n\times n$ stiffness matrix expressed as

$$K = k\begin{bmatrix} 1 & -1 & 0 & \cdots & 0 \\ -1 & 2 & -1 & \ddots & \cdots \\ 0 & -1 & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & 2 & -1 \\ 0 & \dots & 0 & -1 & 1 \end{bmatrix}, \tag{3}$$

where k is the stiffness constant representing disk's bending flexibility.

The blade span angle α as depicted in Fig. 3 is allowed to vary from 0 to sector span angle β. When $\alpha = 0$, an axis-symmetric disk or a ring system is formed while when $\alpha \neq 0$ and $\alpha \neq \beta$, an IBR can be approximated by the discrete IBR model. A heavier axis-symmetric disk or ring system is recovered when $\alpha = \beta$. In the present model, blades are assumed to be lumped masses m_j connected by inertial-less links to disk elements and adjacent blade elements as depicted in Fig. 3. That is, to ensure continuity, a blade element m_j vibrates

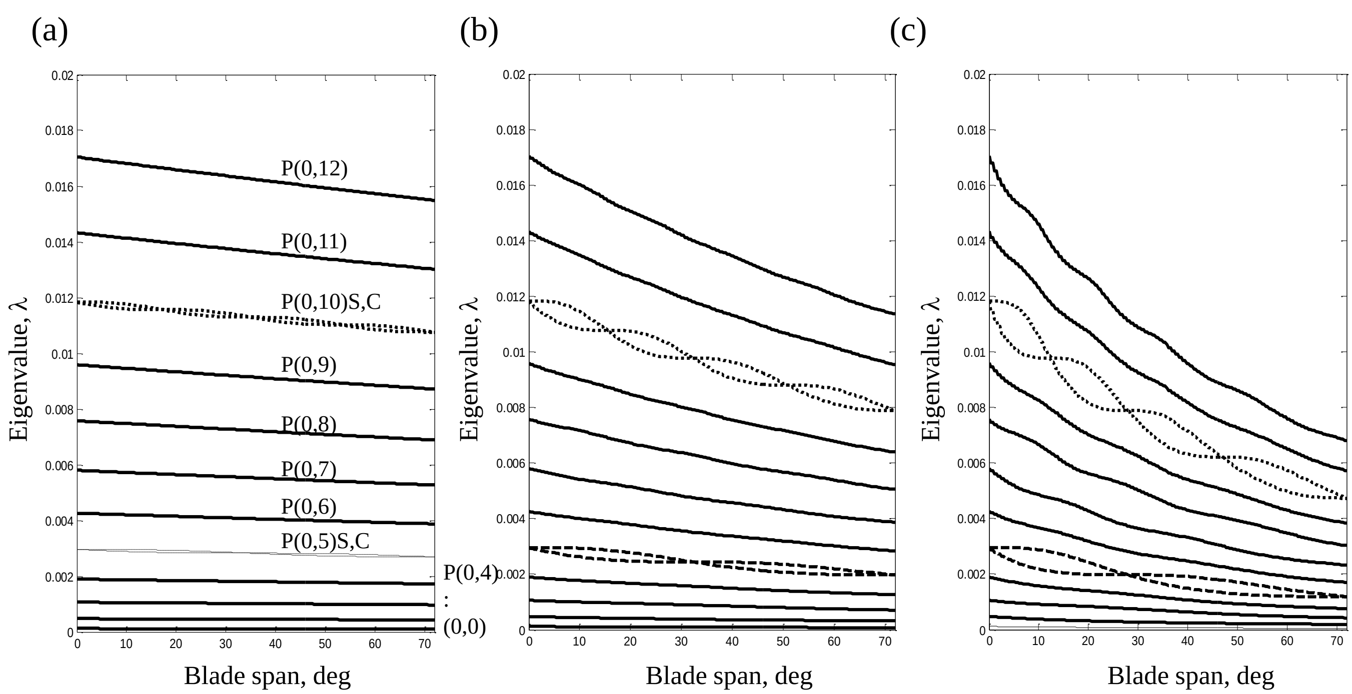

Fig.4 Comparison of eigenvalue loci of several modes of the model IBR as a function of blade to disk inertia ratio m_j/M_i, (a) 10%, (b) 50% and (c) 150%.

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exactly the same as the adjacent disk element M_i and adjacent blade elements m_{j-1} and m_{j+1}. Sufficient number of elements were used for the modeled NF=5 discrete IBR model to ensure convergence of eigenvalue λ. Assuming 40 disk elements were employed over one revolution of 360^o, that is n=40, for the illustrated model, 8 identical, evenly distributed disk mass elements are employed over β=360^o/5= 72^o section span denoted by the dashed arc in Fig. 3. As an example, for each disk mass element M_i=2 within the blade span angle $\alpha = 36^o$, a blade mass element m_j=1is added to it forming the system mass matrix M as

$$M = \begin{bmatrix} M_1 & & & & \\ & M_2 & & 0 & \\ & & M_3 & & \\ & 0 & & M_4 & \\ & & & & M_5 \end{bmatrix}, \tag{4}$$

where M_h (h=1,2,3,4,and 5) are eight by eight subsystem mass matrices given as

$$M_h = \begin{bmatrix} 3 & & & & & & & \\ & 3 & & & 0 & & & \\ & & 3 & & & & & \\ & & & 3 & & & & \\ & & & & 2 & & & \\ & & & & & 2 & & \\ & 0 & & & & & 2 & \\ & & & & & & & 2 \end{bmatrix}. \tag{5}$$

Depending on accuracy, one can certainly expand number of elements used in the discrete model. In the present study, convergence was assured by allocating one disk mass element per each degree in one revolution of 360^o. Likewise, one blade element is used for 1^o within blade span angle β. Standard modal analysis was then executed with the view toward understanding morphing of engivenvalues as function of increasing blade angle and blade to disk inertia ratio m_j/M_i.

CROSSING OF EIGENVALUE LOCI

The blade airfoil angle with respect to the rotor's rotating axis is a key design factor for engine efficiency. In a limiting case when blade is placed at right angle to the rotating axis, $\alpha \rightarrow 0$ and "point mass" assumption is valid. However, the blade's placement angle is normally away from right angle with respect to the rotating axis. Therefore, it is believed that the present model using blade span α along with large blade to disk element inertia ratio can reveal vibration insights of an IBR, at least to the first order.

Of particular interest is the effect of nonlinear migration of eigenvalues in IBRs over large geometric factor such as blade span angle. Using the aforementioned discrete model, several eigenvalues are plotted in Fig. 4 as function of blade span angle

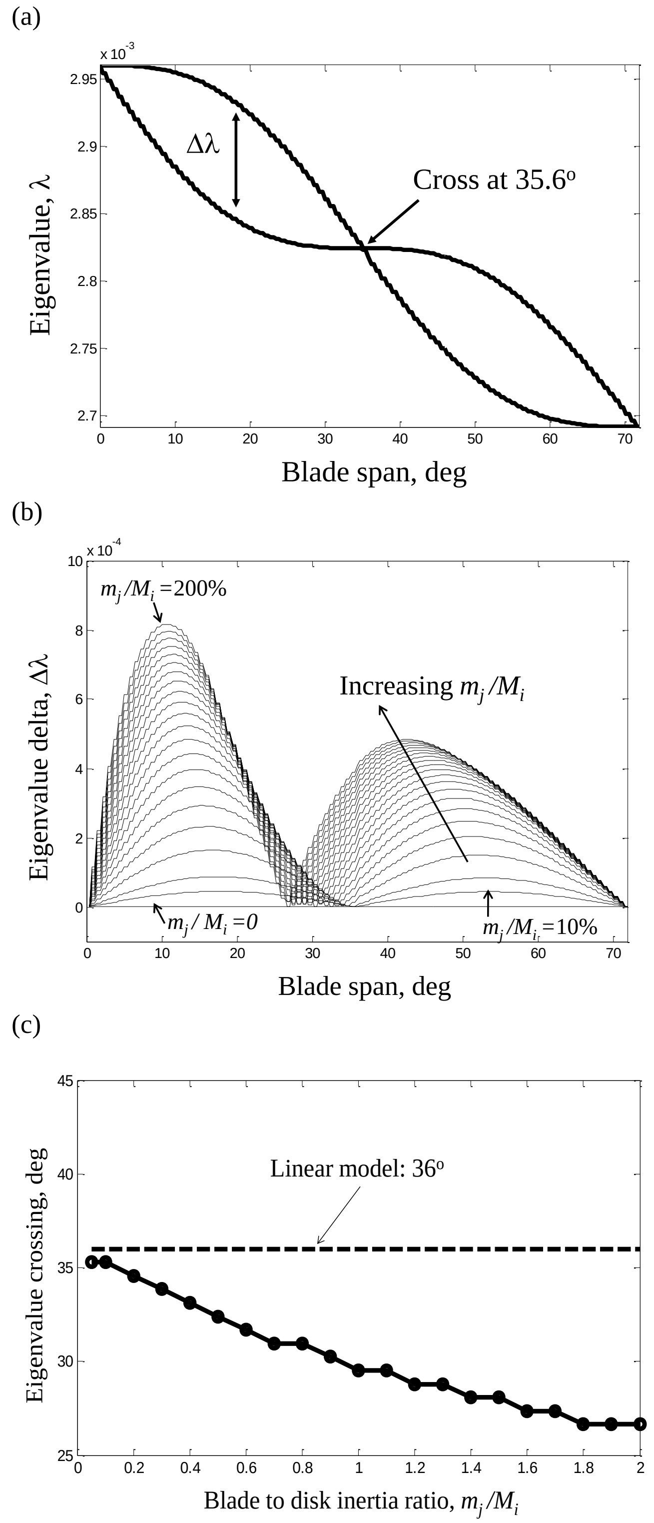


Fig.5 Migration of (a) eigenvalue for $P(0,5)SC$ split doublet mode when NF=5, m_j/M_i =10%, (b) eigenvalue delta as function of blade span and inertia ratio, and (c) $P(0,5)SC$ eigenvalue crossing as function of inertia ratio when NF=5.

α with blade to disk inertia ratio at 10%, 50% and 150% in Fig. 4(a), (b), and (c), respectively. When blade's inertia is relatively small compared to disk's inertia, eigenvalue loci are found to be linearly decreased with increasing blade span α as shown in Fig. 4(a). This agrees very well to the perturbation's prediction [12]. Although it may be hard to observe in Fig. 4(a) due to the same scales being applied in Fig. 4(b) and (c), from the simulation, eigenvalue loci for split doublet $P(0,5)SC$ modes which are denoted as the dashed lines cross at 35.6^o while for $P(0,10)SC$ modes, the dotted lines, their loci cross at 17o, 37^o, and 53^o. Quantitatively, these numbers agree extremely very well with the perturbation's prediction as stated in equation (1).

When blade to disk inertia ratio increases, the eigenvalue loci are found to nonlinearly decrease with increasing blade span α for all repeated and split double frequency modes. From Fig. 4(b) and (c), one can observe that the order of nonlinearity increases when heavier blades are placed on the underlying axis-symmetric disk. Obviously, the analytical perturbation model is deficient to capture such nonlinear phenomena.

To clearly demonstrate the difference between what predicted by the perturbation model and that by the discrete model, eigenvalue delta, $\Delta\lambda$, which is the frequency separation between split sine and cosine modes at blade span angle α, is defined. As shown in Fig. 5(a), when α = 0, 35.6^o, and 72^o, $\Delta\lambda$=0. Its value increases and diminishes between the crossing points. In Fig. 5(b), $\Delta\lambda$ is plotted as a function of blade span angle α at various blade to disk element inertia ratios for the $P(0,5)SC$ modes in the model NF=5 IRB. It is noted that when the inertia ratio m_j/M_i increases, the eigenvalue separation, the $\Delta\lambda$, increases accordingly. From Fig. 5(b), it is also noted that the crossing point of the split doublet sine and cosine eigenvalues, where $\Delta\lambda$=0 as α varies, decreases in degree when the inertia ratio increases. Plotting the crossing point values against blade to disk inertia ratio with dashed line denoted as the asymptote predicted by the perturbation model in Fig. 5(c), it is interesting to note that the deviation of the crossing point from what approximated by linear model seems to be "linearly" increases as the blade to disk inertia ratio increases. The same observation is found true for higher order split double modes such as $P(0,10)SC$, and $P(0,15)SC$ in the same NF=5 IBR as shown in Fig. 6.

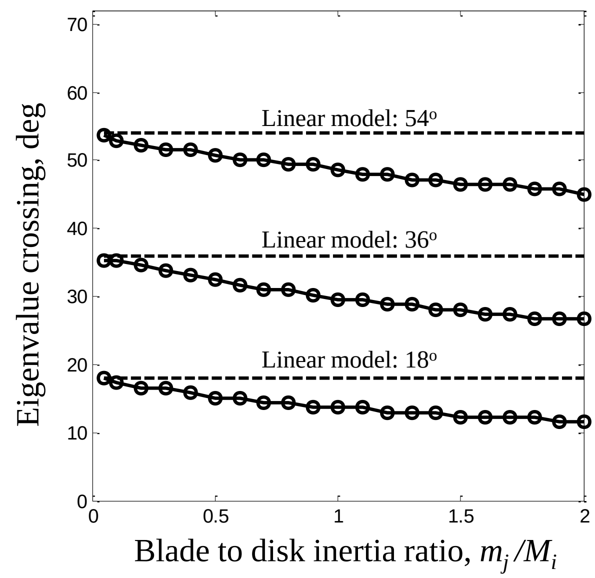


Fig.6 $P(0,10)SC$ eigenvalue crossing as function of inertia ratio predicted by linear model (– –) and the discrete model (–o–) in the model NF=5 IBR.

VEERING AND CLUSTERING OF EIGENVALUE LOCI

It is found that eigenvalue loci can veer and cluster with each other when large blade angle and blade to disk inertia ratio are applied. As a demonstrative example, eigenvalue-loci of NF=9 IBR are plotted against blade span angle as shown in Fig. 7 when blade to disk inertia ratio is significantly large, say at 200%. Split $P(0,9)$SC doublet's eigenvalues cross at α=11^o instead of 20^o (180^o / NF= 180^o / 9 = 20^o) predicted by linear perturbation model. It is interesting to see in the figure that eigenvalue-loci of $P(0,14)$ and $P(0,13)$ repeated doublet modes denoted by the dotted lines veer but not cross two times when α varies from 0 to α=β=360^o / 9 = 40^o. It is also noted that no veering or crossing occur between $P(0,4)$ and $P(0,5)$ repeated doublet modes. However, they tend to bounce and then veer or merge again when blade span is approaching 40^o. Certain modes such as the $P(0,8)$, $P(0,10)$ modes seem to be clustered together and morph in a similar manner as blade span angle increases.

This veering phenomenon has not been observed and reported in the literature when large geometry such as the blade span angle is considered for structure vibrations. The aforementioned veering and clustering can be otherwise explained by the repeated doublet modes' wave number content using the rule of $|n\pm k|=NF, 2NF, 3NF$, etc developed in [11-13]. A checkerboard diagram following this rule is constructed in Table 1 for the NF=9 IBR case. For $P(0,14)$ repeated doublet mode, it has base wavenumber at n=14 and contaminated wavenumbers at k=4, 5, 13, etc. For $P(0,13)$ repeated doublet mode, its base wavenumber is n=13 while its contaminated wavenumbers are at k= 4, 5, 13, etc. As commented by Perkins and Mote in [15], veering of eigenvalues can occur when modes have similar modal content which can eventually lead to sudden exchange of mode shapes. Obviously for the $P(0,14)$ and P(0,13) repeated double modes of the present model, since they have not only similar but also exactly the same model wavenumber content, they definitely would veer or bounce and then merge when structure geometric factors are changed. As an illustrative example of using the checkerboard diagram as shown in Table 1 to assist identification of modes that would veer, as denoted in Table 1, veering occurs for the modes when four wavenumber squares are packed adjacent with each other to form a bigger square whereas crossing of modes occurs then wavenumber squares cross with each other. For the present example, wavenumber squares cross when n=9 while veering occurs for the n=4, 5 and n=13, 14 repeated double mode pairs.

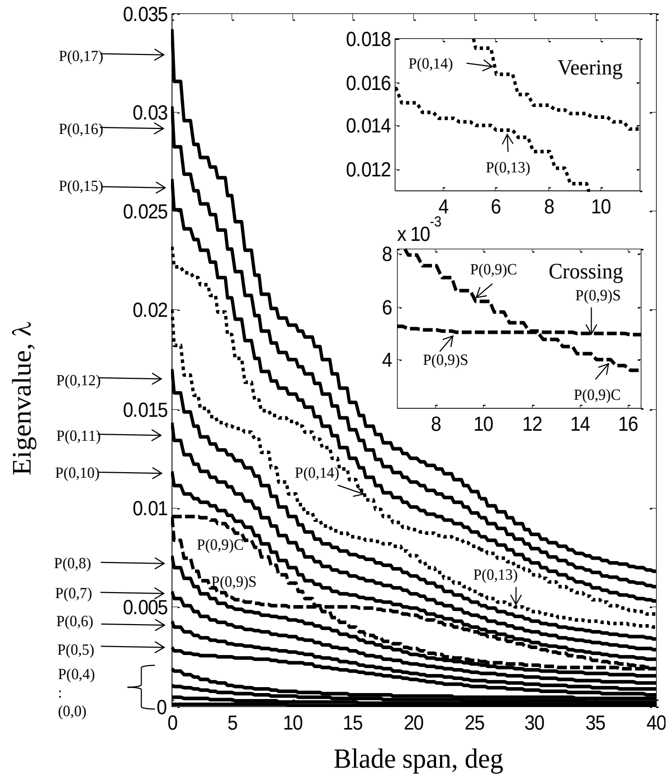
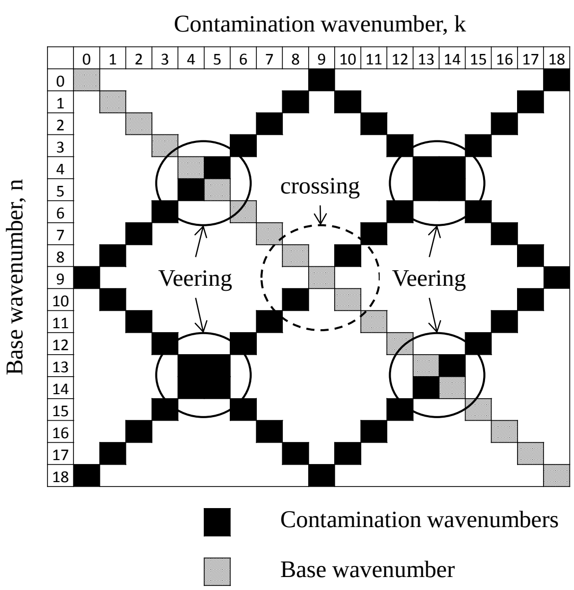


Fig.7 Example of nonlinear eigenvalue loci veering and clustering, NF=9, m_j/M_i=200%.

Table 1 Checkerboard diagram showing base, n and contaminated, k wavenumbers for NF=9 IBR.



The linear perturbed model developed in [11-13] didn't predict such mode veering and identification when large structure geometry is considered for in IBR vibration analysis. The identification methodology discussed here can add to the existing literature on vibration veering as well as research field of IBR structure vibrations.

In Fig. 7 certain eigenvalue-loic are grouped or clustered when α is changed. For instance, P(0,8) and P(0,10) loci are alike while loci for P(0,12) and P(0,15) modes are alike. Based on the checkerboard diagram in Table 1, P(0,8) has wavenumbers at 1, 8, 10, 19, etc and P(0,10) has wavenumbers at 1, 8, 10, 17, etc. For P(0,12) mode, it has wavenumbers at 3, 6, 12, 15, 24, etc while for P(0,15) mode, wavenumbers are at 3, 6, 12, 15, 21, etc. They have "similar" but not exactly the same wavenumbers as P(0,13) and P(0,14) pair. Therefore, these {P(0,8), P(0,10)} and {P(0,12), P(0,15)}, etc repeated doublet pairs are clustered in groups which tend to veer but blocked by crossing split doublet P(0,9)SC and veering repeated doublet {P(0,13), P(0,14)} pairs.

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CONCLUSION

In this paper, a discrete model was developed for free vibrations of integrally bladed rotors. The capabilities and effectiveness of the discrete model are validated by finite element simulations and calibrated experimental modal analysis. Migration of natural modes is examined with consideration of large blade span angle geometry in the model IBR structure. Through parameter studies using the discrete model, the following conclusion remarks are reached.

(a) Eigenvalue-loci for split doublets in an IBR are found to morph nonlinearly as a function of large blade span angle.
(b) Crossing points of the split doublets are found to be linearly dependent of large blade span angle.
(c) Eigenvalue-loci of repeated doublets with exactly the same wavenumber content are found to veer in a nonlinear fashion as a function of blade span angle.
(d) Eigenvalue-loci for repeated doublets having similar wavenumber content are found to cluster with each other and morph nonlinearly with respect to blade span angle when blade to disk inertia ratio is significantly large.

These four findings have archival research value for the study of vibrations of integrally bladded rotors. They also have engineering implications in designing integrally bladed rotors to avoid forced resonant responses.

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