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DETERMINATION OF CRACK INITIATION ON L-1 LP STEAM TURBINE BLADES, PART 2: COMPUTATIONAL ANALYSES

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ABSTRACT

After ten years operation, one L-1 blade of steam turbine with large output power suddenly fell down. All 6 LP rotors (two machines) were checked and many cracks on L-1 blades were found. Due to economic reason, new blades were made with same geometry, but with better material in term of yield limit. Some L-1 stages were made from blades of original material, the rest was made from new material blades. Also the tip-timing measurement was installed on two L-1 stages to monitor blades. The investigation of blade cracks reason started.

After one year of smooth operation there was an inspection. Surprisingly, the cracks were indicated again. The blades from new material had relatively more cracks than the original blades.

This part describes some computational analyses which were done. For example, the modal analysis and forced vibration of mistuned bladed disk and of whole rotor train with attached L-1 blades, CFX and CFD calculations were done.

The measurement of rotor torque, damping, Tip-Timing, experimental modal analysis of blades and rotor train and material test are presented in Part 1 – [7].

Application of both approaches (experimental and computational) rejected some hypotheses and revealed which hypotheses should be analyzed in a deeper way. The unstalled flutter has been identified as the most probable blade failures.

INTRODUCTION

The history of crack detection: one L-1 free-standing blade suddenly failed after ten years of active operating on power station with large output power. Two last stages were damaged. The follow-up inspection (using non-destructive examination called faced array) found many cracks on L-1 blades on all 6 LP rotors (two machines). Due to time and economic reasons, the fastest overhaul was to made geometrical identical blades from better material (from yield limit point of view). Some stages were bladed by original blades, some stages by new blades. The rotor grooves for new blades were egalized. I.e., the grooves were machined more accurately. Simultaneously the tip-timing system was placed on two L-1 stages for possible investigation of the crack initiation reasons.

The laboratory evaluation of the cracking indicated the failure mechanism as high cycle fatigue. The fracture surfaces contained striation corresponding to the number of turbine start-ups. The cracks were situated in the first neck of the fir-tree root (Fig. 1), where the maximum stress from the first and second bending mode is concentrating. Several surface conditions were observed within the hooks that could have contributed to crack initiation, which include machining grooves, fretting, scratches, and possible pitting.

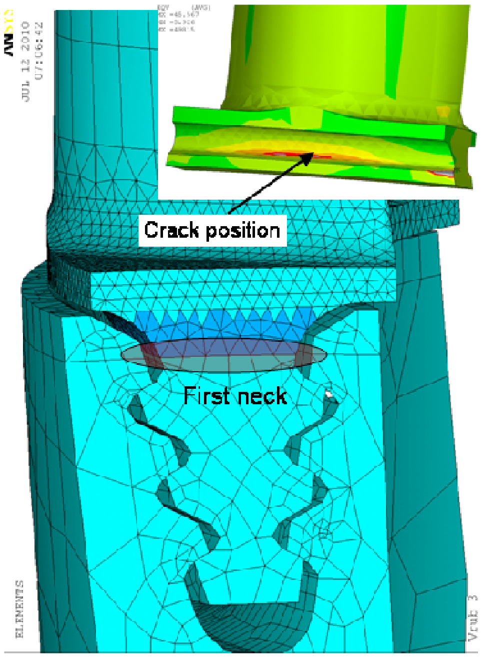

Figure 1. Coarse model of root attachment and position of crack initiation

One of the crack theory was the resonant vibration. The natural frequencies were recalculated (the blades were designed in 1986) and measured in balancing tunnel using a telemetry system. But the natural frequencies were far away from possible rotor excitation frequencies (all multiples of 50 Hz up to 400 Hz).

Also the rotor torsional vibration can be the blade failure causation [10]. There was strain gage telemetry system installed between last LP part and generator to obtain data of torsional vibration.

These measurement and some computational analyses were done throughout the whole year, when the machine worked with new and old blades. After one year, the inspection surprisingly found that blades from new material (T552) had cracks. The original blades (AK1TD) were almost without cracks. The cracked blades were localized in groups. The tip-timing record changes of natural frequencies and excessive blade vibration only during start up. New commercial tip-timing measurement with better resolution was installed. During the second LP rotors repair, several calculations were done. Several computational analyses are described in this paper: modal analysis and forced vibration of mistuned disk, torsional forced vibration of rotor-train with all L-1 blades and flutter analysis. Other probably cause, like fretting, creep or low cyclic fatigue due to start-ups were eliminated due to experimental and computational results.

MISTUNING

One of hypothesis was bladed disk mistuning and that the excessive blade vibrations should be excited either by torsional vibrations of rotor or by nozzle passing frequency. The goal of next subsections is to verify conclusions in [11] – the amplitudes for mistuned model are approximately 20% higher than for tuned model.

Each blade made by Skoda goes through the quality inspection control. The natural frequency, weight and static moment are measured blade by blade. This information was used to calculate correlation between blades with/without crack, their frequency and static moment. There was no correlation found between those parameters and damaged blades (see Part 1). The mentioned relationships do not look like the ones reported in mistuning studies.

Next step was to create the model of a mistuned bladed disk with 90 blades. This model was firstly excited by electromagnetic torque moment measured on the generator. Secondly, the mistuned blisk (bladed disk) was excited by nozzle passing frequency.

Mistuned disk -Torsional excitation

A torsional excitation of the rotor train together with torsional vibrations of the train have also been analyzed in order to confirm or reject these phenomena as the cause of low pressure blade cracks. The torsional excitation of the train, which has been taken into account, is produced by the generator through its electromagnetic torque that torsionally strains the train (experimental results see in Part 1). The measured frequencies involved in the time series of the electromagnetic torque are around 170 Hz (measurement is in Part 1).

The question is, if this torsional moment can excite mode shapes with more nodal diameters (ND) than 0 due to mistuning of blades. This mistuning (due to manufacture tolerances) can really change the natural frequencies and corresponding mode shapes, see below.

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The model of tuned and mistuned blisk was created, full model without cyclic symmetry conditions. At first for clear interpretation, the mistuned model was hardly mistuned. The purple blades on Fig. 5 are tuned, seven (randomly chosen) red blades have the 1.075 higher density, four grey blades have 0.914 lower density. The location was chosen so that the mistuned blades don't make periodic structure. The mistuning can be done by blade elasticity modulus change, but results are similar to blade density change. The boundary conditions of disk correspond to actual state.

The modal analysis of the tuned and mistuned disk was done. First family frequencies of both models are in the graph in Fig. 2 (for mistuned model only frequencies corresponding to original purple blades). The mode shapes of mistuned model are not diametrically different for original and changed blades, see Fig. 3. On the left side, there are mode shapes, where the majority of normal blades vibrates with 0 ND (left up) and with 1 ND

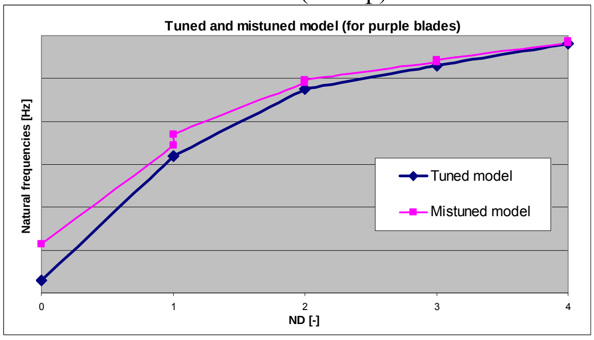

Figure 2. First family natural frequencies of tuned and mistuned model

(left down). On the right side, there are modes shapes corresponding to mass-different blades vibration.

These tuned and mistuned models were harmonically excited on the outer disk radius by torsional moment amplitude M = 225 kN.m. This moment is much higher than real measured axial torque, see Part 1. The excitation frequency was in range from 170 to 190 Hz with step 0.05 Hz.

The forced vibration of tuned bladed disk is displayed on Fig. 4. The transparent blue planes express the computed natural frequency for 0 - 2

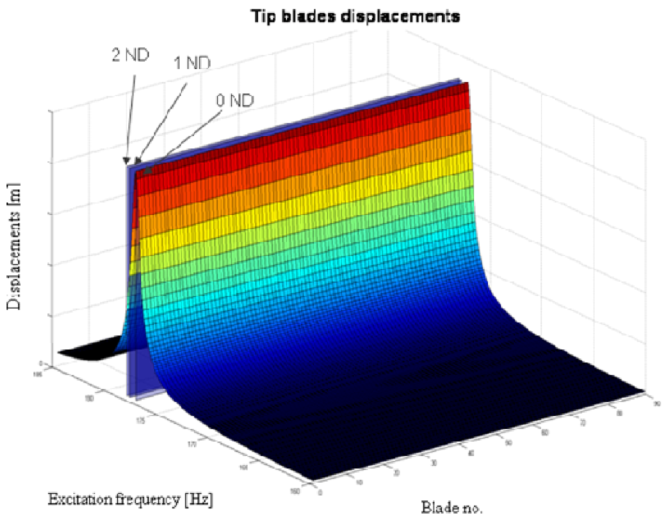

Figure 4. Tip blades amplitudes: tuned blisk forced vibration by torsional harmonic moment

nodal diameters (ND) depicted in Fig. 2 – blue line. The torsional moment can excite only 0 ND for tuned model as is mentioned in [1]-[4].

For mistuned bladed disk, the situation with nodal diameters is more complicated, but the results given in above mentioned references are also valid here. The majority of blades (normal weight) can be excited by torsional moment only at 0 ND, see Fig. 5. Other nodal diameters of these tuned blades are not excited by disk torsion vibrations. Planes labeled as 1a ND, 1b ND, 2a ND, 2b ND correspond to mistuning blisk natural frequencies (Fig. 2 – purple line). The mistuned blades (both

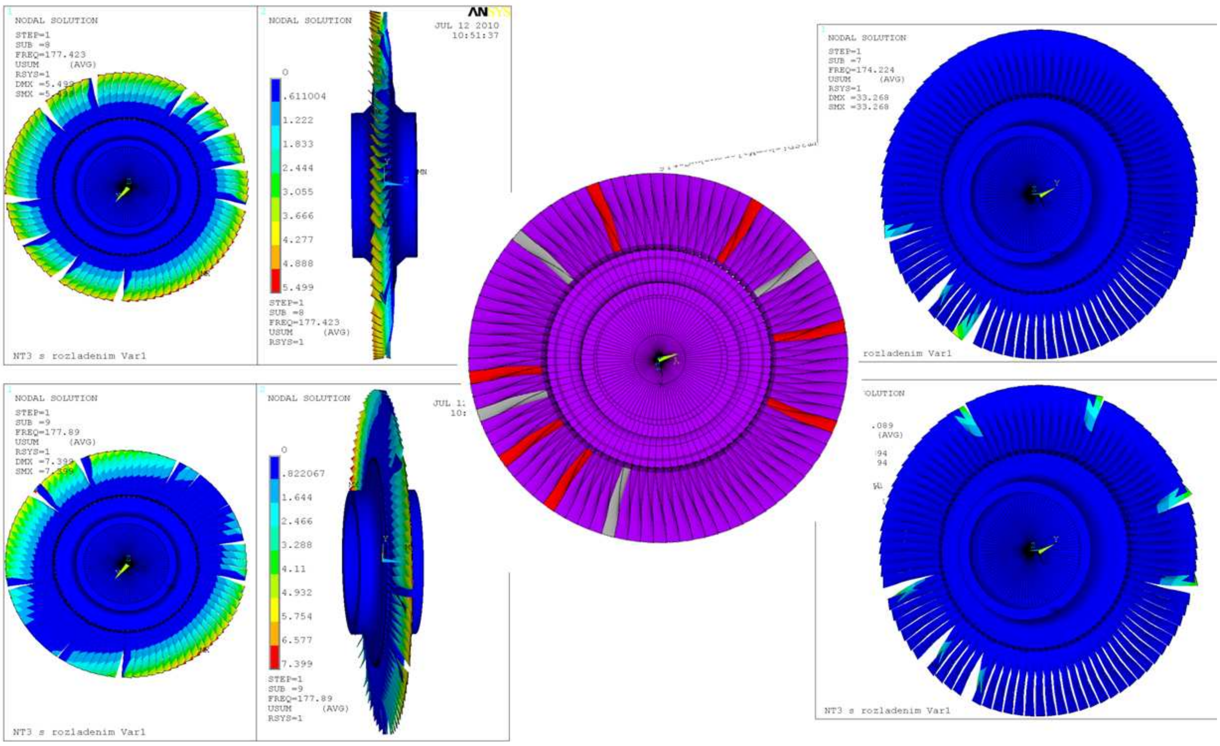

Figure 3. The mistuned model (in centre) and some mode shapes

heavier and lighter) have on these frequencies minimal amplitudes. The heavier blades vibrate with same phase delay and with same amplitude on lower frequency corresponding to their natural frequency, i.e. the forced vibration can be described as “0 ND”. Other ND diameters were not excited by torsional moment. The same situation occurs for lighter blades. The maximal amplitude of this mistuned model is 13% higher than for tuned model.

The results for this subsection:

- Numerical calculation of mistuned bladed disc (full model) results in natural frequencies splitting (for duplex mode shapes).
- The torsional moment really can excite only 0 nodal diameter of majority same blades. That corresponds to theory.
- The torsional moment excites same mistuned blades in shape which can be described as 0 nodal diameter.

The real measured torsional moment on generator was 85times lower than computed. When the computed amplitudes of blades vibration are recalculated, the corresponding stresses in root neck are too small to invoke HCF.

Mistuned disk - forced vibration from nozzles

New model of mistuned bladed disk was done for nozzle excitation. This model should verify that mistuned blisk vibrates dominantly with ND determined from SAFE diagram. This model created in MATLAB is simplified drawn on Fig. 7. Periodic structure is formed by N segments. Each segment $M1_i$ is connected by $k3_i$ to frame. The disk stiffness is expressed by $k1_i$. The blade mistuning by stiffnesses $k2_i$ and masses $m2_i$ (i=1, 2,… 90 blade) simulates the real measured mistuned blisk. This mistuning has been detected by tip-timing, see Part 1 and Fig. 6. The number of stator nozzles n_s is 64. The excited node diameter should be then 90- n_s = 26.

The harmonic analysis was done with

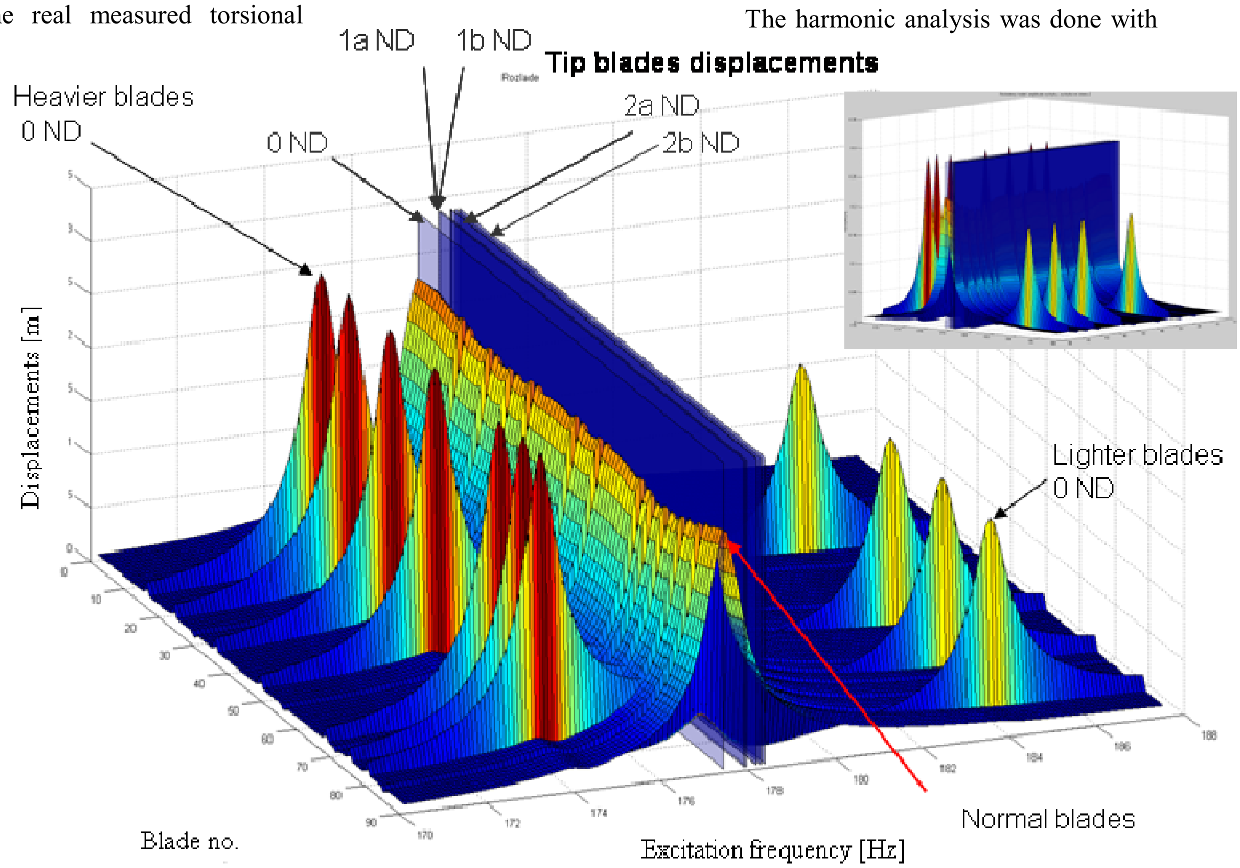

Figure 5. Tip blades amplitude: mistuned blisk forced vibration by torsional harmonic moment

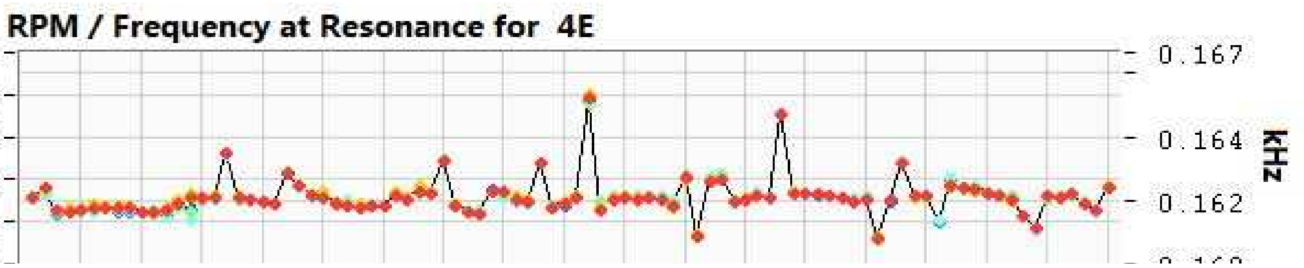

Figure 6. Real bladed disk mistuning meassured by tip-timing

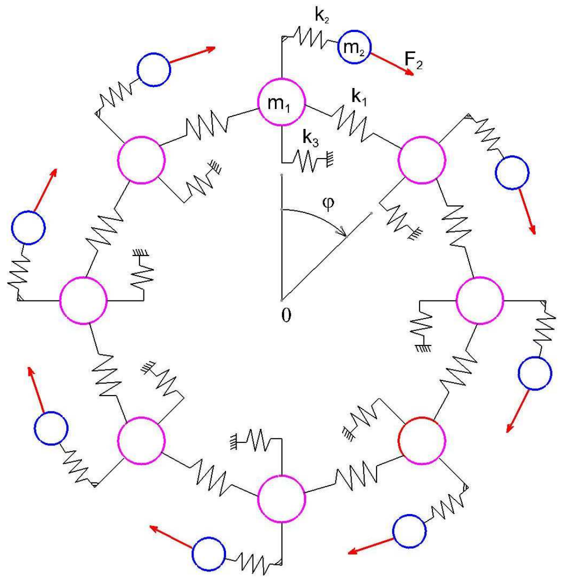

Figure 7. Model in MATLAB for real mistuned disk

excitation from n_s nozzles. The amplitude-frequency characteristic is depicted on Fig. 8 for all blades. The amplitudes for mistuned model are 12% higher than for tuned model. That corresponds to [11]. The forced vibration modes were analysed by Fourier decomposition to find out, which nodal diameters are present in forced response. 64 nozzles excited dominantly 26 nodal diameters, see Fig. 9.

Because amplitudes of mistuned disk were to low to invoke HCF, the hypothesis of mistuned blades as the crack reason was rejected.

MODAL AND FORCED VIBRATION ANALYSES OF ROTOR TRAIN WITH FULL MODEL OF L-1 BLADES

One hypothesis was also the torsional vibration of rotor train (torsional moment from generator) which can invoke the blade kinematic excitation (see Fig. 10). The torsional vibration of rotor is transmitted by disc torsional vibration up to blade root. That invokes blades movement. This blades jerking can cause additional bending stress.

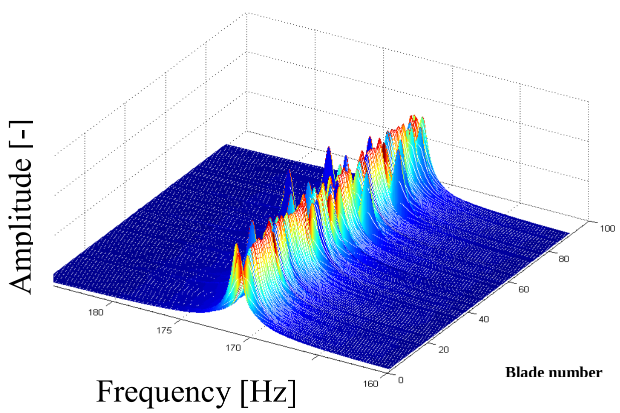

Figure 8. Amplitudo-frequency characteristics for real mistuned blisk model

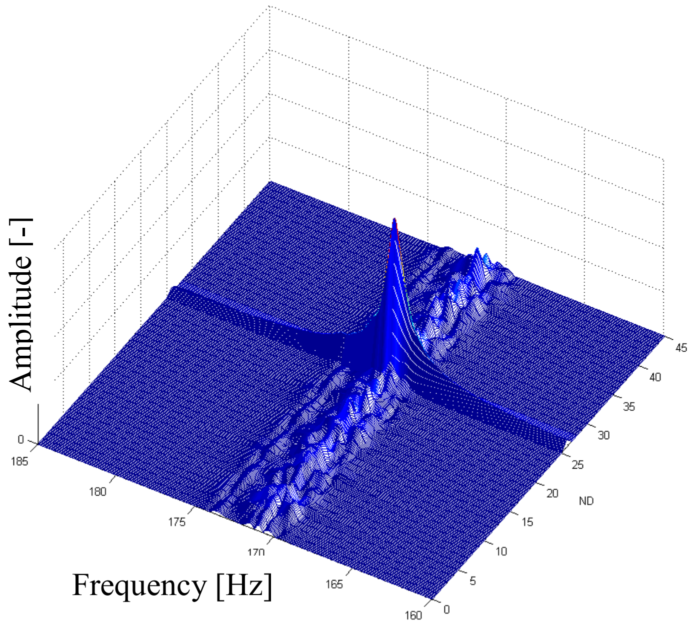

Figure 9. Fourier decomposition of harmonic forced response (mistuned blisk)

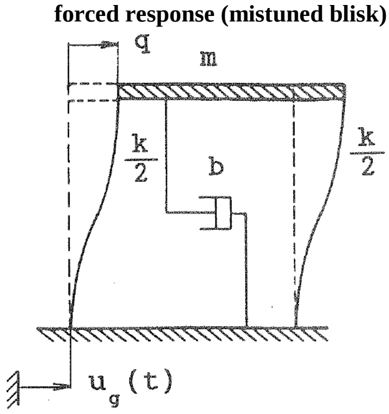

Figure 10. Scheme of kinematic excitation – movement of the frame (rotor shaft)

The worst situation should be when the natural frequency of bladed disk is same as the natural frequency of .whole rotor train. Therefore the model of rotor-train with all stages with L-1 blade was created and the modal and forced vibration analyses were done.

The purpose of this part is to calculate the forced vibration of blades placed on rotor train. The rotor is excited by generator torsional moment.

The model of rotor train involves:

- HP – LP – LP – LP – Generator – Exciter of generator,
- elastic modulus change of individual rotor parts due to different temperature and different material,
- centrifugal load 3000 rpm,
- influence of blades and disks masses, inertia moments etc.,
- torsional moment M= 10^3 N applied on generator,
- rotor is modeled by beam elements,

- DOFs of rotor are only in rotation around axial axis,
- 3D model of real L-1 blade on all 3 LP rotors, 6 stages
- 3 translation DOF in every blade node,
- response is investigated by harmonic analysis,
- relative damping is 0.2% (measured in Part 1),
- root contact areas are coupled with control rotor nodes (see below).

The rotor train is modeled by beam elements. Only torsion displacements are assumed. Each rotor-beam element has its own elastic modulus due to different temperature and material. The blade root contact areas have coupled circumferential displacements with control node of the corresponding rotor node, other displacements are fixed. This coupling ensures that the torsional vibrations of rotor are transformed as kinematic movement of blade root and vice versa. The full model of rotor train with all L-1 blades (6 stages→540 blades) is too computational consuming – see Fig. 11 in centre. But the torsional excitation can excite only 0 ND on the bladed disk as was above demonstrated.

Therefore the below described simplification was done. The blading of one L-1 stage is concentrated in one representative blade, which has same geometry and same FE model, but its mass is n-times higher, i.e. n=90 for 90 blades of one stage. The elastic modulus must be also n-time higher because of identical natural frequencies of blades and because of real forced displacements. This simplification ensured same physical properties of the original blading and of the representative blade (Fig. 11 – left up). This simplification is valid for torsional shapes of rotor train. The initial model with 18 millions DOF is decreased to 211 thousands DOF. The modal analysis of this size can be real-time solved as well as harmonic response analysis for wide excitation frequency range. Two models of rotor train were solved. Firstly, the real model with correct elastic modulus of rotor parts was calculated. Secondly, the elastic module of rotor parts were changed to invoke the worst situation in term of natural frequencies. That means, that the natural frequency of rotor train is change to be same as natural frequency of bladed disk corresponding to 0 ND.

Original model of rotor train

This model is original in meaning of rotor elastic modulus. The scaled frequencies of this model are written in Tab. 1 (the scaling is done for real natural frequency to hide). Red colored natural

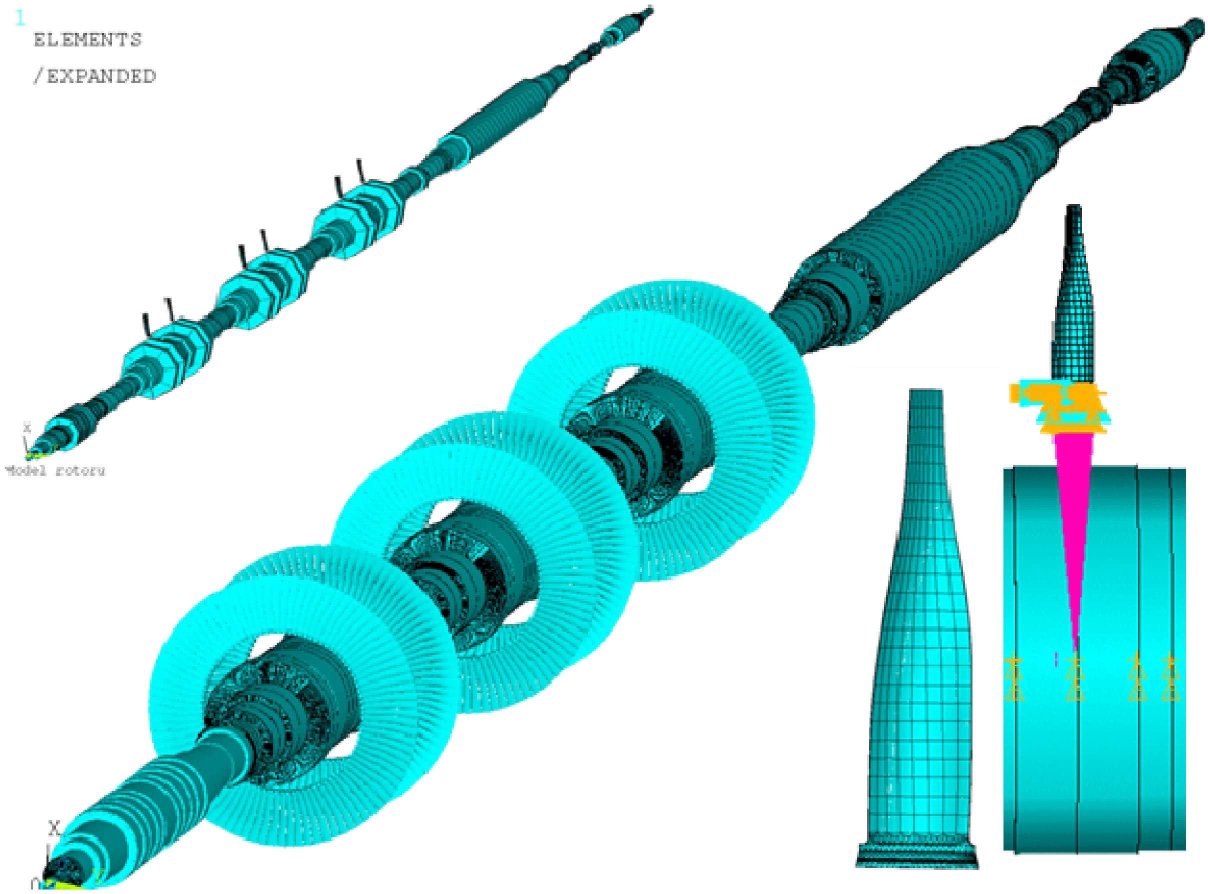

Figure 11. In centre - full model of rotor train with all L-1 blades; left up – simplified model; right down - 3D model of L-1 representative blade and blade coupling to rotor node

Natural frequencies of rotor train with L-1 blades [Hz]
10.0
18.9
21.7
29.3
36.6
122.7
148.4
151.1
154.3
155.3
174.3
176.3
177.4
177.3
180.4
183.1
188.8
190.7
250.3

Table 1. Scaled natural frequencies of rotor train with modeled L-1 blades

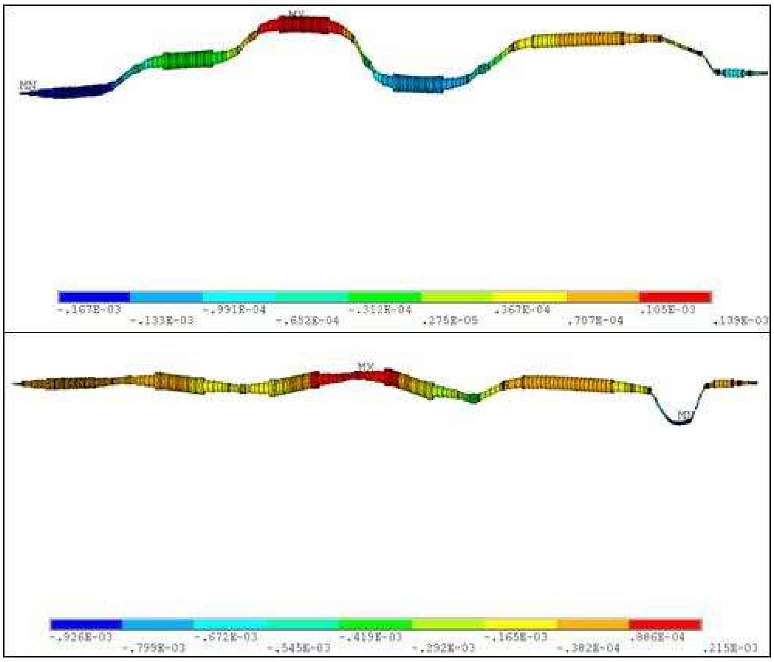

Figure 12. Mode shapes of rotor train with modeled L-1 blades - torsional displacements

frequencies correspond to mode shapes, where L-1 bladed disks vibrate with higher amplitudes. Moreover, these frequencies are not presented in the model without L-1 models. The black eigenfrequecies correspond to frequencies of rotor train without detailed modeled blades. In Fig. 12, some mode shapes of rotor train are displayed (torsional displacement).

The measurement on power station determined the torsional vibration between generator and last LP rotor – the frequency of this vibration was 166.8 Hz, the amplitude of transmitted torsional moment was measured by strain gauges – 2.63 kN.m, see Fig. 13. The vibration frequency 169.5-170.5 Hz of L-1 blades was observed from tip-timing on two different stages. The frequency difference between torsional excitation and natural blade frequency is 2.7 Hz.

The harmonic analysis was done for torsional moment M=1 kN.m. The smaller moment for analysis is in final evaluation multiply by 2.63 because of linear type of solved task. The excitation frequency range is from 150 Hz to 190 Hz with step 0.05 Hz. The axial amplitudes of blades tips are drawn below in Fig. 12. The label UZ_1*i* means axial amplitude of *i*-th L-1 stage, numbered from HP rotor.

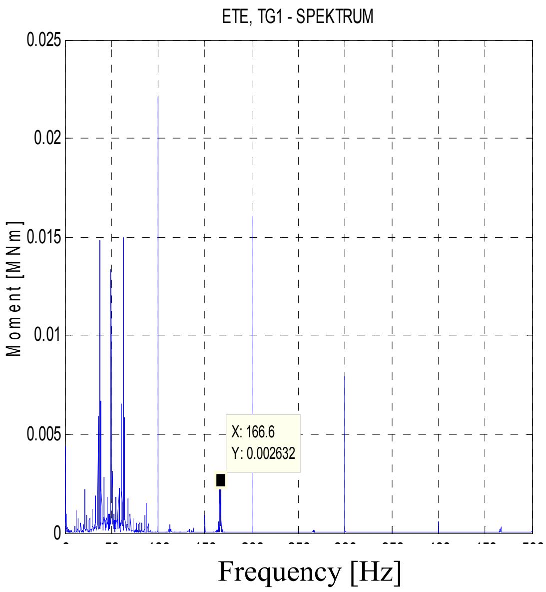

Figure 13. Measured torsional moment on generator for maximal power output

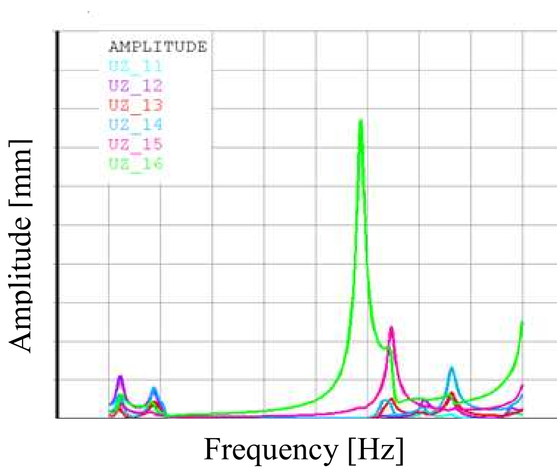

Figure 14. Axial amplitudes of L-1 blades tips - original model

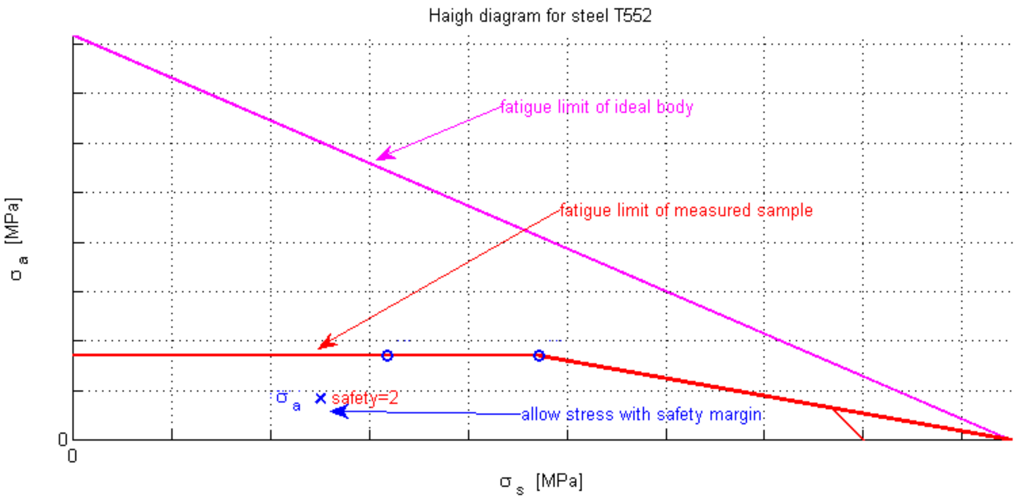

Figure 15. Haigh diagram of T552

The radial stress for maximal blade amplitude is in root neck 2.3 MPa, torsional moment was 1 kNm. That results in real dynamic component of radial stress 2.3*2.6=5.98 MPa. This value has big safety factor in Haigh diagram (Fig. 15). Detailed describing of Haigh diagram for new blade material is in [7].

Retuned model

The frequency difference between rotor-train with and without detailed modeled blade stages is up to 20 Hz (Tab. 2 – first column - frequency 155 and 174 Hz,). It means that mode shapes corresponding to excessive vibration of L-1 blades are frequency far from torsional mode shapes of whole rotor train. Due to change of elastic modulus of rotor (or rotor density change), the mentioned difference was deleted, i.e. the natural frequencies get closer, see Tab. 2, where some examples of retuned system are presented.

E_new/E	1	1.3	1.32	1.325
Eigenfrequency [Hz]	122.65	139.82	140.89	141.16
	148.41	162.59	163.19	163.33
	151.11	164	164.51	164.63
	154.33	165.76	166.14	166.24
	155.25	175.59	**176.31**	177.5
	174.29	176.51	**176.55**	176.36
	176.26	176.59	**176.72**	176.55
	177.44	176.74	177.22	176.73
	177.26	186.22	187.06	187.27
	180.44	190.43	191.31	191.53
	183.1	195.75	196.76	197.05
	188.76	213	214.87	215.25
	190.67	214	215.63	216.03

Table 2. Model retuning by elastic modulus change

The harmonic analysis described for original model of rotor train has been done for this retuned model. The axial amplitudes of blades displacements are plotted in Fig. 16.

In fact, the obtained amplitudes of blade tip displacements are lower than in previous original

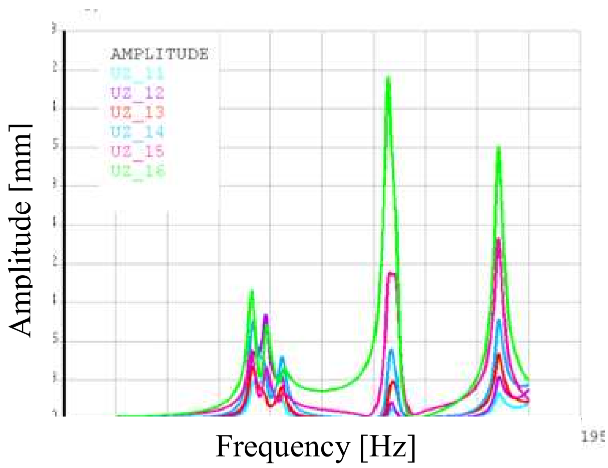

Figure 16. Axial amplitudes of L-1 blades tips – retuned model

model. The maximal stresses from kinematic excitation for this retuned model is only 2.6 MPa for torsional moment M=2.3 kNm. The presumption was that the second model should have higher amplitudes, but it wasn't verified. Three mode shapes, which are frequency-close (Tab. 2 – next to last column), create together a vibration shape which is not excited by moment placed on generator.

The forced vibration of whole rotor train for one excitation frequencies is in Fig. 17. The result from this part is that the measured torsional moment and corresponding rotor torsional vibration can not be the blade failure causation.

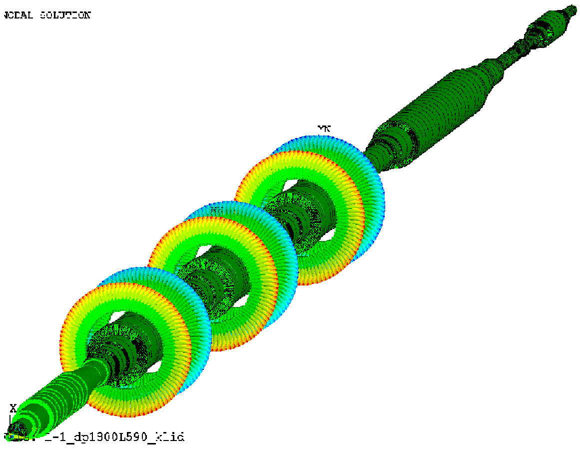

Figure 17. Axial amplitudes of whole rotor train with L-1 blades

FLUTTER ANALYSIS

Flutter is a state of self-excited oscillation of a lifting surface caused by the aerodynamic forces of the surrounding flow. For flutter to occur, the airfoil must move periodically (usually in one of its natural modes of oscillation), with the resulting oscillating pressure field of the surrounding fluid supplying energy into the airfoil. This energy corresponds to aerodynamic damping of the fluid-blade system; if this energy supply is positive, the aerodynamic damping is negative and vice versa. Negative aerodynamic damping indicates instability of the blade with respect to flutter if mechanical damping is neglected. Flutter conditions may lead to high cycle fatigue and subsequent blade failure, in particular for thin and highly loaded blades. In steam turbines, the long low-pressure L-0 and L-1 stage buckets are potentially vulnerable to this type of self-excited oscillation.

The measured blade vibrations are small for low output but for higher output power the blade vibrations grow up. The blade vibrations, respective stress in root neck is mostly too small to induce HCF. But for short, random intervals the excessive vibrations occur for groups of blades. That corresponds to unstall flutter.

CFD Analysis

The freestanding, unshrouded low-pressure L-1 stage buckets of a large steam turbine were investigated for susceptibility to flutter by means of a three-dimensional, transient, RANS finite volume numerical analysis. The calculation was performed in the commercially available code ANSYS CFX 12.1 [9] using the methodology described by Mathias et al. [12]. The computational domain spanned over one half of the tuned bladed disc assembly (180 degrees, 45 blade passages); this setting allowed computing all cases for which the nodal diameter n is an even number. A coarse mesh of approximately 20,000 nodes per passage was used. The mesh around each blade moved periodically according to the first bending mode shape as determined by modal analysis of the blade. The deformation was prescribed for all blades with an inter-blade phase angle defined by the nodal diameter. The nodal diameter and the inter-blade

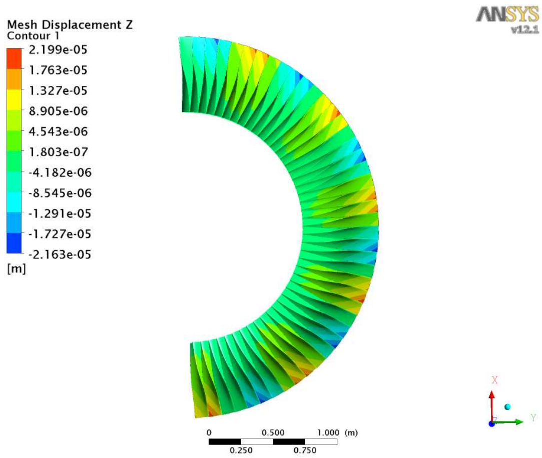

Figure 19. The modeled blades with mesh displacement in the axial (Z) direction at one point in time, nodal diameter n = -12

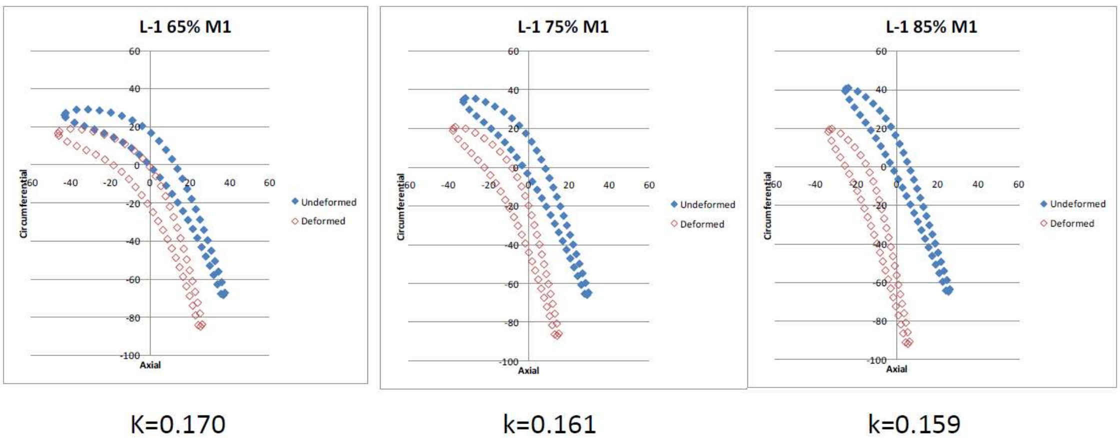

Figure 18. Left side - pitching axis of the first mode shape in 65, 75 and 85 % of blade length

phase angle varied in the individual calculations. Fourteen runs were performed with nodal diameter set to -44, -32, -30, -28, -24, -20, -16, -12, -8, 0, +10, +16, +30 and +44. An example of the blade displacements at one point in time is shown in Fig. 11.

The boundary conditions of inlet total pressure, total temperature and velocity angle and exit static pressure represented the blade row operating at full load. With pressure ratio approximately 0.35, the mass flow rate through the blade row was 165 $kg.s^{-1}$.

Based on the time-dependent pressure and displacement data obtained from the calculation for each node on the blade surface, the work W transferred from the flow to the blade per one cycle of the blade motion was determined as

$$W = \sum_{steps} \sum_{faces} p\vec{A} \cdot \vec{x},$$

where p is the pressure on a face on the blade surface, $\vec{A}$ is the area normal vector of the face, $\vec{x}$ is the relative displacement vector in one time step and the summation takes place over all faces and all twenty-four time steps of a blade motion cycle. This work was evaluated for each of the 45 blades and over six full blade motion cycles. The results were processed to obtain the average value from all blades and cycles in consideration together with the respective standard deviation. The logarithmic decrement □, which represents the stability of the blade-fluid system in the absence of mechanical damping, was calculated as

$$\delta = -\frac{W}{2E},$$

where E is the maximum kinetic energy of the mode shape.

The results of the analysis are summarized in Fig. 20. The dependence of logarithmic decrement on nodal diameter indicates that the system is close to stability limit over a range of negative nodal diameters approximately from $n = -30$ to $n = -20$. However, the average aerodynamic damping remains positive for all nodal diameters examined. A more detailed CFD analysis including a calculation with a refined grid is necessary to confirm the instability.

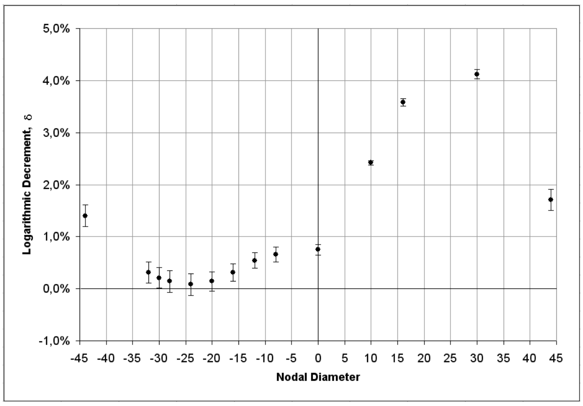

Figure 20. Logarithmic decrement plotted against the nodal diameter. The error bars correspond to one standard deviation of the data in each direction

SOLUTION

The flutter is conditioned not only by the aerodynamic forces of working medium but also by the natural frequencies and mode shapes of blades. The LP rotors should work up to year 2015 when new LP rotors with higher efficiency will be delivered. Because the new blades have crack after one year operation, some change have to be done.

The blade can be frequency retuned by mass reduction. The blade shortening was not accepted due to efficiency decreasing as well as unknown steam stream behind L-1 stage.

The final solution is the change on every other blade (Fig. 21) so that the frequency change increases about 5 Hz. Therefore, the flutter depending on original blade natural frequency can not be evolved. The aerodynamic coupling and damping are changed too [8]. The steam stream behind L-1 stage does not change a lot, and the influence on L-0 is minimal.

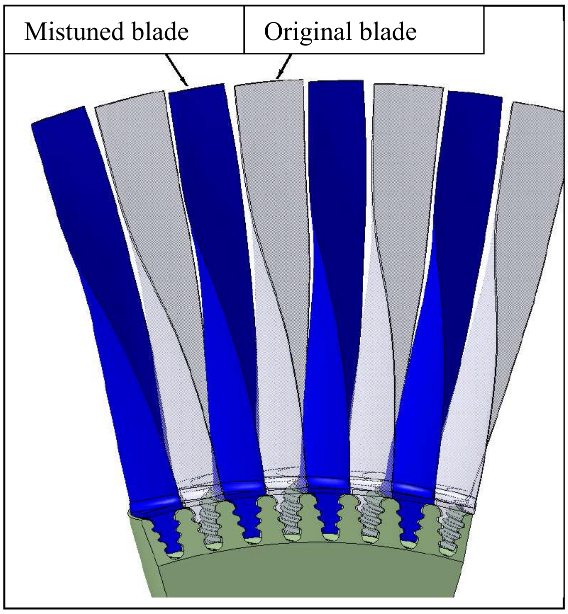

Figure 21. Solution – mistuning more than 5 Hz of every other blade

CONCLUSION

Due to repeated failures of L-1 blade of steam turbine with large power output, the different possible reasons of root cracks were investigated.

At first, the mistuning effect was rejected because of numerical results for real mistuned bladed disk (torsional disk excitation and nozzle passing frequency were calculated). Moreover, the cracks have been found on blades having different values of the first bending mode natural frequencies.

At second, the forced vibration of whole rotor train with modeled L-1 blades was done; the actuating moment was placed on generator. This theory was also rejected because of low stress amplitudes, which were excited.

Finally, the cause of the blade cracks seems to be the unstalled flutter, which is typical for a high flow rate in the area of low pressure parts of steam turbines. The possibility of this type of flutter has been proved computationally indicated by a CFD analysis and its presence has been confirmed by the tip-timing measurement as well. It has also been confirmed by the tip-timing measurement that cracked blades are gathered to groups. The largest vibration amplitudes of the blades have been observed along fast output power changes. Vibration amplitudes of the blades can be described by the Gaussian probability distribution with a pretty high significance level (identified in virtue of the Lilliefors goodness of fit test). This means that they have a character of random variable, which implies that an excitation force cannot have any form of periodic wave. The conclusion is that the system of the bladed disc is found on stability margin due to aero-dynamical forces which cause the system being unstable (negative damping). This phenomenon is known as the unstalled flutter.

The reason of blade cracks is unstalled flutter, which is typical for high flow rate of last LP stages. The CFX analysis and tip-timing measurement confirm the presence of flutter. Tip-timing confirms blade vibration (corresponding to cracks distribution) separated into groups. The behaviour of blades with large amplitude can be described like oscillation. The conclusion is that these blades are on flutter stability margin due to aero-dynamical forces which cause unstable damping and other influences are too small to invoke HCF. The flutter conditions will be avoided due to mistuning of every other blade.

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