Proceedings of ASME Turbo Expo 2011
GT2011
June 6-10, 2011, Vancouver, British Columbia, Canada

GT2011-45391

Reduced Order Model of a Bladed Rotor with Geometric Mistuning: Comparison Between Modified Modal Domain Analysis and Frequency Mistuning Approach

by

Yasharth Bhartiya and Alok Sinha
Department of Mechanical and Nuclear Engineering
The Pennsylvania State University
University Park, PA 16802

Abstract

Mistuning has traditionally been modeled through the changes in Young's moduli of blades, or equivalently through perturbations in the stiffness matrices associated with blades' degrees of freedom. Such a mistuning is termed as Frequency Mistuning because it alters the blade alone frequencies without altering the mode shapes component associated with the blades. Many reduced order models have been developed for frequency mistuning [1-7]. Although frequency mistuning has been developed for Young's Modulus mistuning, it is applied to geometric mistuning in the literature. In this paper frequency mistuning is applied to a geometrically mistuned system and the results from Subset of Nominal Modes (SNM) [5] technique, a reduced order model based on frequency mistuning, are compared with those from Modified Modal Domain Analysis (MMDA). It is shown that frequency mistuning analysis is unable to capture the effects of geometric mistuning in general, whereas MMDA provides accurate estimates of natural frequencies, mode shapes and forced response.

Nomenclature

n	Number of sectors
np	Number of POD features
b_0	Average thickness of the blades
b_l	Actual thickness of blade #l
ξ_l	Mistuning parameter for blade #l
E_0	Young's modulus of tuned/actual blades
E_m	Equivalent young's modulus of blades for frequency mistuning.
Φ_0	Mode shapes of the nominal tuned bladed disk
Φ_1	Mode shapes of tuned bladed disk with geometry perturbed along 1st POD feature

INTRODUCTION

Mistuning is a term adopted to designate the small blade-to-blade variations in geometric and material properties, which are unavoidable in all practical bladed disks due to manufacturing and assembly tolerances and non-uniform wear during service. The fundamental blade mistuning problem stems from the fact that unavoidable (but generally small) blade-to-blade variations cause simultaneous and dependent perturbations in mass and stiffness matrices of each blade, which has a dramatic effect on the vibration behavior of a bladed disk system. Computationally efficient algorithms that analyze perfectly periodic structures using the theory of cyclic symmetry have been developed to study the vibration characteristics of bladed rotors. But in presence of mistuning, the cyclic symmetry property is lost. Due to mistuning leading to breakdown in cyclic symmetry, modeling just one sector is not sufficient; a full bladed disk model is needed. Modern industrial finite element models of a full bladed disk can be on the order of millions of degrees of freedom. Even in today's date of advanced computing, the use of full 360 degree models to perform Monte Carlo simulations is infeasible. Therefore, reduction techniques are used to generate reduced-order models (ROM) from the tuned finite element models and geometric mistuning definitions for a frequency range of interest.

Mistuning has traditionally been modeled through the changes in Young's moduli of blades, or equivalently through perturbations in the stiffness matrices associated with blades' degrees of freedom. Such a mistuning is an approximation of actual mistuning because it does not capture the simultaneous perturbations in mass and stiffness matrices due to perturbations in geometry. Such a mistuning is commonly referred to as "Frequency Mistuning". A consequence of Frequency Mistuning is that it does not alter the blades' mode shapes, but only the blade alone frequencies. For Frequency Mistuning, reduced order models [1 - 7] have been developed which represent the solution as a weighted sum of the modes of the nominal tuned system. Such an assumption works because Frequency Mistuning does not alter the mode shapes associated with the blades. But actual (geometric) mistuning leads to simultaneous perturbations in mass and stiffness matrices, which alter the mode shapes associated with the blades, hence the subset of nominal modes assumption [5] is no longer valid and the accuracy of these models is reduced. In case of lightly damped structures like integrated blade rotors or blisks, this inaccuracy can lead to large errors in the predicted forced response of the mistuned system.

Modified Modal Domain Analysis (MMDA) [8] is a breakthrough approach for modeling mistuned bladed disks in the presence of simultaneous perturbations in mass and stiffness matrices, i.e. geometric mistuning. It has been shown [8] that MMDA is able to capture the effects of geometric mistuning even in case of large mistuning. The algorithm works by identifying the independent geometric features which result in mistuning using proper orthogonal decomposition (POD) of perturbations in blade geometries. The approximate solution is then obtained by projecting the true solution on the vector space containing modes from the tuned average geometry (nominal modes) and the modes from tuned geometry perturbed along the POD features (non-nominal modes).

Subset of Nominal Modes (SNM) [5] is a reduced order model based on frequency mistuning. SNM and MMDA are similar in the sense that both use nominal modes to form the bases. The difference arises in the use of non-nominal modes in MMDA. Due to the use of additional set of modes, the cost of conducting MMDA analysis is higher as compared to SNM. Due to relatively higher cost of MMDA as compared to SNM it is preferable to identify scenarios where nominal mode solutions may work and where they do not. Hence a comparison study has been undertaken in this paper, where SNM and MMDA both have been applied to a geometrically mistuned system and the results have been presented to recognize the advantages of using the non-nominal modes over just the nominal mode approximation.

SNM and MMDA: Comparison

SNM and MMDA algorithms have been summarized in the Appendix. In this section, both the techniques are applied to a bladed disk with geometric mistuning. The disk considered by Sinha[8] (Figure 1) is considered again. The number of sectors or blades (n) is 24.

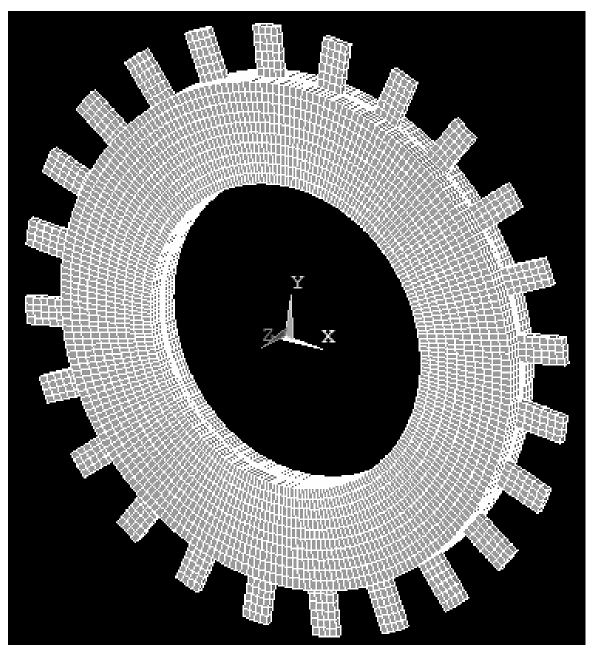


Figure 1: Finite element model of a bladed disk

Mistuning has been introduced by varying the thicknesses of the blades. The thickness of the blade #l is given by:

$$b_l = b_0(1+\xi_l);\ l = 1,2,\cdots,n \qquad (1)$$

where ξ_l is the fractional change in blade thickness. Values of ξ_l are generated by the Matlab routine 'randn' [9] and shown in Figure 2. Mean and standard deviation of this random mistuning pattern are -0.0024 ($\cong 0$) and 0.017, respectively.

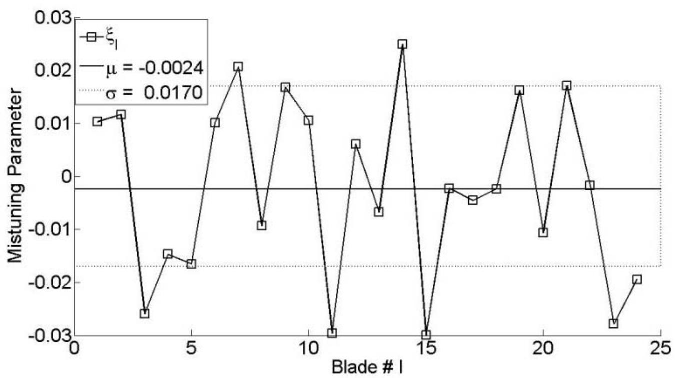


Figure 2: Mistuning pattern for blade thicknesses

A sector analysis of the nominal tuned sector is run and the natural frequencies (Figure 3) and mode shapes are calculated. The first step for frequency mistuning analysis is to calculate the Young's moduli of the equivalent blades so that the natural frequency of the equivalent blade matches the natural frequency of the actual mistuned blade. The Young's moduli for the blades can be calculated using equation A.1. Since for same geometric mistuning (in this case, change of thickness), the ratio of the mistuned and tuned natural frequencies depends upon the modes of excitation (the ratio for bending modes would be different from that of torsional modes), it is essential to identify the range of frequency for modal analysis and then identify the blade mode shapes dominant in that range of interest. Once the dominant blade mode shapes are identified then the equivalent Young's moduli of the blades can be calculated using equation A.1.

Most of the papers dealing with mistuning have employed Young's modulus mistuning in the reference full order finite models and the results from the reduced-order models have matched exactly with the full order models. Feiner and Griffin [7] applied their reduced order model (FMM technique) to a geometrically mistuned system and showed excellent accuracy of the reduced order model, but they dealt with a very small value of geometric mistuning (0.2% standard deviation in the changes in lengths of blades) and an isolated family of modes. Here SNM technique (on which FMM technique is based) is applied to a geometrically mistuned disk with comparatively larger value of geometric mistuning (1.7% standard deviation in thickness changes of the blades) in order to study its accuracy under larger geometric mistuning values.

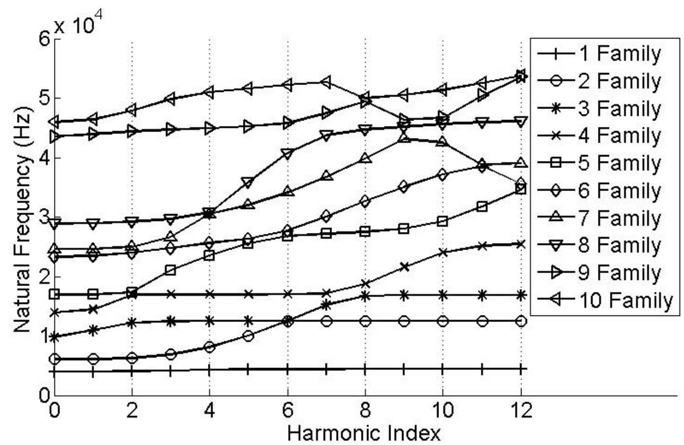
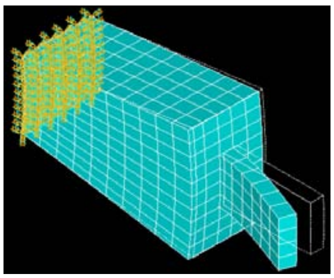
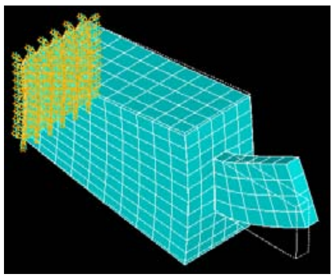


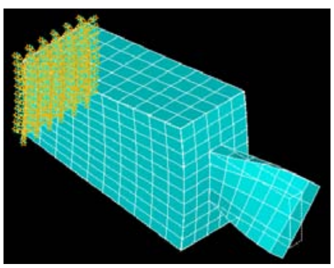
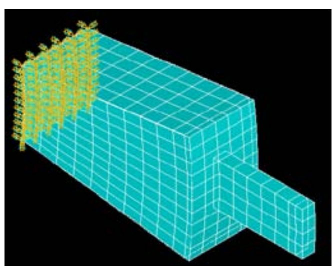
Figure 3: Natural Frequencies vs. Harmonic Index for the first 10 families of the nominal tuned bladed disk

Figure 3 shows the natural frequencies of the nominal bladed disk assembly for different families of modes. As observed from the figure, two types of regions exist; (i) regions with isolated family of modes in a narrow frequency band where the primary energy is stored in the blades; for example, family 1. (ii) regions with overlapping families spanning a larger frequency bands where the primary energy is stored in the disk; for example, families 4, 5 and 6. From frequency mistuning point of view these two regions are different in the sense that for isolated families only a single blade mode shape is present in the region and natural frequency of the mistuned blades for that mode can be used to calculate the equivalent Young's moduli of the blades for frequency mistuning. But in the other region where multiple families overlap, multiple definitions of equivalent frequency mistuning exist depending upon the family of modes used to calculate the equivalent Young's moduli of the blades. Both the cases for the frequency mistuning have been considered in this study.

As shown in Figure 3, the first family of modes is an isolated family of modes. Figure 4a shows the 0 harmonic index sector mode shape of the first family. As observed from the figure, the energy in the sector is primarily stored in first bending (FB) mode of the blade, hence the frequency mistuning based on the first bending mode of the blade clamped at the base is generated to apply SNM analysis in the frequency band associated with the first family of the sector modes. Figure 5a shows that the 1st mode of the blade clamped at the base is the first bending mode. Figure 3 also shows that in frequency band near 22 kHz. families 4, 5 and 6 overlap. The 0 harmonic index sector mode shapes for families 4, 5 and 6 are shown in figure 4 (b), (c) and (d) respectively. The mode shapes for the 4th, 5th and 6th families show that the dominant blade mode shapes for the three families are lateral bending (LB), torsion (T) and elongation (E) respectively.

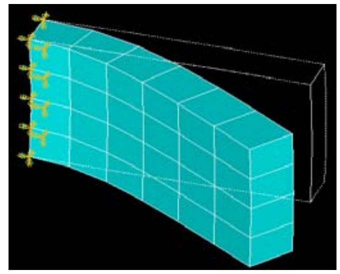
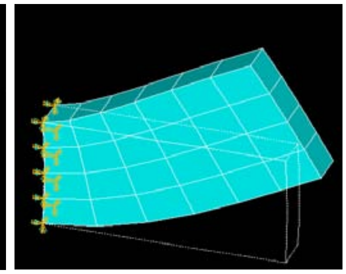
(a) 1st Family (b) 4th Family

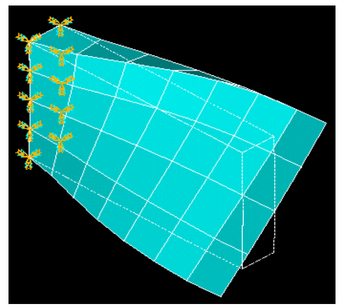
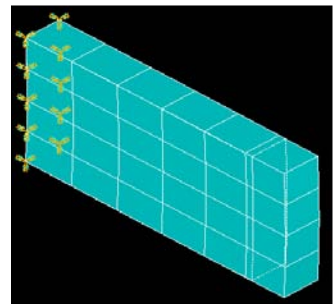
(c) 5th Family (d) 6th Family

Figure 4: 0 Harmonic Index sector mode shapes of the 1st, 4th, 5th and 6th Families

The modal analysis of tuned blade clamped at base shows that the 2nd, 3rd and 5th modes are the lateral bending, torsion and elongation modes respectively (Figure 5). Since the 3 families of blade mode shapes are present in the frequency band around 22 kHz., frequency mistuning is created for each of the 3 cases, i.e. equivalent Young's moduli for lateral bending, torsion and elongation and the results are presented.

(a) 1st mode (b) 2nd mode

(c) 3rd Mode (d) 5th Mode

Figure 5: 1st, 2nd, 3rd and 5th modes for blade clamped at base

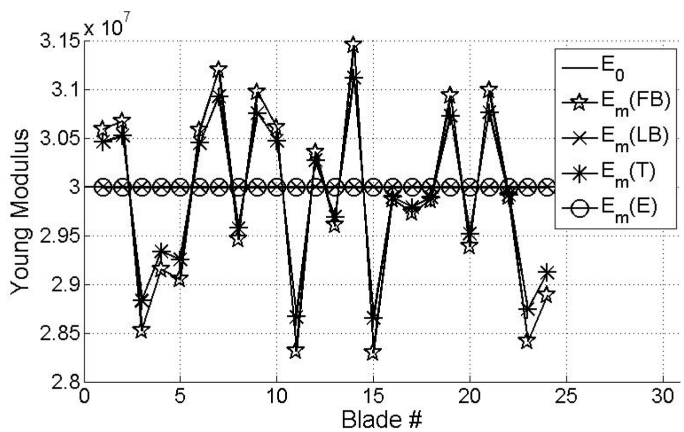


Figure 6: Equivalent Young's moduli (*psi*) of the blades for first bending (FB), lateral bending (LB), torsion (T) and elongation (E) blade mode shapes

Figure 6 shows the equivalent Young's moduli of the blades calculated for first bending, lateral bending, torsion and elongation modes of the blades. As observed from Figure 6, the equivalent Young's moduli of the blades as calculated for the first bending and torsional modes follow the pattern of actual geometric mistuning parameter as plotted in Figure 2. This suggests that the first bending and torsional modes are sensitive to the changes in thicknesses of blades and significant changes in bladed disk assembly mode shapes are expected for the modes dominated by first bending and torsional blade mode shape components. On the other hand, the equivalent Young's moduli of the blades estimated for the lateral bending and elongation modes of the clamped blades have a constant mean value of 3×10^7 psi, which suggests that lateral bending and elongation modes are insensitive to changes in thickness of blades and the assembled bladed disk modes dominated by lateral bending or elongation blade mode component are not expected to change. It should also be noted that since no perturbation in the Young's moduli of blades is observed for frequency mistuning based on lateral bending or elongation modes, such frequency mistuning will not capture any mistuning effects and the natural frequencies and mode shapes estimated from the SNM analysis will match with the natural frequencies and mode shapes of the nominal system.

Finite element models for these cases of frequency mistuning are generated and SNM analyses are performed on the basis of first 240 tuned modes. MMDA analysis is also performed for the mistuned bladed disk assembly for which number of POD features, $np = 1$ and 240 modes are used for both Φ_0 and Φ_1 in equation (A.9). Natural frequencies and mode shapes of the mistuned bladed disk assembly are also generated from finite element analysis of full rotor in ANSYS to compare the accuracy of the two reduced order models.

Figure 7 shows deviations in the first 24 natural frequencies estimated via MMDA, SNM and ANSYS analysis. As observed from the Figure, SNM is unable to capture the deviations in natural frequencies due to geometric mistuning with standard deviation equal to 1.7%.

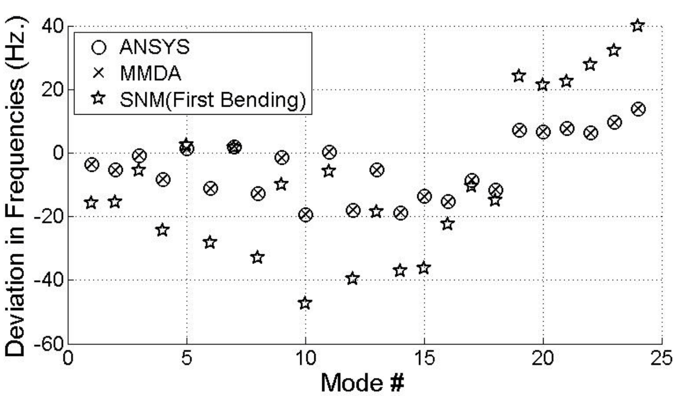
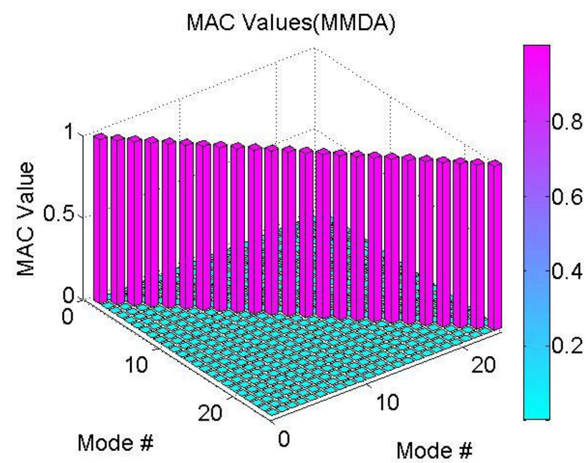
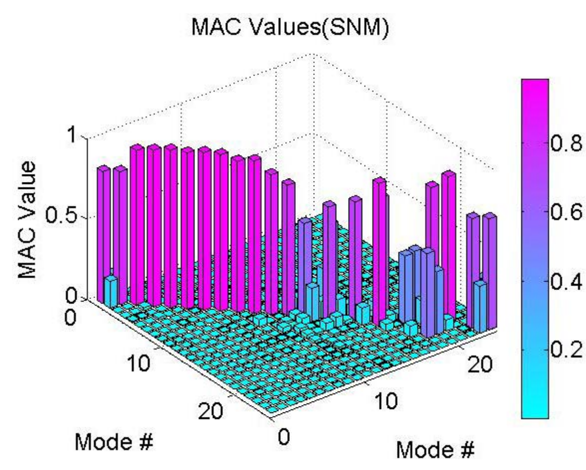


Figure 7: Deviations in frequencies estimated via reduced order models (MMDA and SNM) for the first bending family

Next the mode shapes from the reduced order models (MMDA and SNM) are compared with the mode shapes from the full rotor ANSYS analysis using Modal Assurance Criterion (MAC) [10]. MAC values for the mode shapes estimated via reduced order models are plotted in Figure 8. The values closer to 1 on the diagonal suggest that the mode shapes estimated from the reduced order model are identical to the reference mode shapes (mode shapes from full rotor ANSYS analysis), whereas the values closer to 0 on the diagonal suggest that the estimated mode shapes from the reduced order model are orthogonal to the reference mode shapes.



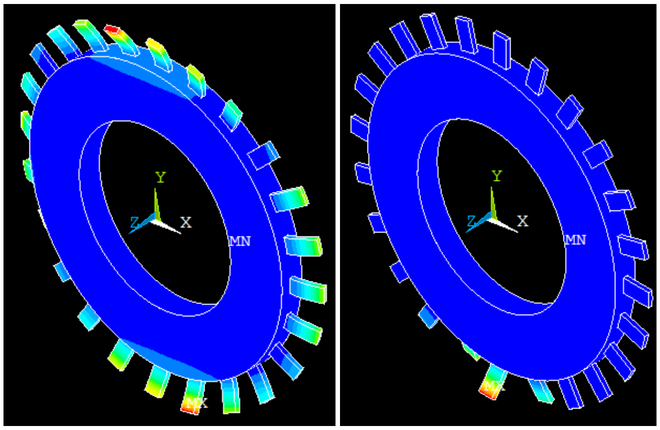
(a) MMDA



(b) SNM (First Bending)

Figure 8: MAC values for the first 24 modes calculated via reduced order models (MMDA and SNM) for the first bending family

The observation of MAC values for mode shapes estimated via MMDA suggests that MMDA is able to capture the mode shapes exactly. On the other hand MAC values for the modes estimated via SNM suggest that the technique is able to capture mode shapes for modes 1-12, but shows large errors in estimated mode shapes for modes 13- 24.



(a) Mode #5 (b) Mode #19

Figure 9: Mode shapes of mistuned bladed disk assembly

A closer look at the mode shapes of the bladed disk assembly shows that the first 12 modes do not show significant mode localization (for example mode #5 in figure 9a) and are hence similar to the modes of the nominal tuned bladed disk assembly. For this reason, nominal mode approximation is sufficient to estimate the first 12 mode shapes of the bladed disk assembly. On the other hand, modes 13-24 show significant mode localization (for example mode #19 in figure 9b) and are different from the mode shapes of the nominal tuned bladed disk assembly. In this case, the nominal mode approximation of the mistuned modes is not sufficient and an additional set of non-nominal modes is required to form a suitable basis for the mistuned mode shapes.

Similar analysis for comparison between SNM and MMDA is also performed for frequency band near 22 kHz. Figure 10 shows the deviations in frequencies estimated via MMDA, SNM and full rotor ANSYS analysis. As observed from the figure, MMDA is able to capture the effects of geometric mistuning exactly whereas errors are observed in the frequency deviation estimates from SNM analyses. The observation of deviations in frequencies in Figure 10 shows large values of frequency deviation for modes 73-89, whereas small deviations for modes 90-110 and then large and small frequency deviations inter-mixed for modes 111 to 120.

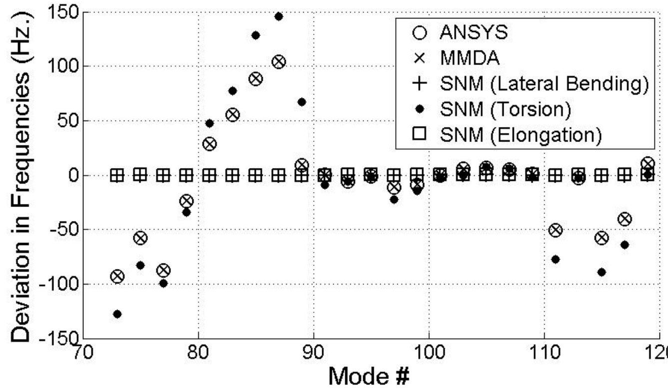


Figure 10: Deviations in frequencies estimated via reduced order models (MMDA and SNM)

A closer look at the mode shapes of the mistuned bladed disk assembly shows that modes 73-89 are blade dominated torsional mode shapes with significant mode localization. Since the torsional mode shapes are sensitive to the changes in thicknesses of the blades, the mode shapes 73-89 of the mistuned bladed disk assembly are significantly different from the mode shapes of the nominal tuned bladed disk assembly, which results in large frequency deviations. On the other hand, for modes 90-110, lateral bending, torsion and elongation modes are all present. A closer look at these mode shapes shows that the torsional mode shapes present in the range are disk dominated with small or no mode localization, hence they are not significantly altered by the mistuning. The elongation and lateral bending mode shapes present in the range are not disk dominated, but since the lateral bending and elongation mode shapes are not sensitive to the changes in the thicknesses of the blades, these mode shapes are also not altered due to mistuning. This results in small or no deviations in frequencies for modes 90-110. For modes 111 to 120, blade dominated 2nd bending modes (7th family) are also present along with disk dominated torsional and elongation modes, all of which do not show significant mode localization, hence are similar to the nominal tuned bladed disk assembly. Therefore for torsional and elongation modes in the range, significant deviation in frequency is not observed. For modes corresponding to 2nd bending in the range (modes 111, 115 and 117) , although the mode shapes are not localized, they are blade dominated and since the natural frequency of the bending mode is sensitive to the thickness of the blade, significant shift in natural frequency is observed for modes corresponding to 2nd bending modes. This phenomenon is similar to what is observed for the 1st bending family in figures 7 and 8, where modes 1-12 are not localized but significant deviation in natural frequencies is

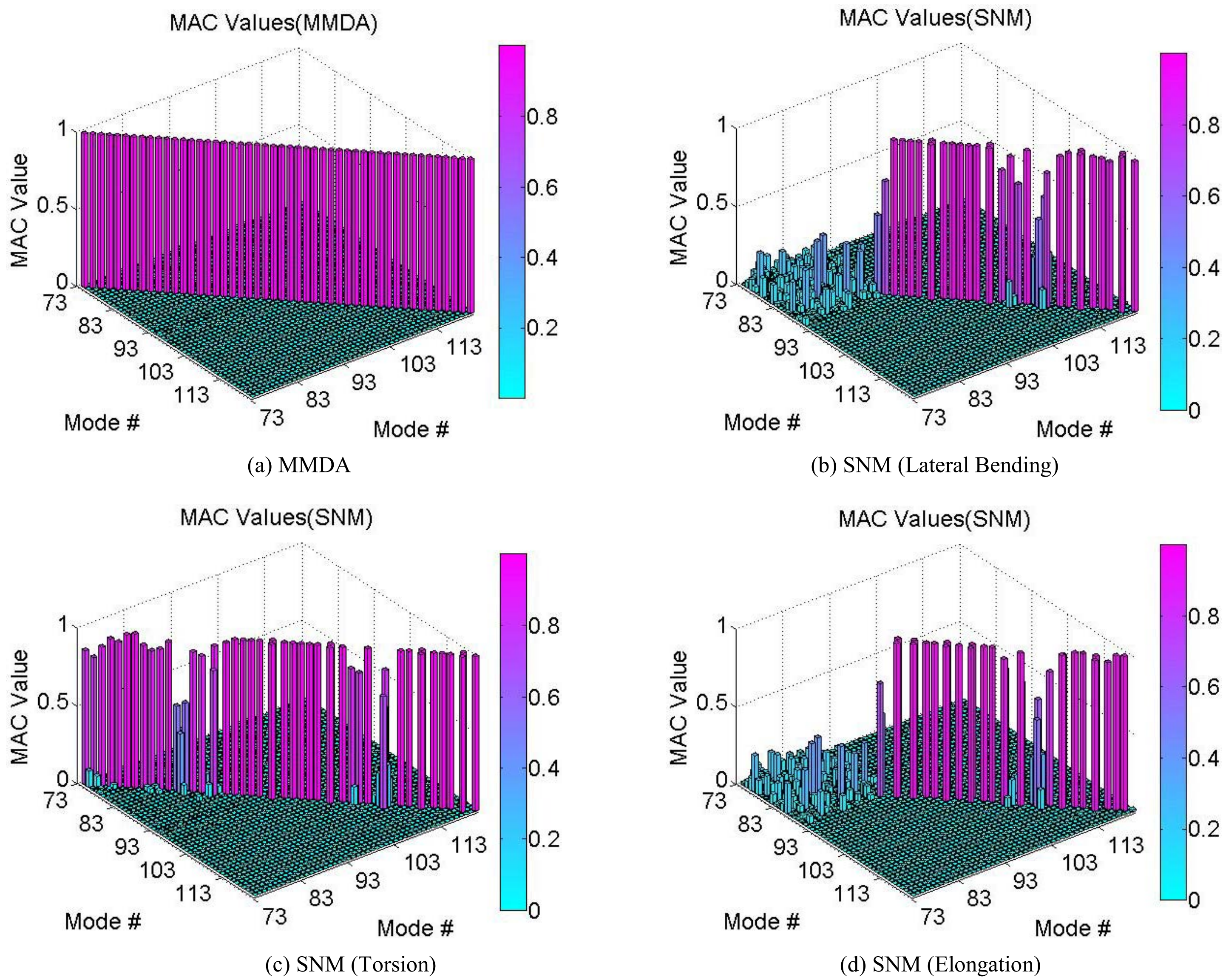


Figure 11: MAC values for modes 73-120 calculated via MMDA, SNM (Lateral Bending), SNM (Torsion) and SNM (Elongation)

observed. This analysis is also confirmed by the MAC values plotted for modes 73-120 for MMDA and frequency mistuning based on lateral bending, torsion and elongation blade modes (Figure 11). As discussed earlier, frequency mistuning based on lateral bending or elongation modes does not capture geometric mistuning and the mode shapes estimated via SNM analysis match with those of the nominal tuned system. Hence MAC values closer to 1 in Figures 11b and 11d suggest that the mistuned mode shapes are similar to the mode shapes of the nominal system, whereas MAC values closer to 0 suggest that the mode shapes are significantly altered from the mode shapes of the nominal system. MAC values in Figure 11a show that MMDA is able to estimate the mistuned modes accurately.

Next the harmonic response of the bladed disk assembly is estimated via each reduced order model and compared with full rotor ANSYS analysis. The differential equations of motion for the bladed disk assembly can be written as:

$$M\ddot{\mathbf{x}} + C\dot{\mathbf{x}} + K\mathbf{x} = \mathbf{f}(t) \tag{2}$$

where M, K, C and $\mathbf{f}(t)$ are mass matrix, stiffness matrix, damping matrix and forcing vector, respectively. Using reduced order modeling transformation (equation (A.3) for SNM, and equation (A.8) for MMDA), the reduced-order equations of motion can be written as:

$$M_r \ddot{\mathbf{y}} + \Phi^H C \Phi \dot{\mathbf{y}} + K_r \mathbf{y} = \Phi^H \mathbf{f}(t) \qquad (3)$$

Equation (3) can be solved by first performing the modal analysis, and then using mode superposition technique to get the harmonic response.

The mistuned blade disk assembly (Figure 1) is excited by a harmonic force corresponding to engine order (EO) 6. Since the harmonic forcing function corresponds to 6th engine order, in order to study the accuracy of harmonic response for the first family of modes, excitation frequencies are chosen to be within ±3 percent of the mean excitation frequency of 4386.3 Hz (4386.3 Hz is the tuned natural frequency corresponding to 6th harmonic index for 1st family). The damping ratio in each mistuned mode is taken to be 0.001. Figure 12 shows the normalized maximum amplitude (*nma*) of the bladed disk assembly, which is defined as the ratio of the maximum amplitude in the bladed disk assembly at a given frequency to the maximum amplitude of the nominal tuned assembly at resonance, i.e.

$$nma = \frac{\|\mathbf{a}\|_\infty}{\max\limits_{\omega} \|\mathbf{a}_t\|_\infty} \qquad (4)$$

where $\mathbf{a}$ and $\mathbf{a}_t$ are amplitudes vectors for mistuned and nominal tuned bladed disk, respectively. An *nma* value greater than 1 indicates that a blade's response is higher than that of the nominal system at resonance. *nma* is also calculated for ANSYS analysis of full (360 degree) model to compare the accuracy of the reduced order models. As observed from Figure 12, MMDA is able to capture the effects of mistuning accurately. This is expected because the mode shapes and natural frequencies estimated via MMDA are exact as shown in Figures 7 and 8. On the other hand, mean excitation frequency of 4386.3 Hz falls near the 15th mode of the mistuned bladed disk assembly. As shown in figure 8, the mode shapes 13-24 estimated via SNM are not accurate, hence harmonic response estimates based on SNM analysis is not expected to be accurate in the frequency band where these modes are excited. This behavior is also observed in Figure 12.

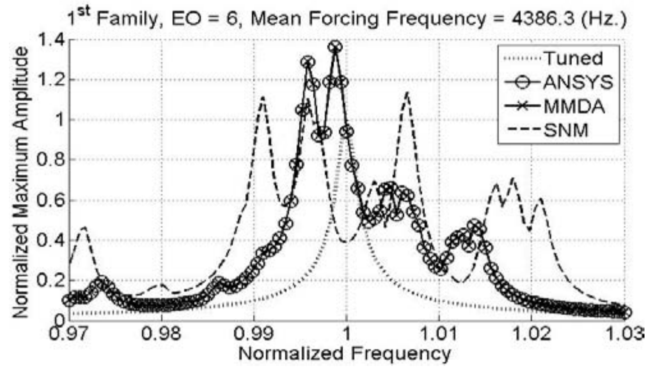


Figure 12: Normalized maximum amplitudes estimated via reduced order models (MMDA and SNM) for the first bending family. Engine order = 6, Mean excitation frequency = 4386.3 Hz

To get the estimates of the worst case scenario, it is important to compute the normalized peak maximum amplitude (*npma*) defined as:

$$npma = \frac{\max\limits_{\omega} \|\mathbf{a}\|_\infty}{\max\limits_{\omega} \|\mathbf{a}_t\|_\infty} = \max_{\omega} nma \qquad (5)$$

The error in *npma* is defined as the difference in *npma* values as estimated via reduced order model and the actual *npma* values estimated via full rotor ANSYS analysis. The error in *npma* values estimated via MMDA is -1.08e-3%, whereas for *npma* values estimated via SNM is -22.42%, which suggests that SNM is not suitable for *npma* estimates for the first bending family.

The frequency deviation analysis in Figure 10 and MAC values in Figure 11 show that the natural frequencies and mode shapes for modes 73-89 are significantly different from the mode shapes of the nominal system, whereas for modes 90-120, deviations in mode shapes are small. In order to study the accuracy of the reduced order models for both the regions of high and low deviations, the system is excited by a harmonic forcing function corresponding to 6th engine order excitation, within ±3 percent of the mean forcing frequencies of 17001.5 Hz and 26788.4 Hz, natural frequencies corresponding to 6th harmonic index of the 4th (lateral bending) and 5th (torsion) family in Figure 4 respectively. The frequency band of 16491.4 Hz to 17511.5 Hz (mean excitation frequency 17001.5102 Hz) excites modes between 73 and 89, whereas frequency band between 25984.8 Hz and 27592.1 Hz excites modes in the range 95-124, which include elongation family modes. The damping ratio in each mistuned mode is again taken to be 0.001. Figures 13 and 14 show the normalized maximum amplitudes (*nma*) estimated via ANSYS analysis of full (360 degree) model and from the different reduced order models. SNM results are only presented for torsional modes of blade vibration. Since the lateral bending and elongation modes are not sensitive to the changes in thicknesses of the blades, SNM results based on these modes are same as responses of the nominal tuned system. Here again it is observed that MMDA estimates match exactly with the full order model ANSYS estimates, whereas *nma* estimates based on SNM analysis differ from the *nma* estimates from the ANSYS analysis. In Figure 13, peak value of *nma* predicted by SNM (torsion) is close to its actual value; however, frequency spectrum is quite different. In Figure 14, actual response is quite close to that of a nominal tuned system because modes (95 – 124) in the frequency band are similar to the mode shapes of the nominal tuned system.

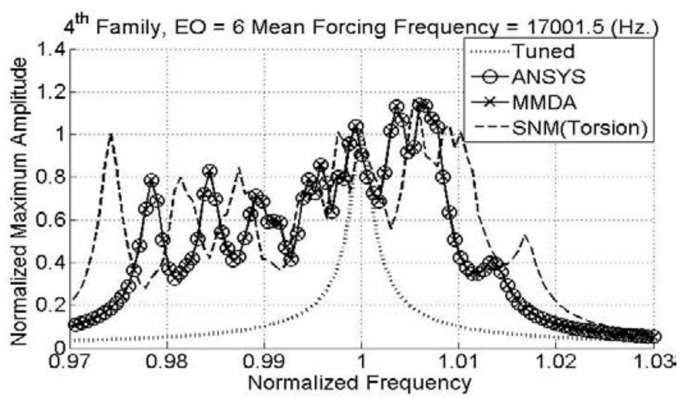


Figure 13: Normalized maximum amplitudes estimated via MMDA and SNM (Torsion) for 4th Family. Engine order = 6, Mean excitation frequency = 17001.5 Hz

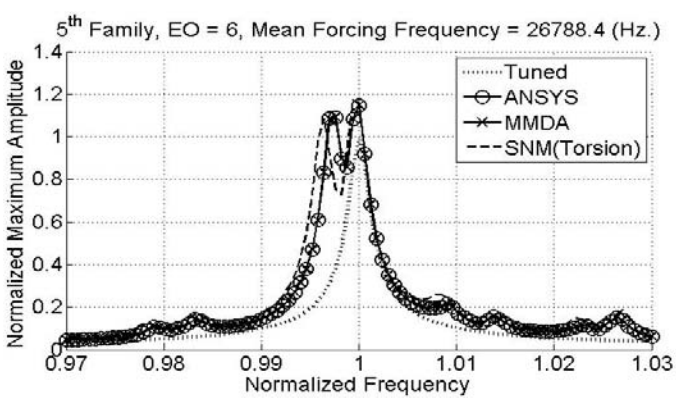


Figure 14: Normalized maximum amplitudes estimated via MMDA and SNM (Torsion) for 5th Family. Engine order = 6, Mean excitation frequency = 26788.4 Hz

The results in the previous section suggest that frequency mistuning fails to capture the effects of geometric mistuning, especially in the frequency bands of interest where the mode shapes are significantly altered by geometric mistuning. Figures 12, 13 and 14 show that harmonic response estimates based on SNM analysis do not match with the actual harmonic response in the frequency spectrum, but a look at the normalized peak maximum amplitude (*npma*) from SNM analysis suggests that the errors in *npma* estimates are small. This observation could be misleading, as it may suggest that SNM analysis can be employed for calculating *npma* values. Here, the amplitude magnifications obtained for this mistuning pattern are small with a maximum amplitude amplification of 1.4. In order to verify SNM's ability to accurately estimate *npma* values even in cases of large amplitude magnification, a worst case mistuning pattern is obtained by using the constrained minimization of a nonlinear objective function in MATLAB (*fmincon*) [9]. The inverse of the peak maximum amplitude in the frequency band of interest is the nonlinear function of the mistuning parameters that is minimized. As it has been shown by Bhartiya and Sinha [11], MMDA analysis based on 2nd order approximations of the perturbations in mass and stiffness matrices provide an accurate reduced order model, which can be used to quickly generate reduced order matrices without any expensive computations; it is used to perform modal analysis of the mistuned system generated for each new set of mistuning parameters. The natural frequencies and mode shapes thus obtained are then used to calculate the peak maximum amplitude at each iteration during the maximization process. The worst case mistuning parameters values are calculated to maximize *npma* for 6th engine order excitation of the first bending family. Excitation frequencies are again chosen to be within ±3 percent of the mean excitation frequency of 4386.3 Hz (4386.3 Hz is the tuned natural frequency corresponding to 6th harmonic index for 1st family). The damping ratio in each mistuned mode is again taken to be 0.001. The mistuning parameters values for the worst case mistuning and the normalized maximum amplitude (*nma*) are plotted in Figures 15 and 16, respectively. SNM analysis is also performed for the system and *nma* values based on SNM analysis are also plotted in Figure 16.

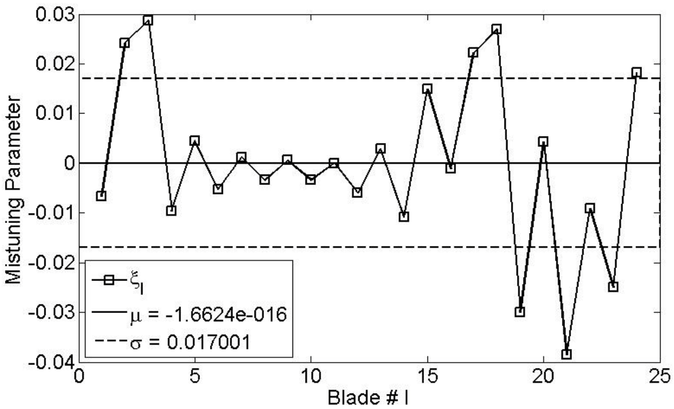


Figure 15: Mistuning pattern for *npma* maximization for first bending family and 6th EO excitation

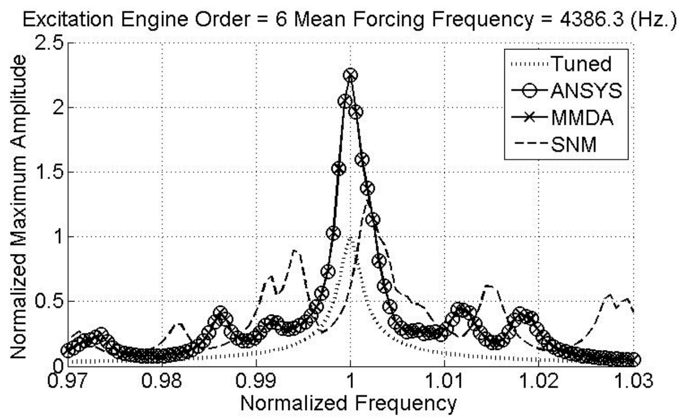


Figure 16: Normalized maximum amplitude estimated via reduced order models (MMDA and SNM) for the worst case mistuning pattern. Engine order = 6, Mean excitation frequency = 4386.3 Hz

Figure 16 shows that *npma* value of 2.23 is obtained for the worst mistuning pattern. It also shows that MMDA provides exact estimates of the maximum amplitudes even this case of large amplitude magnification, whereas the *npma* value estimated via SNM analysis is 1.29 which differs significantly from the true *npma* value. Figure 16 clearly shows that it is possible to get large errors in *npma* values estimated via SNM analysis as well; hence SNM analysis cannot be used to reliably estimate *npma* values.

Conclusion

A study has been performed to compare the results from MMDA [8] and Frequency Mistuning analysis SNM [5] for a case of geometric mistuning, for both the (i) regions of isolated families of modes, and (ii) regions of multiple families overlap. It has been clearly shown that MMDA provides accurate results for all cases where as Frequency Mistuning is unable to provide accurate results for geometric mistuning in general. The ability of frequency mistuning to capture the effects of geometric mistuning depends on the region of interest. For frequency bands where the mode shapes are either disk dominated or blade dominated with no significant mode localization, the mistuned mode shapes are similar to mode shapes of the nominal tuned bladed disk assembly and as a result, errors in SNM results may not be large. In cases where amplitude amplification due to geometric mistuning is not high, the peak maximum amplitude predicted by SNM is comparable to its actual value. However, the SNM frequency spectrum of the response is inaccurate. For a geometric mistuning pattern for which the peak maximum amplitude is high (2.23), the peak maximum amplitude predicted by SNM is 1.29. This result clearly suggests that the SNM can miss the cases of "worst" geometric mistuning pattern.

Acknowledgments

This research has been supported by funds from the Air Force Research Laboratory (AFRL) and GUIde Consortium at Duke University, Durham, NC. The findings, opinions and recommendations expressed therein are those of authors and are not necessarily those of Duke University or the AFRL or the GUIde Consortium Members.

References

[1] Castanier, M. P., Ottarsson, G. and Pierre, C., "A reduced-order modeling technique for mistuned bladed disks," *Journal of Vibration and Acoustics*, vol. 119, no. 3, pp. 439–447, 1997.

[2] Bladh, R., Castanier, M. P. and Pierre, C., "Component-Mode-Based Reduced Order Modeling Techniques for Mistuned Bladed Disks – Part I: Theoretical Models," *Journal of Engineering for Gas Turbines and Power*, vol. 123, no. 1, pp. 89–99, 2001.

[3] Bladh, R., Castanier, M. P. and Pierre, C., "Component-Mode-Based Reduced Order Modeling Techniques for Mistuned Bladed Disks – Part II: Application," *Journal of Engineering for Gas Turbines and Power*, vol. 123, no. 1, pp. 100–108, 2001.

[4] Yang, M. T. and Griffin, J. H., “A Reduced-Order Approach for the Vibration of Mistuned Bladed Disk Assemblies,” *Journal of Engineering for Gas Turbines and Power*, vol. 119, no. 1, pp. 161–167, 1997.

[5] Yang, M. T. and Griffin, J. H., “A reduced-order model of mistuning using a subset of nominal system modes,” *Journal of Engineering for Gas Turbines and Power*, vol. 123, no. 4, pp. 893–900, 2001.

[6] Yang, M. T. and Griffin, J. H., “A normalized modal eigenvalue approach for resolving modal interactions,” *Journal of Engineering for Gas Turbines and Power*, vol. 119, no. 3, pp. 647–650, 1997.

[7] Feiner, D. M. and Griffin, J. H., “A fundamental model of mistuning for a single family of modes,” *Journal of Engineering for Gas Turbines and Power*, vol. 124, no. 4, pp. 597–605, 2002.

[8] Sinha, A., “Reduced-Order model of a bladed rotor with geometric mistuning,” *ASME Journal of Turbomachinery*, Vol. 131, No.3, July 2009, 031007. (Also, GT2007-27276)

[9] MATLAB, 2004, MathWorks, Inc., Natick, MA.

[10] Allemang, R. J., “Modal Assurance Criterion – Twenty Years of Use and Abuse,” *Sound and Vibration*, August, pp. 14 -21, 2003.

[11] Bhartiya, Y. and Sinha, A., “Reduced Order Modeling of a Bladed Rotor with Geometric Mistuning via Estimated Deviations in Mass and Stiffness Matrices”, to be submitted to *ASME International Conference on Multibody Systems, Nonlinear Dynamics and Control (IDETC/MSNDC 2011)*, Washington, D.C., August 2011.

APPENDIX

Mistuning Modeling and SNM

For frequency mistuning the mistuning is simulated by the changes in the Young’s moduli of the blades. The representation of actual mistuning in terms of frequency mistuning (equivalent changes in Young’s moduli of blades) involves the following steps:

1. Determination of natural frequency (f_t) of the blades with average geometry and Young’s modulus E_0 clamped at base (Figure A.1a).
2. Determination of natural frequency (f_m) of mistuned blades with Young’s modulus E_0 clamped at base (Figure A.1b).
3. Calculation of equivalent Young’s modulus for a blade with average geometry such that the natural frequency (f_t^{eq}) of the blade is same as the natural frequency of the mistuned blade (f_m), i.e. $f_t^{eq} = f_m$ (Figure A.1c). The equivalent Young’s modulus can be calculated as:

$$E_m = E_0\left(\frac{f_m}{f_t}\right)^2 \tag{A.1}$$

Thickness = b_0
Natural Frequency = f_t
Young’s Modulus = E_0

Figure A.1a: Tuned blade

Thickness = $b_0(1+\xi_1)$
Natural Frequency = f_m
Young’s Modulus = E_0

Figure A.1b: Mistuned blade

Thickness = b_0
Natural Frequency = f_m

Young’s Modulus = $E_m = E_0\left(\dfrac{f_m}{f_t}\right)^2$

Figure A.1c: Blade for Frequency Mistuning

The finite element model of the bladed disk with the blades modeled as represented in Figure A.1c is used to generate the mass and stiffness matrices of the mistuned bladed disk assembly.

Let $M^{freq} = M_t$ and K^{freq} be the mass and stiffness matrices of the equivalent frequency mistuned bladed disk assembly (Note that the mass matrix of the bladed disk assembly is same as the mass matrix of the assembly with mean geometry and Young's modulus E_0 because the geometry and density of the blades represented in Figures A.1a and A.1c are same).

Then the equations of motion for the system can be written as:

$$M_t \ddot{\mathbf{x}} + K^{freq}\mathbf{x} = 0 \tag{A.2}$$

The idea behind the SNM technique is that the solution **x** can be represented as a weighted sum of the modes of the nominal assembly, i.e.

$$\mathbf{x}(t) = \Phi_0 \mathbf{y}(t) \tag{A.3}$$

where Φ_0 is the set of modes for the nominal tuned assembly.

Substituting equation (A.3) in equation (A.2) and pre-multiplying with Φ_0^H (complex conjugate transpose of Φ_0), the equation of motion can be written as:

$$M_r^{SNM}\ddot{\mathbf{y}} + K_r^{SNM}\mathbf{y} = 0 \tag{A.4}$$

where

$$M_r^{SNM} = \Phi_0^H M_t \Phi_0 \tag{A.5}$$

and $$K_r^{SNM} = \Phi_0^H K^{freq}\Phi_0 \tag{A.6}$$

The eigenvalue problem associated with equation (A.4) can be solved to get the mode shapes and natural frequencies of the mistuned bladed disk assembly.

MMDA

In MMDA [8], the exact mass and stiffness matrices of the mistuned system are used to describe the vibration of the assembly. Also in addition to the set of nominal modes, a set of mode shapes obtained from tuned bladed disk with geometries perturbed along the POD features (non-nominal modes) are also used to form the bases of solution. Let *M* and *K* be the mass and stiffness matrices of the mistuned bladed disk assembly. The equations of motion can be written as:

$$M\ddot{\mathbf{x}} + K\mathbf{x} = 0 \tag{A.7}$$

The solution **x** can be written as:

$$\mathbf{x}(t) = \Phi\mathbf{y}(t) \tag{A.8}$$

where

$$\Phi = [\Phi_0 \quad \Phi_1 \quad . \quad . \quad . \quad \Phi_{np}] \tag{A.9}$$

Φ_0: set of tuned modes of the system with blades having the mean geometry.

Φ_l: set of tuned modes of the system with blades having perturbed geometry along l^{th} POD feature, $l = 1,\ldots,np$.

Then the reduced order model is given by:

$$M_r^{MMDA}\ddot{\mathbf{y}} + K_r^{MMDA}\mathbf{y} = 0 \tag{A.10}$$

where

$$M_r^{MMDA} = \Phi^H M\Phi \tag{A.11}$$

and $$K_r^{MMDA} = \Phi^H K\Phi \tag{A.12}$$

The eigenvalue problem associated with equation (A.10) can be solved to get the mode shapes and natural frequencies of the mistuned bladed disk assembly.