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FORCED RESPONSE OF BLADED DISKS WITH DAMPING MISTUNING

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ABSTRACT

The effect of mistuning on the vibration of bladed disks has been extensively studied in the past 30 years. Most of these analysis typically cover the case of small variations of the elastic characteristics (mass and stiffness) of the blades. In this work we study the not so common case of the forced response of a stable rotor with damping mistuning.The Asymptotic Mistuning Model (AMM) is used to analyze this problem. The AMM methodology provides a simplified model that describes the effect of blade to blade damping variation, and gives precise information on the underlying mechanisms involved in the action of damping mistuning.

INTRODUCTION

Turbomachinery bladed disks are designed to be cyclic structures, that is, to have all its sectors perfectly identical. In practice, however, this is obviously not the case, and there are small unavoidable differences between sectors due to the tolerances in the manufacturing and assembling processes, and to the use wear. These small imperfections are referred to as "mistuning", and their effect on the dynamic response of the rotor constitutes a very important subject since they can give rise to considerably higher vibration levels and, therefore, higher risk of High Cycle Fatigue failure.

The mistuning effect on bladed disks vibration have been extensively studied since the 70's, see, e.g., the reviews by Slater et al. [1] and by Castanier and Pierre [2], and the more recent presentation by Ewins [3]. The well known main conclusions about the consequences of mistuning can be briefly summarized as follows: (i) mistuning can give rise to a high increase of the forced response vibration levels, and (ii) mistuning has a stabilizing effect on the aeroelastic instabilities, that is, it tends to reduce flutter.

The above mistuning results correspond to small variations of the elastic characteristics (mass and stiffness) of the blades, which is the situation considered in the vast majority of mistuning studies. In this paper the different case of damping mistuning is analyzed, that is, the case of a bladed disk with a sector to sector variation of the damping of the blades. This damping variation is always present in realistic situations and is typically due to the scatter present in material damping values, and also to the variability of the conditions on friction and interface joints.

Lin and Mignolet [4] statistically analyzed the effect of damping mistuning on the forced response of bladed disks using a simplified 1 DOF per sector model, and they concluded that damping mistuning can lead to variations in the blades' vibration amplitude similar to that found with mass/stiffness mistuning. In the more recent work of Siewert and Stüer [5] a Reduced Order Model (ROM) derived from a complete bladed disk with damping mistuning is analyzed. The results from the ROM are first successfully compared with those from a detailed FEM simulation of the forced response of the mistuned rotor, and then the ROM is used to statistically analyze the effect of damping mistuning. They confirm that the magnitude of the resulting amplification factors is comparable to those from mass/stiffness mistuning, but the required damping mistuning levels are much higher $\sim 50\%$. Also, from the figures in their paper, it can be clearly

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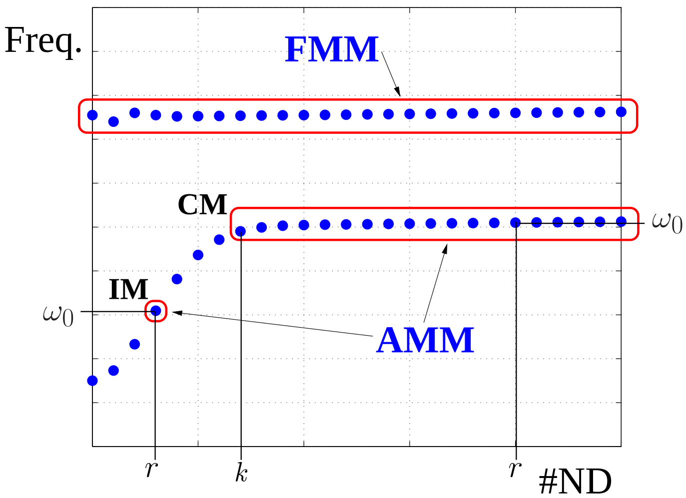

Figure 1. Sketch of tuned natural vibration frequencies vs. number of nodal diameters for a bladed-disk. The FMM covers the case of the forcing of a modal family with very similar frequencies. The AMM can describe also the forcing of other modal configurations, like isolated modes (IM) and clustered modes (CM).

appreciated the interesting fact that damping mistuning does not produce any noticeable frequency splitting.

The objective of this paper is to use the Asymptotic Mistuning Model (AMM) to analyze the effect of mistuning damping on the forced response of bladed disks. The idea is not to perform a statistical analysis of the effect of the different mistuning patterns, but to use AMM to identify the relevant parameters and the key mechanisms involved in the action of damping mistuning.

The AMM can be regarded as an extension of the Fundamental Mistuning Model (FMM) [6–8] for the very frequent case in which all modes of the family do not share the same frequency (see Fig. 1), and it is obtained by means of a fully consistent asymptotic expansion procedure from the complete mistuned bladed disk model. The AMM has been already used by the authors for the study of the optimal intentional (mass/stiffness) mistuning patterns for the stabilization of aerodynamically unstable rotors [9] and for the analysis of the forced response mistuning amplification [10], and its accuracy has been successfully checked against high fidelity FEM simulations [11, 12].

In this paper we present a complete derivation of the AMM for the forced response of a general realistic bladed disk with damping mistuning. And we then compare the AMM results with those from a 1 DOF per sector lumped model (Fig. 2), for two forcing cases not covered by the FMM: isolated mode and clustered modes (labeled IM and CM, respectively, in Fig. 1).

NOMENCLATURE

$\mathbf{M}$	Mass matrix.
$\mathbf{K}$	Stiffness matrix.
$\mathbf{C}$	Damping matrix.
$\mathbf{F}$	External forces.
$\mathbf{X}$	DOF displacements vector.
TW	Traveling wave.
N	Number of sectors.
$\mathbf{Z}$	Traveling wave mode shape.
$\mathbf{P}$	Change of basis matrix from TW to displacements.
A_j	Traveling wave mode amplitude.
AF	Amplification factor.
δ_j	Damping amplitude at each sector.
δ_m	Average damping.
δ_k^F	Fourier coefficients of the damping distribution.
r, ω	Engine order and angular frequency of the forcing.
ω_0	Tuned natural angular frequency of the excited TW.
d	Mistuning damping amplitude.
Δ	Mistuning correction matrix.
ω_a	Active mode tuned natural angular frequency.
D, D_{ij}	Mistuning coefficients in the TW basis.
f_r	Forcing coefficient.
ω_p, ω_c	Spring frequencies of the model in Fig. 2.
c	Damping coefficient of the model in Fig. 2.
f	Forcing amplitude of the model in Fig. 2.

DERIVATION OF THE SIMPLIFIED MODEL (AMM)

In this section, the derivation process of the simplified model is briefly explained. A more detailed derivation can be found in the references [9] and [10].

The starting point is the motion equation of the FEM discretization of a forced bladed disk model with N identical sectors,

$$\mathbf{K} \cdot x + \mathbf{M} \cdot \ddot{x} = f(t), \tag{1}$$

where x and f are, respectively, the displacement and the force vector.

If it is assumed that the forcing takes the form of a traveling wave excitation with angular frequency ω and engine order r and that, consequently, the response can be written as a complex mode shape $\mathbf{X}$ with angular frequency ω, a time independent system is obtained

$$\left[\mathbf{K} - \omega^2 \cdot \mathbf{M}\right] \cdot \mathbf{X} = \mathbf{F},$$

where the mass $\mathbf{M}$ and stiffness $\mathbf{K}$ matrices are cyclic symmetric matrices

$$\mathbf{K}=\begin{bmatrix} K & K_c & 0 & \cdots & K_c^T \\ K_c^T & K & K_c & & 0 \\ 0 & \ddots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & \vdots \\ K_c & 0 & \cdots & K_c^T & K \end{bmatrix}, \mathbf{M}=\begin{bmatrix} M & M_c & 0 & \cdots & M_c^T \\ M_c^T & M & M_c & & 0 \\ 0 & \ddots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & \vdots \\ M_c & 0 & \cdots & M_c^T & M \end{bmatrix}, \quad (2)$$

once the vector $\mathbf{X}$ is arranged by sectors

$$\mathbf{X}=\begin{bmatrix} X_1 \\ \vdots \\ X_j \\ \vdots \\ X_N \end{bmatrix},$$

with the vector X_j containing the displacements of the DOF associated with sector j

In order to take into account the sector to sector mistuning damping in our model, a damping matrix $\mathbf{C}$ is included

$$\left[\mathbf{K}+\mathrm{i}\mathbf{C}-\omega^2\cdot\mathbf{M}\right]\cdot\mathbf{X}=\mathbf{F}, \quad (3)$$

which is a block diagonal matrix with blocks proportional to K

$$\mathbf{C}=\begin{bmatrix} \delta_1\cdot K & 0 & \cdots & \cdots & 0 \\ 0 & \delta_2\cdot K & \ddots & & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & 0 \\ 0 & \cdots & \cdots & 0 & \delta_N\cdot K \end{bmatrix}. \quad (4)$$

Note that, as a first approximation, only linear in-sector damping is considered in our model, and that the mistuning damping distribution is given by the damping coefficients $\delta_j, \quad j=1,\dots,N$.

It is important to highlight that the variation of the damping δ_j from sector to sector is of the order of the damping itself. This is a completely different situation with respect to the case of mass/stiffness mistuning where the mistuning is just a small variation around the tuned value.

In order to have a more clear understanding of the effect of the damping nonuniformity it is convenient to transform the system eq. (3) into the basis of the traveling wave natural modes of the undamped system:

$$\left(\begin{bmatrix} \Omega_1^2-\omega^2 I & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \Omega_N^2-\omega^2 I \end{bmatrix}+i\Delta\right)\begin{bmatrix} A_1 \\ \vdots \\ A_j \\ \vdots \\ A_N \end{bmatrix}=\begin{bmatrix} 0 \\ \vdots \\ P_r^H\cdot F \\ \vdots \\ 0 \end{bmatrix} \quad (5)$$

The resulting mistuning correction matrix in the traveling wave basis is then given by:

$$\Delta=\mathbf{P^H}\cdot\mathbf{C}\cdot\mathbf{P}, \quad (6)$$

where $\mathbf{P}$ is the transformation matrix from the basis of traveling waves to the basis of physical displacements,

$$\mathbf{P}=\frac{1}{\sqrt{N}}\begin{bmatrix} P_1 e^{i(2\pi 1/N)1} & \cdots & P_N e^{i(2\pi N/N)1} \\ \vdots & & \vdots \\ P_1 e^{i(2\pi 1/N)j} & \cdots & P_N e^{i(2\pi N/N)j} \\ \vdots & & \vdots \\ P_1 e^{i(2\pi 1/N)N} & \cdots & P_N e^{i(2\pi N/N)N} \end{bmatrix} \quad (7)$$

After inserting the above expression into eq. (6), the mistuning correction matrix Δ takes the form

$$\Delta=\begin{bmatrix} \Delta_{11} & \Delta_{12} & \cdots & \Delta_{1N} \\ \Delta_{21} & \Delta_{22} & \cdots & \Delta_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \Delta_{N1} & \Delta_{N2} & \cdots & \Delta_{NN} \end{bmatrix} \quad (8)$$

where

$$\Delta_{kj}=\sum_{s=1}^{N}\delta_s\cdot\left(P_k^H\cdot K\cdot P_j\right)\cdot e^{i\left(\frac{2\pi(j-k)}{N}\right)s} \quad (9)$$

The structure of the blocks Δ_{kj} can be described more easily if the discrete Fourier transform of sector damping distribution δ_j is used:

$$\delta_j=\sum_{k=1}^{N}\delta_k^F e^{i\left(\frac{2\pi k}{N}\right)j}. \quad (10)$$

Note that, since δ_j is a real distribution, its Fourier coefficients must verify

$$\delta_k^F = \overline{\delta_{-k}^F}, \tag{11}$$

and the sector averaged damping δ_m is given by

$$\delta_m = \frac{1}{N}\sum_{j=1}^{N}\delta_j = \delta_N^F \tag{12}$$

If we take the Fourier expression for the damping distribution (10) into equation (9), the blocks of the mistuning matrix Δ can be finally written as

$$\Delta_{kj} = \delta_{k-j}^F \cdot \left(P_k^H \cdot K \cdot P_j\right). \tag{13}$$

From the above equation it can be concluded that the effect of the mistuning is to couple the natural TW modes, and that it is precisely the harmonic $k-j$ of the damping distribution which is responsible for the coupling of the TW with k and j nodal diameters. This coupling effect means that other TW modes different from the TW with number of nodal diameter equal to the engine order r appear in the forced response of the bladed disk.

In order to see which TW modes are more relevant we now make use of the fact that the damping δ_j is small (it induces a temporal decay in the unforced response of the system that takes place in a time scale much slower than the oscillation period), and we derive the AMM retaining only the dominant terms in eq. (5). Note that, to have a maximum tuned response amplitude normalized to 1, the forcing term in the right side of eq. (5) has to be of size δ_m and, therefore, it is also a small term.

When the forcing angular frequency ω with engine order r is close to resonant tuned frequency ω_0 (corresponding to a TW with r nodal diameter) two types of TW modes can be distinguished:

1. Passive modes that, in first approximation, do not have any effect on the forced response amplitude. These modes have frequencies ω_{kj} that are not close to the resonance angular frequency ω_0, in other words, for the passive modes$\left|\omega_{kj}-\omega_0\right|$ is large as compared with the damping terms. Note that the subindices k and j represent respectively the nodal diameter and the mode shape. The equations corresponding to these modes in the system (5) are of this form:

$$(\omega_{kj}^2-\omega^2)A_{kj} = \sum_{i,h=1}^{N,m}(\text{small terms})A_{ih} + (\text{small forc.}) \tag{14}$$

Since we are forcing near resonance, ω is close to ω_0, and then $\left|\omega_{kj}-\omega\right| \sim \left|\omega_{kj}-\omega_0\right|$ and the coefficient in left hand side of the equation is large as compared to the damping and forcing terms. Consequently, the right hand side of the above equation can be neglected, and gives:

$$A_{kj} \simeq 0 \tag{15}$$

2. Active modes that take part in the forced response. These modes have angular frequencies ω_a close to the resonance angular frequency ω_0. Their resulting equation takes the following form:

$$(\omega_a^2-\omega^2)\cdot A_a + i\cdot\sum \delta_{aa'}A_{a'} = F_a \tag{16}$$

with

$$\delta_{aa'} = \delta_{a-a'}^F \cdot (Z_a^H \cdot K \cdot Z_{a'}) \tag{17}$$

where Z_a, $Z_{a'}$, a and a' are, respectively, the indexes of the active modes A_a and $A_{a'}$, ω_a is the frequency of the active mode, and the forcing F_a is equal zero for the modes with nodal diameter different from the engine order r.

Finally, after removing the passive modes from eq.(5), the AMM is obtained

$$\begin{bmatrix} d_k & & \cdots & \\ & d_{k+1} & i\cdot D & \\ \ddots & \ddots & \ddots & \vdots \\ \vdots\ i\cdot D^H & \ddots & d_{-(k+1)} & \vdots \\ & \cdots & & d_{-k}\end{bmatrix}\begin{bmatrix} A_k \\ A_{k+1} \\ \vdots \\ A_{-(k+1)} \\ A_{-k}\end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ i\cdot\delta_m\cdot f_r \\ \vdots \\ 0\end{bmatrix}. \tag{18}$$

The size of this linear system is equal to the number of active TW, and the variables $A_k, A_{k+1}, ..., A_{-k}$ are their amplitudes. The forcing coefficient f_r is set to

$$f_r = (Z_r^H \cdot K \cdot Z_r) \tag{19}$$

in order to have tuned maximum response 1, the diagonal terms are given by

$$d_k = (\omega_k^2-\omega^2) + i\cdot\delta_m\cdot(Z_k^H\cdot K\cdot Z_k) \tag{20}$$

and the off-diagonal elements D are of the form

$$D_{kk'} = \delta_{k-k'}^F \cdot (Z_k^H \cdot K \cdot Z_{k'}). \tag{21}$$

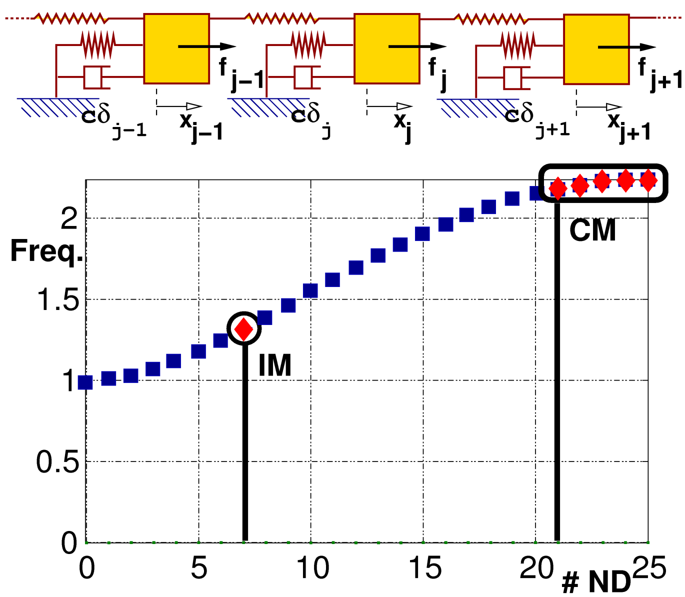

Figure 2. Top: sketch of the simple 1-DOF per sector system. Bottom: tuned frequencies vs. number of nodal diameters (IM: forcing engine order 7, CM: forcing engine order 21).

The resulting AMM written in eq.(18) reduces drastically the size of the problem and it retains only the minimal set of TW that are relevant for the forced response (the active TW modes). The derivation of the AMM requires to know only the tuned vibration characteristics of the system and the damping distribution, and it gives precise information on which Fourier harmonics of the damping distribution induce a coupling between the active TW modes.

APPLICATIONS

In this section the validity of the AMM results is checked using a simple 1-DOF spring-mass model (shown in Fig.2) given by,

$$\ddot{x}_j + \omega_a^2 x_j + \omega_c^2 (2x_j - x_{j+1} - x_{j-1}) + c\delta_j \dot{x}_j = f\mathrm{e}^{\mathrm{i}(\omega t + 2\pi(r/N)j)} + \text{c.c.}, \quad j = 1 \dots N, \tag{22}$$

where c.c. means complex conjugate, $N = 50$ sectors, $\omega_a^2 = \omega_c^2 = 1$, δ_j is the mistuning distribution with average value one ($\delta_m = 1$), c is the damping coefficient, and f is scaled in order to have tuned response equal to 1.

Below the two forcing configurations shown in Fig. 1 are considered, which correspond to EO = 7 (isolated modes) and EO = 21 (clustered modes).

Isolated modes

The TW with $r = 7$ nodal diameters and vibration frequency ω_0 labeled IM in Fig. 1 is well apart from the rest, i.e., its distance to the rest of the frequencies is large as compared with the small damping. In this case, the only active modes are this TW and its symmetric one with $r = 7$ frequency ω_0.

The resulting AMM (see eq. (18)) for this case has only two equations and takes the form:

$$\begin{bmatrix} (\omega_0^2 - \omega^2) + i\delta_m D_0 & i\delta_{2r}^F \cdot D \\ i\overline{\delta_{2r}^F}\,\overline{D} & (\omega_0^2 - \omega^2) + i\delta_m D_0 \end{bmatrix} \begin{bmatrix} A_+ \\ A_- \end{bmatrix} = \begin{bmatrix} i\delta_m D_0 \\ 0 \end{bmatrix} \tag{23}$$

where:

$$D_0 = Z_{+r}^H \cdot K \cdot Z_{+r} \quad \text{and} \quad D = Z_{+r}^H \cdot K \cdot Z_{-r}, \tag{24}$$

and the response of the system consists of the superposition of the two TW modes with index $\pm r$

$$X_j = (Z_{+r} A_{+r} \mathrm{e}^{\mathrm{i}(2\pi r/N)j} + Z_{-r} A_{-r} \mathrm{e}^{-\mathrm{i}(2\pi r/N)j}), \quad \text{for } j = 1, \dots, N. \tag{25}$$

Note that, from the above system, it can be concluded that only the harmonic $2r = 14$ of the damping mistuning distribution has a noticeable effect on the vibratory response of the system.

If the rescaled frequency

$$\Delta\omega = \frac{\omega_0^2 - \omega^2}{\delta_m \cdot D_0}, \tag{26}$$

and damping mistuning

$$d\mathrm{e}^{\mathrm{i}\phi} = \frac{\delta_{2r}^F}{\delta_m} \frac{D}{D_0}, \tag{27}$$

are used, the AMM simplifies to

$$\begin{bmatrix} \Delta\omega + i & id\mathrm{e}^{\mathrm{i}\phi} \\ id\mathrm{e}^{-\mathrm{i}\phi} & \Delta\omega + i \end{bmatrix} \begin{bmatrix} A_+ \\ A_- \end{bmatrix} = \begin{bmatrix} i \\ 0 \end{bmatrix}. \tag{28}$$

The vibration amplitude of blade $j = N$ can be written as

$$\text{Amplitude} \propto |A_{+r} + A_{-r}|, \tag{29}$$

and, after solving the AMM eq. (28), the amplification factor due to damping mistuning can be finally expressed as

$$\mathrm{AF} = \frac{\left|[1-i\Delta\omega] - de^{-i\cdot\phi}\right|}{\left|[1-i\Delta\omega]^2 - d^2\right|}. \tag{30}$$

Note that, for the tuned case $d = 0$, the maximum value of the above AF is 1.

In order to find the maximum amplification due to damping mistuning the values of the expression for the AF (eq. (30)) are explored as a function of the three parameters: $\Delta\omega$, d, and ϕ.

The size of the scaled damping mistuning is limited to

$$d = |\frac{\delta^F_{2r}}{\delta_m}||\frac{D}{D_0}| < 0.5$$

since: (i) to avoid the unphysical possibility of having negative in-sector damping δ_j the Fourier coefficients must verify

$$|\frac{\delta^F_{2r}}{\delta_m}| < 0.5$$

(see eq. (10)), and (ii)

$$\left|\frac{D}{D_0}\right| \leq 1,$$

as it can be readily obtained from eq. (24) using the fact that $Z_{+r} = \overline{Z_{-r}}$ and K is a real symmetric non-negative matrix.

The resulting maximum values of the amplification factor AF (eq. (30)) over the angle ϕ for any given values of $\Delta\omega$ and d are plotted in Fig. 3. The value of d has to be less than 0.5, so the resulting amplitude factor can be as high as $AF = 2$. Note that this maximum $AF = 2$ is indeed not physically attainable since it would require to have damping $\delta_j = 0$ at some sector (see eq. (10)).

In order to check the above results several simulations of the full mass-spring system given by eq. (22) have been performed with engine order forcing $EO = 7$ and with frequency close to the isolated mode frequency labeled IM in Fig. 2.

In Fig. 4 the mistuning distribution consists of a mean damping plus a pure harmonic with wavenumber $k = 2r = 14$ (see Fig. 4 middle and bottom left plots), and the resulting rescaled mistuning amplitude d is equal to 0.25. The resulting maximum amplitude, 1.332..., is very close to that predicted by the AMM (AF =1.333..., see eq. (30) and Fig. 3), and, as it can be appreciated from the right plot in Fig. 4, the only TW modes involved

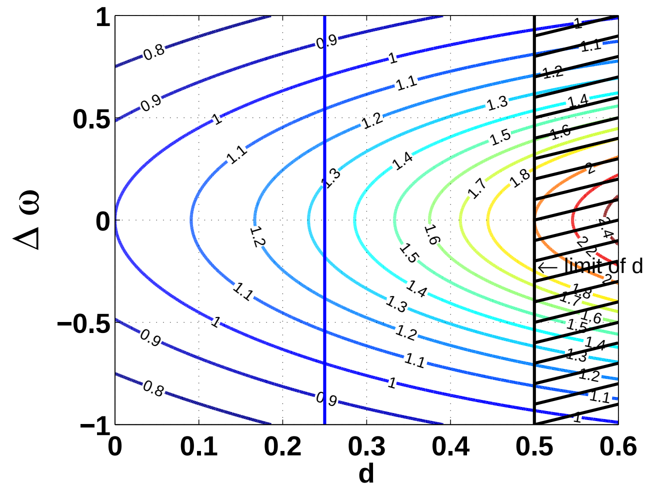

Figure 3. Maximum AF (eq. (30)) as a function of the mistuning amplitude $d \leq 0.5$ and the frequency $\Delta\omega$.

in the mistuned response are those with wavenumber 7 and -7 (again in perfect agreement with the AMM conclusions).

It is interesting to mention that, even though in Fig. 4 it can be seen that there are sectors with damping as low as 0.5 (1 is the reference uniform tuned damping), the amplitude response is not multiplied by a factor of 1/0.5=2 at this sector because the relevant damping is not the local in-sector damping but the damping of the global active TWs involved in the response of the system.

If more harmonics with random amplitude (see Fig. 5) are added to the above mistuning distribution the response of the system remains almost unchanged (as predicted by the AMM) although some sectors have now even smaller damping values.

Finally, if the harmonic component with wavenumber 14 is removed from the damping mistuning distribution, the resulting response amplitude is identical to the tuned one (see Fig. 6), without any noticeable mistuning effect, which confirms again the AMM results. This complete lack of effect of the damping mistuning also happens in the case plotted in Fig. 7, where the damping mistuning pattern consists of mean value and a pure harmonic with $k = 19$, different from the only active harmonic predicted by the AMM $k = 14$.

In summary, the damping mistuning effect in the forcing of a TW with frequency well apart from its neighbours (i.e. with a frequency gap large as compared with the damping) can produce an amplification of the forced response amplitude only when damping pattern contains the Fourier harmonic with wavenumber equal to twice the engine order of the forcing.

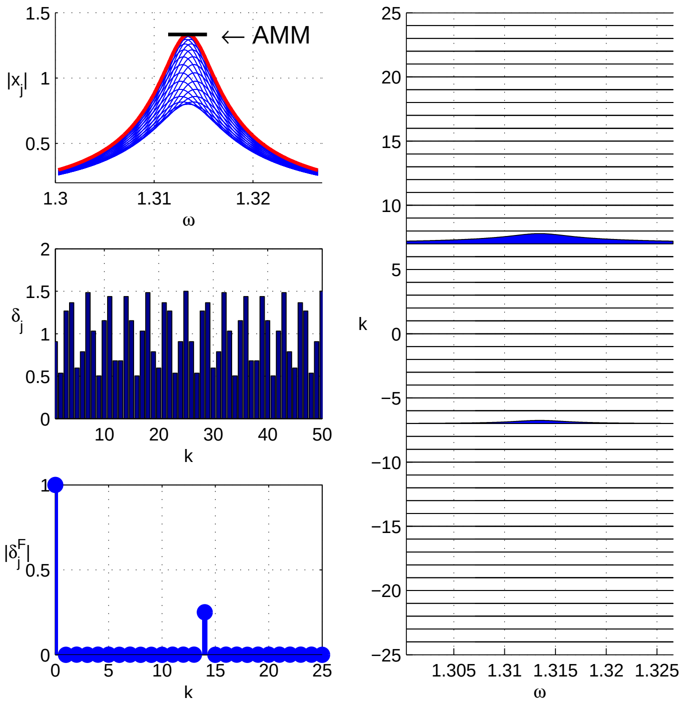

Figure 4. Response of the system in fig. 2 to a forcing with engine order 7 (tuned TW frequency $\omega_0 = 1.3135\ldots$, and $c = 0.01/\omega_0$) and a damping mistuning pattern composed of a mean value plus a pure harmonic with wavenumber 14 (AMM prediction for maximum response). The damping mistuning amplitude d is equal to 0.25. Top left: displacements $|x_j|$ vs. forcing frequency (max $|x_j| = 1.332...$). Middle left: damping mistuning distribution δ_j. Bottom left: amplitude of the Fourier modes of the damping mistuning distribution. Right: amplitude of the TW components of the response vs. forcing frequency (wavenumber in the vertical axis).

Clustered modes

If now the system is forced with an engine order r such that the corresponding tuned TW has a frequency ω_0 on the horizontal part of the frequency distribution of the modal family (see the encircled modes labeled CM in Fig. 1), then there are more than two active modes.

This is the case when the mass-spring system given by eq. (22) is forced with a forcing engine order $r = 21$ and with a frequency close to the frequency of the clustered modes labeled CM in Fig. 2. The symmetric TWs with negative wavenumber have to be also taken into account, and thus the cluster in Fig. 2 actually contains 9 TW modes, with wavenumbers: $21, 22, 23, 24, 25, -24, -23, -22, -21$.

The AMM description, according to eq. (18), can now be written as 9×9 linear system

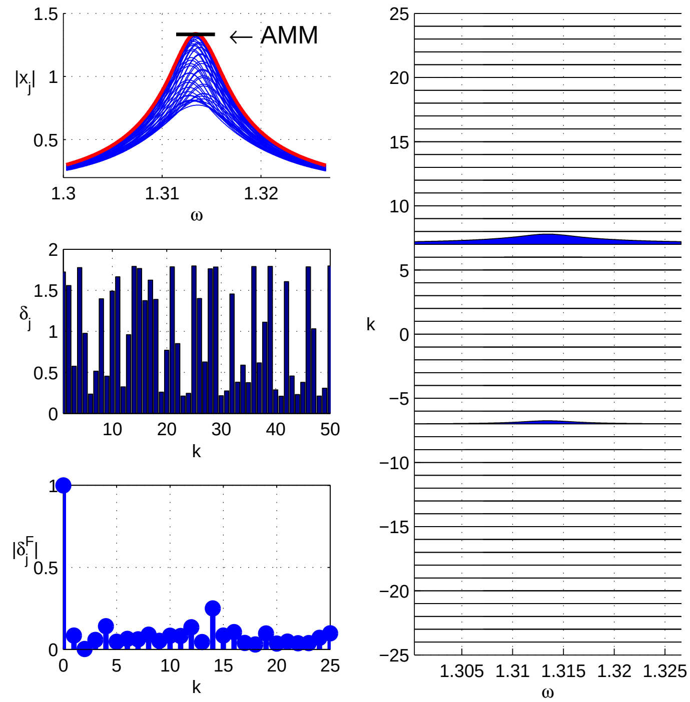

Figure 5. Same as in Fig. 4 but with the rest of the harmonics added to the damping mistuning pattern with random amplitude (max $|x_j| = 1.338...$).

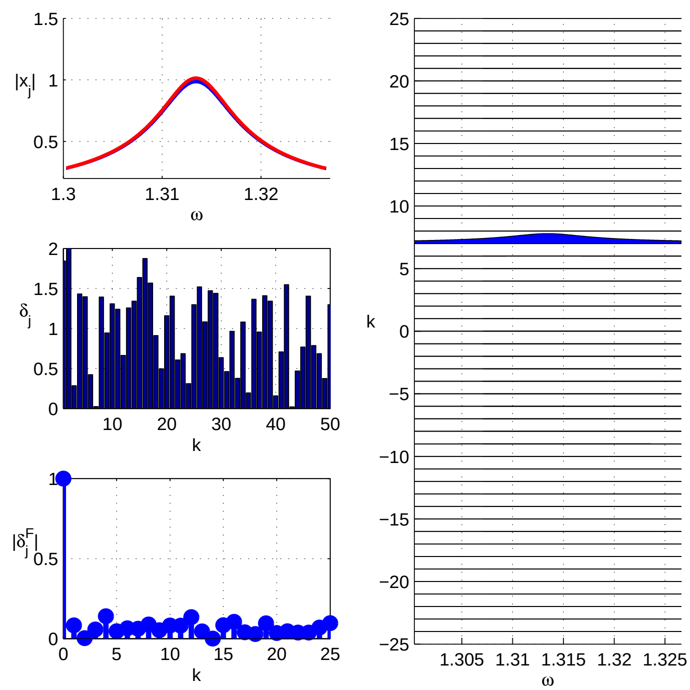

Figure 6. Same as in Fig. 4 but with the rest of the harmonics added to the damping mistuning pattern with random amplitude and zero harmonic with wavenumber 14 (max $|x_j| = 0.992...$).

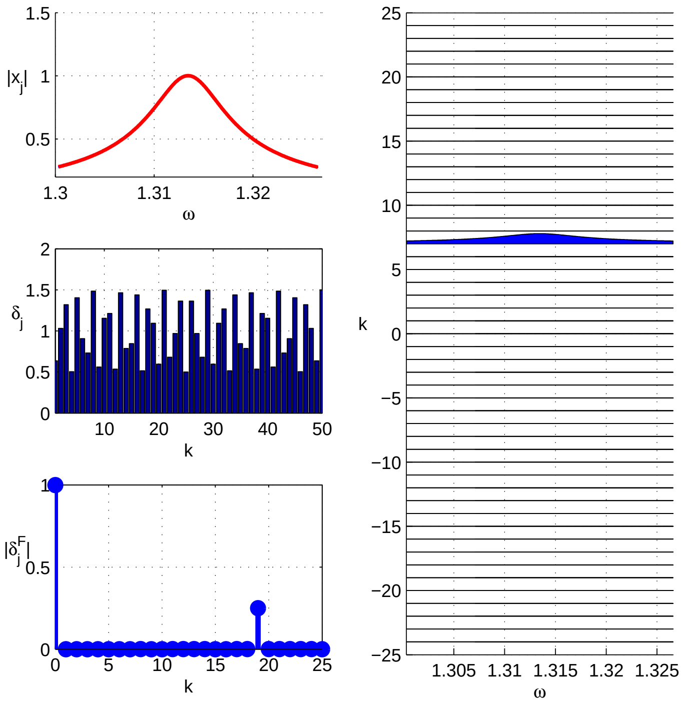

Figure 7. Same as in Fig. 4 but with a pure harmonic with wavenumber 19 (max $|x_j| = 1.001\ldots$).

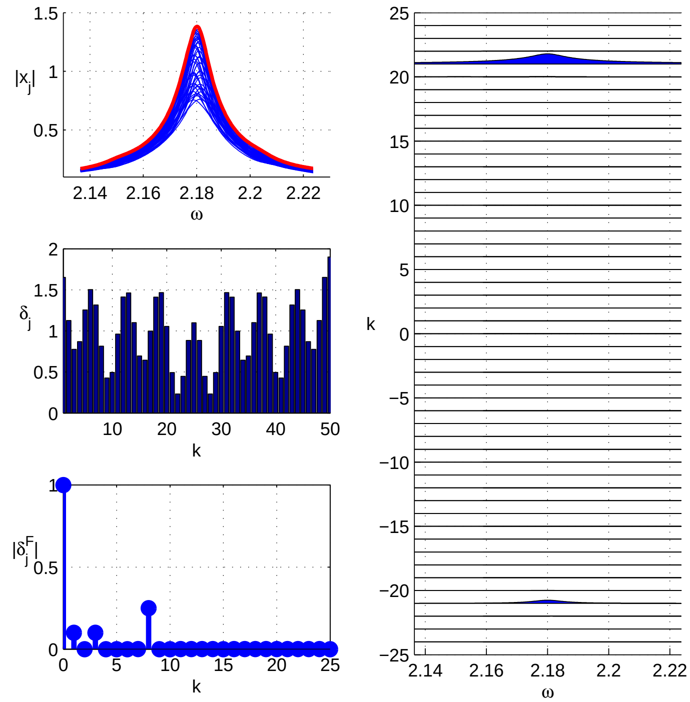

Figure 8. Response of the system in fig. 2 to a forcing with engine order 21 (tuned TW frequency $\omega_0 = 2.1830\ldots$ and $c = 0.03/\omega_0$) and a damping mistuning pattern composed of a mean value plus harmonics with wavenumber 1, 3 and 8. Top left: displacements $|x_j|$ vs. forcing frequency (max $|x_j| = 1.382\ldots$). Middle left: damping mistuning distribution δ_j. Bottom left: amplitude of the Fourier modes of the damping mistuning distribution. Right: amplitude of the TW components of the response vs. forcing frequency (wavenumber in the vertical axis).

$$\begin{bmatrix} d_{21} & & & & \\ & d_{22} & & iD & \\ & & \ddots & & \\ & iD^{\mathrm{H}} & & d_{-22} & \\ & & & & d_{-21} \end{bmatrix} \begin{bmatrix} A_{21} \\ A_{22} \\ \vdots \\ A_{-22} \\ A_{-21} \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ \mathrm{i}\delta_m f_r \\ \vdots \\ 0 \end{bmatrix}, \tag{31}$$

where, according to eq. (21), the only damping mistuning harmonics that appear in the off-diagonal terms are $\delta_1^F, \delta_2^F, \ldots, \delta_8^F$ (which are the only that effectively couple the 9 active TWs $A_{21}, \ldots, A_{-21}$). And the vibration amplitude of blade $j = N$ is in this case given by

$$\text{Amplitude} \propto |A_{21} + A_{22} + \cdots + A_{-22} + A_{-21}|, \tag{32}$$

The resulting AMM is again much simpler than the original problem, but it is still too complicated to obtain an explicit expression for the amplification factor as it was done in the previous section. So, in this case, the results of the simulations of the lumped system (22) will be used only to check the validity of some qualitative AMM conclusions.

The AMM indicates that only the damping mistuning harmonics with wavenumber from 1 to 8 actually have effect on the response of the system. In Fig. 8 the response of the lumped system for a damping mistuning pattern with wavenumbers 1,3 and 8 is plotted (all of them active according to the AMM). The resulting maximum response of the system is $1.382\ldots$, much larger than the tuned one. On the other hand, the results shown in Fig. 9 correspond to a damping mistuning pattern with harmonics with wavenumber 13, 16 and 20 (all of them not active according to AMM), and, in this case, there is no amplification of the response of the system (even though there are in-sector dampings as small as 0.25).

The mistuned natural vibration characteristics of the system can also be obtained from the AMM system in eq. (18) by setting the forcing to zero and solving the resulting eigenvalue problem for the ω.

Moreover, if the AMM matrix is premultiplied by the vector $\overline{[A_k, A_{k+1}, \ldots, A_{-(k+1)}, A_{-k}]}$ and the real part is taken, the follow-

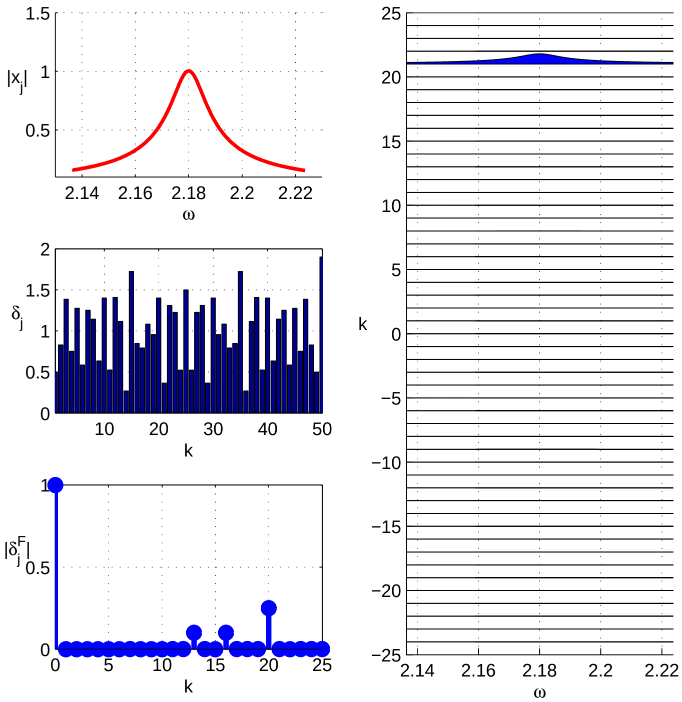

Figure 9. Same as in fig. 8 but for a damping mistuning pattern composed of a mean value plus harmonics with wavenumber 13, 16 and 20 (max $|x_j| = 1.004...$).

ing expression is obtained:

$$\sum_{j=-k}^{k} |A_j|^2(\Delta\omega_j - \mathrm{Re}(\Delta\omega)) = 0, \tag{33}$$

where

$$\omega_j = \omega_0(1+\Delta\omega_j),$$
$$\omega = \omega_0(1+\Delta\omega),$$

and, since the frequencies of the active modes ω_j and the mistuned natural frequencies ω are both very close to the frequency of the directly forced mode ω_0, the higher order corrections in $\Delta\omega_j$ and $\Delta\omega$ have been neglected.

Using the expression above, eq. (33), it can be readily seen that

$$\min_{j=-k...k}(\Delta\omega_j) \leq \mathrm{Re}(\Delta\omega) \leq \max_{j=-k...k}(\Delta\omega_j),$$

and it can be concluded that the mistuned frequencies are in between of those of the tuned active modes. Or, in other words, it can be concluded that, in contrast to what happened with the mass/stiffness mistuning, there is no frequency splitting associated with the damping mistuning. This can be clearly appreciated in Fig. 8, and in the results presented by Siewert and Stüer [5].

CONCLUDING REMARKS

The AMM (Asymptotic Mistuning Model) has been used to analyze the effect of damping mistuning on the forced response of a bladed disk. We focused on the case of linear damping with a pattern that is made of a tuned mean value plus a sector to sector variation that is of the order of the mean damping itself (i.e., not a small modulation over the tuned value). The AMM is systematically derived from the general equations for the forced response of a mistuned bladed disk using an asymptotic expansion procedure that is based solely on the smallness of the damping (as compared with the forced frequency). The AMM has been applied to describe the effect of damping mistuning in two frequent situations: forced response of a pair of isolated modes, and forced response of a group of modes with close frequencies, and the results have been compared with those from the numerical simulation of the mistuned mass-spring system given by eq. (22).

The application of the AMM allows us to draw the following final remarks about the effect of damping mistuning:

1. The AMM reduces drastically the size of the problem to be considered, and the AMM results are in very good agreement with those from the mass-spring system given by eq. (22).
2. Damping mistuning can increase the forced response of the system. The harmonics of the damping mistuning distribution that couple active TW modes can increase the forced response, but the rest of the harmonics do not produce any effect on the system. In other words, the system has zero sensibility (in first approximation) to the harmonics of the damping mistuning pattern that do not couple active TW modes
3. The presence of small in-sector values of the damping does not necessarily implies an amplification of the forced response (see Figs. 6 and 9). This is because the relevant damping value is that of the global active TW modes involved in the mistuned response, and not that of the local in-sector damping.
4. From Fig. 3 it can be clearly appreciated that the amplification factor appears to grow monotonously with the size of the damping mistuning. This rules out the possibility of using intentional damping mistuning as a way to reduce the amplification factor.
5. Lin and Mignolet concluded in ref. [4] that damping mistuning can lead to a severe increase in the forced response amplitude, which can be larger than that resulting from mass/stiffness mistuning. This is in agreement with our re-

sults: in the case of isolated modes, the maximum amplification due to damping mistuning is $AF_{max} = 2$ (see Fig. 3), while for mass/stiffness mistuning the maximum amplification is lower $AF = (1+\sqrt{2})/2 = 1.207\ldots$ (see [10]).
6. And finally, it is interesting to highlight that, as opposed to what happens with the mass/stiffness mistuning, the damping mistuning amplification of the forced response does not produce any appreciable frequency splitting (see Fig. 8 and [5]).

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REFERENCES

[1] J.C. Slater, R.G. Minkiewicz, and A.J. Blair. Forced response of bladed disk assemblies - a survey. *The Shock and Vibration Digest*, 31:17–24, 1999.

[2] M.P. Castanier and C. Pierre. Modeling and analysis of mistuned bladed disk status and emerging directions. *Journal of Propulsion and Power*, 22(2):384–396, 2006.

[3] D.J. Ewins. The mistuned blade vibration problem revisited. *Twelfth International Symposium on Unsteady Aerodynamics, Aeroacoustics and Aeroelasticity of Turbomachines*, October 2009.

[4] C.C. Lin and M.P. Mignolet. Effects of damping and damping mistuning on the forced response vibrations of bladed disks. *Journal of Sound and Vibration*, 193:525–543, 1996.

[5] C. Siewert and H. Stuer. Forced response analysis of mistuned turbine bladings. *presented at the International Gas Turbine Institute Turbo Expo, Glasgow, UK, ASME Paper GT2010-23782*, 2010.

[6] D.M. Feiner and J.H. Griffin. A fundamental model of mistuning for a single family of modes. *Journal of Turbomachinery*, 124:597–605, October 2002.

[7] D.M. Feiner and J.H. Griffin. Mistuning identification of bladed disks using fundamental mistuning model–part i: Theory. *Journal of Turbomachinery*, 126:150–158, January 2004.

[8] D.M. Feiner and J.H. Griffin. Misuning identification of bladed disks using fundamental mistuning model–part ii: Application. *Journal of Turbomachinery*, 126:159–165, January 2004.

[9] C. Martel, R. Corral, and J.M. Llorens. Stability increase of aerodynamically unstable rotors using intentional mistuning. *Journal of Turbomachinery*, 130:011005, January 2008.

[10] C. Martel and R. Corral. Asymptotic description of maximum mistuning amplification of bladed disk forced response. *Journal of Engineering for Gas Turbines and Power*, 131:022506, March 2009.

[11] O. Khemiri, C. Martel, and R. Corral. Quantitative validation of the asymptotic mistuned model (amm) using a high fidelity bladed disk model. *presented at the International Gas Turbine Institute Turbo Expo, Glasgow, UK, ASME Paper GT2010-22498*, 2010.

[12] O. Khemiri, C. Martel, and R. Corral. Amm analysis of mistuning effects on bladed disk forced response. *Twelfth International Symposium on Unsteady Aerodynamics, Aeroacoustics and Aeroelasticity of Turbomachines*, October 2009.