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GENETIC ALGORITHM FOR GAS TURBINE BLADING DESIGN

Cleopatra F. Cuciumita, Valeriu A. Vilag, Valentin Silivestru, Ionut Porumbel
COMOTI Romanian Research and Development Institute for Gas Turbines
220D Iuliu Maniu Ave., Bucharest 061126, Romania
Phone +40 21 4340236, Fax +40 21 4340241
email: cleopatra.cuciumita@comoti.ro

ABSTRACT

Designing a gas turbine from scratch has always been an extremely laborious task in terms of obtaining the desired power output and efficiency. Theoretical prediction of the performances of a gas turbine has proven in time to be a compromise between accuracy and simplicity of the calculus. Methods such as the Smith chart are very easy to apply, but to make an exact prediction of the flow in a turbine would lead to an almost infinite number of variables to be considered. A quite precise method of determining total-loss coefficients for a gas turbine, based on a large number of turbine tests, was developed by D.G. Ainley and G.C.R. Mathieson, with an error of the calculated efficiency within 2%. The accuracy of the method has been validated by Computational Fluid Dynamics simulations, included in the paper. Even if it is not a novel approach, the method provides accurate numerical results, and thus it is still widely used in turbine blade design. Its difficulty consists of the large number of man-hours of work required for estimating the performances at each working regime due to the many interdependent variables involved. Since this calculus must be conducted only once the geometry of the turbine is determined, if the results are not satisfactory one must go back to the preliminary design and repeat the entire process. Taking into account all the above, this paper aims at optimizing the efficiency of a newly design turbine, while maintaining the required power output. Considering the gas-dynamic parameters used for determining the preliminary geometry of a turbine, and the influence of the geometry upon the turbine efficiency, according to the procedure stated above, a Monte Carlo optimizing method is proposed. The optimization method consists in a novel genetic algorithm, presented in the paper. The algorithm defines a population of turbine stage geometries using a binary description of their geometrical configuration as the chromosomes. The turbine efficiency is the fitness function and also acts as the mating probability criterion. The turbine energy output is verified for each member of the population in order to verify that the desired turbine power is still within acceptable limits. Random mutations carried on by chromosome string reversal are included to avoid local optima. Hard limits are imposed on optimization parameter variation in order to avoid ill defined candidate solutions. The approach presented here significantly reduces the time between design goal definition and the prototype.

INTRODUCTION

The current turbine design methods contain 3 major sequences: the preliminary design, which is a global calculus at mean radius, the through flow design, which adds the radial dimension, by selecting a radial equilibrium closure and the airfoil design, materialized in a 3D model, which can be subject to a computational fluid dynamics (CFD) flow analysis.

The main objective when designing a turbine is to obtain the highest possible efficiency and the desired power output at the given operating conditions. When running at low efficiency, the losses cause the net power to drop and, in order to compensate, more fuel must be burnt. Higher fuel consumption also implies an increase in the pollutant emissions level (NOx, unburned hydrocarbons, carbon monoxide).

Therefore, every turbine design must include a model for determining the losses that occur. From this point of view, the main disadvantage of CFD commercial software, widely available today, is its place in the design process. This tool, rather expensive in terms of work time in itself, can only be used once the 3D model is available and, if the results in terms of performances are not satisfactory, one must go back, sometimes all the way to the preliminary design. It is fair to say, under these circumstances, that a system of aerodynamic losses estimation is needed in earlier design stages. Such a system can also be enclosed in an optimization algorithm, such as the genetic algorithm presented here, to significantly reduce the number of iterations in the design cycle.

The first reliable methods for the pressure loss calculation in an axial gas turbine emerged through the synthesis of the experimental data obtained by testing a considerable number of turbines under different circumstances [1, 6]. When dealing with the estimation of an axial turbine performance, one must always make a compromise between accuracy and the complexity of the calculus. This led to two different approaches of loss modeling over time.

The first method is extremely simple to apply, modeling the overall losses against stage loading and flow coefficients. Some examples are the Smith chart [1] or the models described by Soderberg, and presented by Horlock [2], and Latimer [3]. However, these methods can be applied only to turbine families with similar characteristics.

A more widely applicable approach breaks down the losses into profile loss, clearance loss, secondary loss and trailing edge loss and considers the influence of a larger number of geometrical and flow parameters. Such an approach was adopted by Muchatarov and Krichakin [4], Craig and Cox [5] and others.

Probably the most comprehensive and complete method of predicting a turbine's performances at design point and off-design conditions, is that due to Ainley and Mathieson [6]. While Muchatarov and Krichakin correlated the profile and secondary loss coefficients directly to the incidence angle, Ainley and Mathieson first estimated the positive stalling incidence. Their original scheme has been revised several times, as test data from different turbines families has accumulated. Dunham and Came (AMDC) [7], followed by Kacker and Okapuu [8] improved the correlations of Ainley and Mathieson regarding the profile, secondary and tip leackage losses coefficients at design point, while Moustapha et al revised the correlations at off-design conditions for profile and secondary losses [9] and with respect to the influence of leading edge geometry upon the profile losses [10]. Most of the revisions were no more than a change in a coefficient value for a better match to the testing values for newly designed turbine families. That is, undoubtedly, a tribute to the wide employment that the original work of Ainley and Mathieson still finds today. It is true that an accurate design is within 0.5% error margins nowadays, while this algorithm provides 2 % accuracy, but once the optimization algorithm is completed, it can always be updated.

The selection of the proper input parameters, such that the best turbine performance to be obtained at the end of the design phase, is never a straightforward task, irrespective of the preferred design method. With the development of modern computers, various numerical approaches to the input parameter selection have been proposed. Such a solution, proposed in this work, is the use of a genetic algorithm to select the set of input parameters that provides the best turbine stage efficiency while maintaining the desired turbine power output.

Genetic algorithms were used in optimization problems related to turbomachinery design as early as the last decade of the 20th century. Thus, in 1996, Selig and Coverstone – Carroll [11] combined a genetic algorithm with a classical inverse design approach to maximize the energy production of wind turbines.

Over the last decade, a larger number of studies involving genetic algorithms applied in turbomachinery design applications, such as hub and shroud shape optimization [12], turbine blade cooling system [13, 14, 15], compressor blading optimization [16, 17, 18, 19], wind turbine power output increase [20], gas turbine engine health monitoring [21], or turbine blade profile optimization, like the one presented here. However, these previous studies, in order to determine each candidate profile performance, coupled the genetic algorithm either with CFD codes [22, 23, 24, 25, 26, 27], which requires a large amount of computational resources, and a significant computational time, while strongly reducing the genetic algorithm population, hence its chances of finding a true optimum point, or with Artificial Neural Networks trained off-line by means of previous CFD simulations [28, 29, 30, 31], with impact on method accuracy if the candidate profile lies outside the initial training space [32].

The novelty of the approach proposed here is the use of a well known, and extensively tested and trusted design method to provide the turbine stage performance to the genetic algorithm, leaving the computationally expensive CFD simulation to be performed only at the end, for the optimum solution, in order to verify the turbine stage performances.

NOMENCLATURE

α	gas flow angle, measured relative to the axial direction	[°]
β	blade angle, measured relative to the axial direction	[°]
M	Mach number	[-]
Re	Reynolds number	[-]
c	blade chord	[m]
t	maximum blade thickness	[m]
t_e	blade trailing-edge thickness, measured normal to the camber-line at trailing edge	[m]
o	blade opening, or throat	[m]
s	blade pitch, or spacing	[m]
e	mean radius of curvature of the convex surface of a blade between the throat and the trading edge	[m]
k	radial tip clearance, or minimum shroud band clearance	[m]
H	annulus height (equal~~s~~ to the blade height if radial tip clearance is zero)	[m]
D	outer diameter of turbine annulus	[m]

d	inner diameter of turbine annulus	[m]
Y_t	total loss coefficient	[-]
Y_p	profile loss coefficient	[-]
Y_s	secondary loss coefficient	[-]
Y_k	clearance loss coefficient	[-]
z	blade loading parameter	[-]
C_L	lift coefficient based on the vector mean velocity	[-]
n_g	the number of seals	[-]
p^*	total pressure	[Pa]
T^*	total temperature	[K]
n	rotational speed	[rpm]
$\dot{M}_g$	mass flow rate	[kg/s]
Δh^*	variation of the total specific enthalpy	[J/kg]
n_{red}	reduced rotational speed	[rpm/ $\sqrt{K}$]
$\dot{M}_{g\,red}$	reduced mass flow rate	[m·s· $\sqrt{K}$]
Δh^*_{red}	reduced variation of the total specific enthalpy	[J/kg/K]

Suffices

1 Inlet to a turbine row

2 Outlet from a turbine row

TURBINE DESIGN ALGORITHM

When designing a new turbine, the first step is to analyze its application. Knowing exactly what the turbine is meant for, one can establish the basic input data. The basic parameters for the algorithm presented in this paper are the gas dynamic parameters of the turbine's inlet and the power output or the total pressure at the turbine's outlet. The algorithm is divided into three main parts.

The first one, as expected, is a preliminary design at mean radius. In the preliminary design, the velocity triangles for the gas expansion in the turbine are determined, so that the required output is obtained. First, the turbine's cross sections are calculated, by approximating the hub radius at the inlet of the first stator in accordance with the application and imposing a critical regime at the stator's outlet. A value for the turbine's efficiency is also predicted using the Smith chart [1]. The total and static gas dynamic parameters are determined using a polynomial approach, and then the corresponding degree of reaction and velocities triangle at each section are computed.

So far, only the axial direction was analyzed. The next phase adds the radial dimension, analyzing the flow in a hub-to-shroud plane, as a grid of streamlines. In order to perform this task, several closures for the radial equilibrium equation are proposed in the literature, from which the user may choose those that better satisfy the problem requirements. This equation, in conjunction with the conservation of mass (continuity equation) enables the velocity triangles to be calculated at any radius.

Having all the above data thus determined, the airfoil design follows. The airfoil geometry at this stage will be represented as a set of two-dimensional sections, corresponding to the radii where the velocity triangles were determined. A minimum of three sections (hub, mean and tip) are required. For the airfoil design, the method of curving a known airfoil was chosen. A standard NACA airfoil mean camber line is curved so that the desired velocity triangles are obtained. Using the new mean camber line and a number of points from the original airfoil, its thickness is then added. For this calculus to be possible, supplementary input is required, such as the chord length, the maximum thickness, the number of blades in a row, and the maximum camber. Although one can only approximate these values form previous experience at this early stage of turbine design, these variables are part of an optimization process, which allows for less accurate initial guesses.

The next step consists in the stacking of the determined airfoils, thus obtaining the 3D model. Having this model, a CFD analysis using dedicated software is usually conducted in order to determine the turbine's performances at the design conditions, and beyond. The main drawback of such an approach is that, in most of the cases, the results are not satisfactory, and a return to the previous design steps~~,~~ sometimes all the way to the preliminary design~~,~~ is often required. This process may be repeated many times before the difference between the actual results and desired ones is small enough.

Having that in mind, the third part of this design algorithm calculates the performances of the turbine using the Ainley and Mathieson method [6] for each airfoil determined at the previous step, and determines the mean values of the performances for the entire turbine. The values are, further in this paper, compared to the ones obtained through CFD for the same conditions, with the purpose of validating the pressure loss coefficients calculation method used here.

The velocity triangles were previously determined using an approximation for the turbine's efficiency. With the known geometry, they are now recalculated. The method developed by Ainley and Mathieson considers the gas flow angle at the outlet of a turbine row as a function of geometry and Mach number, and independent of the incidence angle at the row inlet.

Based on the equations provided by Ainley and Mathieson and by interpolating the experimental data charts in their paper [6], a set of equations have been determined for the purpose of this work. The gas flow angle for the stator blade rows is determined by:

$$\alpha_2 = \alpha_{2i} \tag{1}$$

for Mach numbers between $0 < M_2 \leq 0.5$, by:

$$\alpha_2 = \alpha_{2c} + 4(\alpha_{2c} - \alpha_{2i})(1 - 6M_2 + 9M_2^2 - 4M_2^3) \tag{2}$$

for Mach numbers between $0.5 < M_2 \leq 1$, and, respectively, by

$$\alpha_2 = \alpha_{2c} \tag{3}$$

for Mach numbers $M_2 \geq 1$, where,

$$\alpha_{2c} = -ar\cos\left(\frac{o}{s}\right) \tag{4}$$

$$\alpha_{2i} = 11.1 + 1.14\alpha_{2c} - 4\frac{s}{e}\ . \tag{5}$$

For the rotor blade rows, the gas flow angle relations become:

$$\alpha_2 = \alpha^*{}_{2i} \tag{6}$$

for Mach numbers between $0 < M_2 \leq 0.5$,

$$\alpha_2 = \alpha^*{}_{2c} + 4\left(\alpha^*{}_{2c} - \alpha^*{}_{2i}\right)\left(1 - 6M_2 + 9M_2^2 - 4M_2^3\right) \tag{7}$$

for Mach numbers between $0.5 < M_2 \leq 1$, and

$$\alpha_2 = \alpha^*{}_{2c} \tag{8}$$

for Mach numbers $M_2 \geq 1$, where,

$$\alpha^*{}_{2c} = -ar\cos\left(\frac{o}{s}\left(1 - \frac{k}{H}\right) + \frac{k}{H} + \frac{k}{D_m}\right) \tag{9}$$

$$\alpha^*_{2i} = -arctg\left[\left(1 - \frac{x \cdot k}{H}\frac{\cos\beta_1}{\cos\alpha_{2i}}\right)tg\alpha_{2i} + \frac{x \cdot k}{H}\frac{\cos\beta_1}{\cos\alpha_{2i}}tg\beta_1\right] \tag{10}$$

$$D_m = \frac{(D_1 + d_1) + (D_2 + d_2)}{4} \tag{11}$$

and the constant x = 1.35 for blades with radial tip clearance, respectively x= 0.70 for simple shrouded blades.

The total-loss coefficient is calculated as a sum of profile loss, secondary loss and tip loss coefficients:

$$Y_t = Y_p + Y_s + Y_k \tag{12}$$

The last two coefficients are calculated together. Also, the total loss coefficient is calculated without taking into account the trailing edge thickness influence, for a value of t_e/s of 0.02. Since, in most cases, that’s not to true, a correction is made by means of a multiplication factor plotted against the total loss by Ainley and Mathieson in their work. The loss coefficients are determined with respect to the relative flow parameters

The profile loss coefficient is a complex function of geometrical and flow parameters. Using empirical correlations, Ainley and Mathieson obtained a set of 6 charts allowing its calculation. These graphical dependencies have been transformed into higher order polynomials in order to be integrated in this algorithm. The first one is used to determine the gas flow angle at the exit of a turbine row for a value of the s/c ratio of 0.75, by knowing the true value of this ratio and its correspondent gas flow angle. The gas flow angle at the exit of a turbine row used here is not taking into account the tip clearance. With this result, and also knowing the geometrical angle of the turbine row, the second chart is used to obtain the stalling incidence for s/c = 0.75 and the third chart for the variation given by the actual value of s/c.

In the end, the stalling incidence is the sum of those two, which is, of course, an approximate value, but accurate enough for this work's purposes. The stalling incidence is defined as that incidence at which the profile losses are double those for zero incidence.

The next step is to determine the profile loss coefficient at incidence zero, using the formula:

$$Y_p = \left\{Y_{p(\beta_1=0)} + (\beta_1/\alpha_2)^2\left[Y_{p(\beta_1=-\alpha_2)} - Y_{p(\beta_1=0)}\right]\right\}\left(\frac{5t}{c}\right)^{-\beta_1/\alpha_2} \tag{13}$$

In this formula, the profile loss coefficients, having $\beta_1=0$, respectively $\beta_{1=-}\ \alpha_2$ and the same α_2 and s/c as the actual profile, are obtained using the two polynomials derived from the fourth and fifth charts.

In the end, the last chart is used to determine the profile loss coefficient as a function of profile loss coefficient at incidence zero and the ratio between stalling incidence and actual blade incidence.

After interpolating the graphical dependencies and using the given equations for the secondary and tip clearance loss coefficients, their summed value can be expresses as following:

$$Y_s + Y_k = 0.0334\left(\frac{c}{H}\right)\frac{\cos\alpha_2}{\beta_1}z + B\frac{c}{H}\left(\frac{k}{c}\right)^{0.78}z \tag{14}$$

where,

$$z = \left(\frac{C_L}{s/c}\right)^2\frac{\cos^2\alpha_2}{\cos^3\alpha_m} \tag{15}$$

$$\alpha_m = arctg\left(\left(tg\alpha_1 + tg\alpha_2\right)/2\right) \tag{16}$$

$$\frac{C_L}{s/c} = 2\left(tg\alpha_1 + tg\alpha_2\right)\cos\alpha_m\ . \tag{17}$$

Also, B is a constant, with a value of 0.47 for radial tip clearance and 0.37 for shrouded blades. For the shrouded blades using more than one seal, the tip clearance used in this calculus becomes:

$$k = k_g \cdot n_g^{\ 0.42} \tag{18}$$

where k_g is the geometrical value of the tip clearance and n_g represents the number of seals.

The total loss coefficient in a turbine row is calculated for a Reynolds number of approximately $2x10^5$.

For values different than this one, a correction is applied, as follows:

$$Y_t = \left(Y_p + Y_s\right)\cdot\left(\frac{\mathrm{Re}}{2\cdot10^5}\right)^{-0.2} + Y_k \tag{19}$$

For the proper functioning of this algorithm, a considerable set of input parameters are needed. For the purpose of the optimization algorithm that will be described in the next section of this paper, the input parameters may be split into two groups, as follows.

A first set of input parameters, which are firstly approximated by the user, only to be completely defined at the end of the optimization process, are presented in Table 1. For a turbine stage, which was the case considered in this paper, a number of 27 such variables have been identified.

The parameter variation limits have been chosen such that the middle of the interval to correspond to a known turbine configuration. When designing a completely new turbine, one must rely on their experience or existing literature data in choosing these limits so they would best suit their application.

TABLE 1 OPTIMIZATION PARAMETERS

Name	Units	Limits
Shroud radius for the stator inlet	[m]	0.05 – 0.25
Axial length of the stator	[m]	0.002 – 0.065
Axial length of the rotor	[m]	0.002 – 0.065
Channel divergence angle at the inner diameter of stator row	[°]	0.1 – 1.5
Channel divergence angle at the inner diameter of rotor row	[°]	0.1 – 7.0
Channel divergence angle at the outer diameter of stator row	[°]	0.0 – 8.0
Channel divergence angle at the outer diameter of rotor row	[°]	0.0 – 2.0
Tip clearance for the rotor blade	[mm]	0.1 – 0.9
Number of stator blades	[-]	10 – 50
Number of rotor blades	[-]	10 – 110
Trailing edge radius for the stator blades	[mm]	1.0 – 3.0
Trailing edge radius for the rotor blades	[mm]	1.2 – 3.2
Stator blade chord	[mm]	100 – 650
Shroud rotor blade chord	[mm]	100 – 350
Tip rotor blade chord	[mm]	100 – 350
The positioning of the maximum thickness, for the stator blades	[% chord]	10 – 50
The positioning of the maximum thickness, for the rotor blades	[% chord]	10 – 70
The admissible variation of the positioning of the maximum thickness, for the stator blades	[% chord]	1 – 3
The admissible variation of the positioning of the maximum thickness, for the rotor blades	[% chord]	1 – 3
Stator blades maximum thickness	[% chord]	5 – 35
Shroud rotor blades maximum thickness	[% chord]	5 – 35
Mean radius rotor blades maximum thickness	[% chord]	5 – 20
Tip rotor blades maximum thickness	[% chord]	5 – 15
The abscissa of the gravity center of the profile for the stator blades	[mm]	-10 – 10
The ordinate of the gravity center of the profile for the stator blades	[mm]	-10 – 10
The abscissa of the gravity center of the profile for the rotor blades	[mm]	-10 – 10
The ordinate of the gravity center of the profile for the rotor blades	[mm]	-10 – 10

The second set of data is represented by parameters considered constant throughout the entire design process, independent of the number of iterations as they are imposed by the desired application of the turbine. They are shown in table 2.

The working fluid parameters are only to be used in the preliminary design, in the through flow design being calculated using the polynomial approach.

TABLE 2 CONSTANT PARAMETERS

Name	Units	Value
Total temperature at turbine’s inlet	[K]	1263
Total pressure at turbine’s inlet	[Pa]	969204
Total pressure at turbine’s outlet / Power output	[Pa]/ MW	613580/ 1.091
Mass flow rate	[kg/s]	8.1345
Fuel/air mass flow rates ratio	[-]	0.01816
Rotational speed	[rpm]	22000
Mach number at the turbine’s inlet	[-]	0.1891
Radial equilibrium solution type for each row (stator/ rotor)	[-]	const. enthalpy and geometric angle / const. circulation
The turbine’s efficiency approximation for the mean preliminary design	[-]	0.92
The specific heat capacity at constant pressure for the working fluid	[J/kg/K]	1080.8
The specific gas constant for the working fluid	[J/kg/K]	287.15
The heat capacity ratio for the working fluid	[-]	1.333
The number of streamlines to be considered	[-]	11

OPTIMIZATION GENETIC ALGORITHM

A genetic algorithm (GA) is a Monte Carlo type numerical method based on the paradigm of biological adaptation [33]. Essentially, a genetic algorithm is a homogeneous, irreducible Markov Chain [34] that applies the principle of mutation and selective reproduction on a group of candidate solutions that for the so-called population.

The computer representation of the candidate solutions consists in a binary string, called a chromosome [34], which completely describes the significant characteristics of the candidate solution for the purpose of the optimization problem.

The most important part of a genetic algorithm is the recombination (in GA terms, “the reproduction”) of two candidate solutions’ chromosomes, called parents, into a new chromosome characterizing a so-called “offspring” [34].

During the creation of the new chromosome, like in the natural reproduction processes, a chromosome mutation may occur, that is a reversal of one or more bits in the offspring chromosome.

For each candidate solution, both in the initial solution and among the offspring of each generation, a fitness function, describing the function that is being optimized by the GA, must be computed.

Finally, the candidate solutions that will form the next generation are selected from the current population with a probability of selection depending on the candidate solution's fitness [34].

As a GA progresses, the population fitness improves, but the rate decreases since the individuals that form the population are more and more similar (or even identical). The value of the fitness function in such a case may be the desired global optimum, but it may also be only a local optimum value. Even though, through mutation, the GA will eventually find the global maximum, the total computational time will be much shorter if the GA run is stopped when no changes occur during a significant number of generations, and the run is restarted for several times, recording the best fitness solution for each run.

The GA used here starts by defining minimum and maximum limits for the 27 optimization parameters in Table 1, thus defining the acceptable solution space. Even though this is a limitation for the GA, such a restriction of the solution space is aimed at eliminating non-physical, or unrealistic solutions (e.g. stages with thousands of blades, zero tip clearances, or paper thin blades). The solution space limits used for the GA runs presented here are given in the last column of Table 1.

The initial population is created within these limits, its size being an optimization parameter, to be studied in the later sections of this paper. For each parameter, a random number is sampled from a Gaussian distribution having the mean at the middle of the interval defined by the limits. The variance of the Gaussian distribution is an optimization parameter, to be studied in the later sections of this paper. Any random numbers outside the imposed limits are discarded. The random number is, then, normalized through a linear transformation a brought into the [0; 1] interval, zero corresponding to the minimum value, and one to the maximum value. The first four digits after the decimal point will form the parameter value characterizing the current candidate solution. The chromosome is formed by the concatenation of the binary form of the 27 optimization parameter values determined as described above.

Once the optimization parameters values are known the turbine stage power and efficiency is determined for each member of the population, using the performance evaluation algorithm described previously. If the turbine power is lower than an acceptable value (1 MW in this study), the population member is "condemned" (its efficiency value is set to zero).

Due to the nature of the problem, some candidate solutions will be ill defined, i.e. some combinations of optimization parameters do not allow the performance evaluation algorithm describe previously to be completed, as some of the iterative parts it contains become divergent). In such a case, the diverging loop is broken and the performance evaluation stops. If this occurs during initial population evaluation, a new chromosome is randomly created through the process described earlier, as the initial population size must be the one specified at the start of the GA.

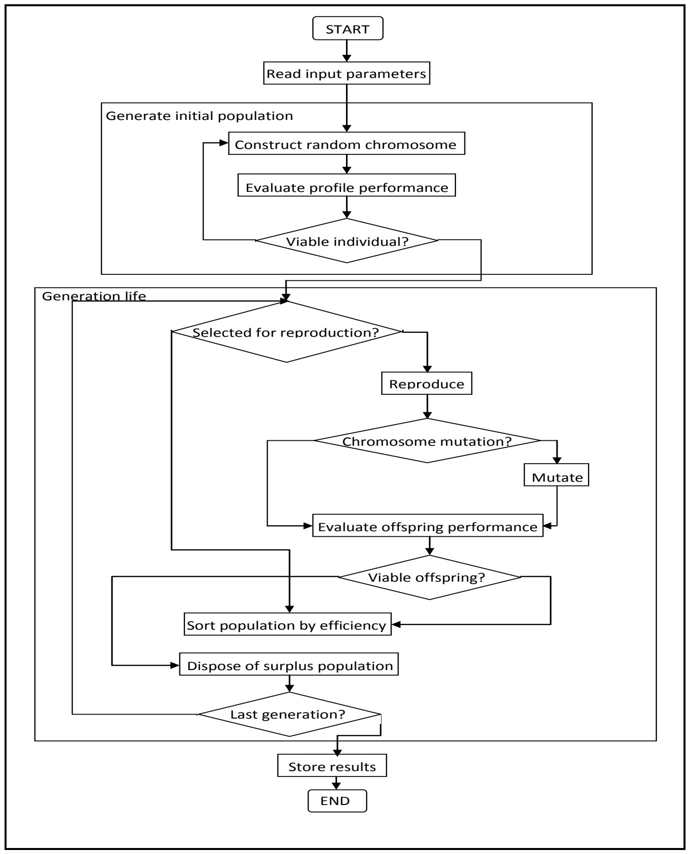

FIGURE 1. GENETIC ALGORITHM DIAGRAM

For each member of the population, with a probability determined by the corresponding efficiency, reproduction may occur at each generation. For this, a "mate" is randomly selected from the remaining population and an offspring chromosome is created by concatenating binary optimization parameter values inherited from either of the two parents, with equal probability.

During the reproduction step, a mutation in the offspring chromosome may occur with a probability set is this study to 0.5 %. This translates numerically into the reversal of a two bits string in the chromosome, at a randomly selected position. Mutations that translate into optimization parameter values outside of the initial range render the offspring as "condemned" and no efficiency evaluation will be performed in the next stage. For each viable offspring, the turbine power and efficiency are computed.

If a diverging loop in the performance evaluation algorithm occurs during offspring evaluation, that the particular offspring is deemed as not viable, its power and efficiency are set to zero, and will die before the next generation starts, as it will be shown next. The parents' chance for reproduction is lost for the current generation.

Finally, the current population is ranked by efficiency and a number of individuals equal to the initial population size will form the population for the new generation, while the rest of the candidate solutions are discarded ("killed"). The chromosomes providing the maximum efficiency and the maximum turbine power are recorded. The diagram describing the algorithm is presented in Figure 1.

OPTIMIZATION RESULTS

The previously described GA and turbine design algorithm coupled together in that the design algorithm is used as fitness evaluation algorithm by the GA , have been used to find the optimal set of parameters that provide the biggest turbine efficiency on condition that the turbine power remains over 1 MW.

Two GA parameters were considered significant for the final results, the population size and the variance of the Gaussian distribution that governs the chromosomes of the initial population, controlling the chromosome variability in the initial population.

To assess the impact of these parameters, a parametric study was carried on, according to Table 3. The first line and the first column of Table 3 indicate the tested parameter combinations, while inside Table 3 is shown the number of generations for each case, determined as the point where no change was recorded in the maximum efficiency, or the maximum power, for over 20 previous generations. A number of 10 runs for each case were carried on, for the initial conditions specified in Table 2.

TABLE 3 GA PARAMETRIC STUDY

	***SIG* = 5**	***SIG* = 10**	***SIG* = 50**	***SIG* = 100**
***NPOP* = 100**	300	300	300	300
***NPOP* = 1000**	500	500	500	500
***NPOP* = 10000**	800	800	800	800

In Table 3, *NPOP* is the population size, while *SIG* is related to the Gaussian distribution variance σ by the equation:

$$\sigma = \frac{X_{\max} - X_{\min}}{SIG} \tag{20}$$

where X_{min} and X_{max} are, respectively, the minimum and maximum limits imposed o the current optimization parameter by Table 1.

Figure 2 presents the time evolution of the largest efficiency of the generation for the 10 runs carried on for *NPOP* = 100 and *SIG* = 5.

The results show that the same maximum efficiency is reached by all runs, with a difference in value of maximum 0.1%, and the situation is similar for the other cases, not shown here due to space constraints. Since different variance values provide a different maximum efficiency, the convergence of the 10 runs to the same value for the case in Fig. 2 indicates the even though it is not the global optimum of the solution space, the obtained maximum efficiency is the local maximum that characterizes the given variability in the initial population, so the stop criterion is correct and the results are significant. Each series represented in Fig. 2 is a distinct run of the GA.

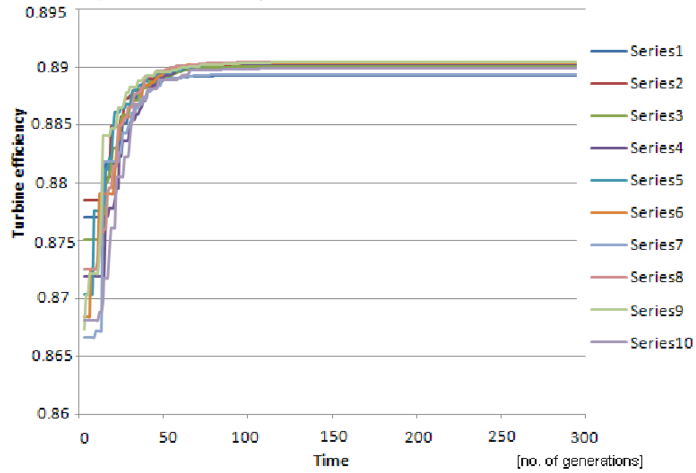

FIGURE 2. TIME EVOLUTION OF THE LARGEST EFFICIENCY FOR for *NPOP* = 100 and *SIG* = 5

Figs. 3, 4 and 5 present the time evolution of the largest efficiency of the generation, averaged over the performed runs, for, respectively, a population size of 100, 1,000 and 10,000.

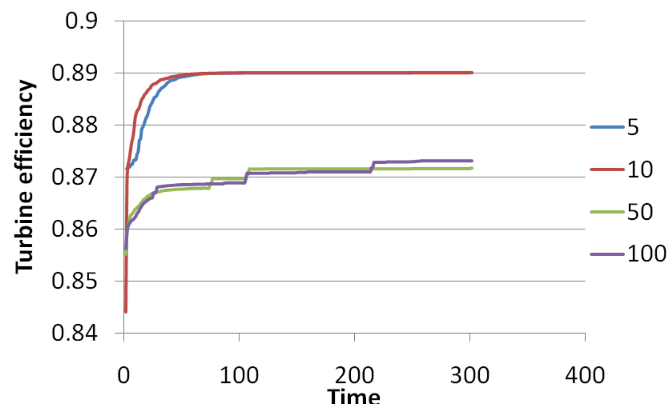

FIGURE 3. TIME EVOLUTION OF THE MEAN EFFICIENCY FOR for *NPOP* = 100 and *SIG* = 5, 10, 50, 100

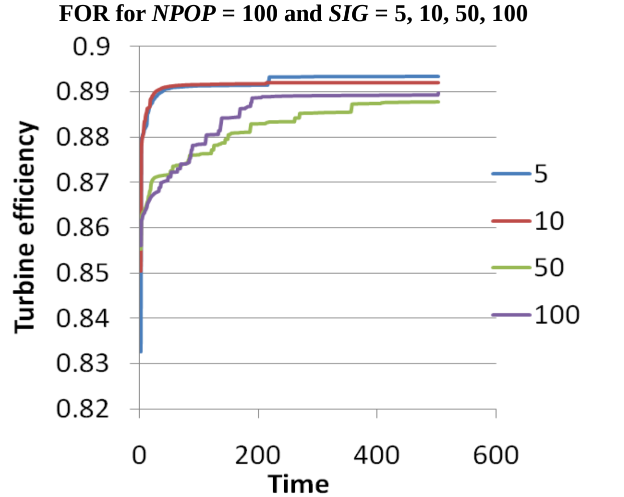

FIGURE 4. TIME EVOLUTION OF THE MEAN EFFICIENCY FOR for *NPOP* = 1000 and *SIG* = 5, 10, 50, 100

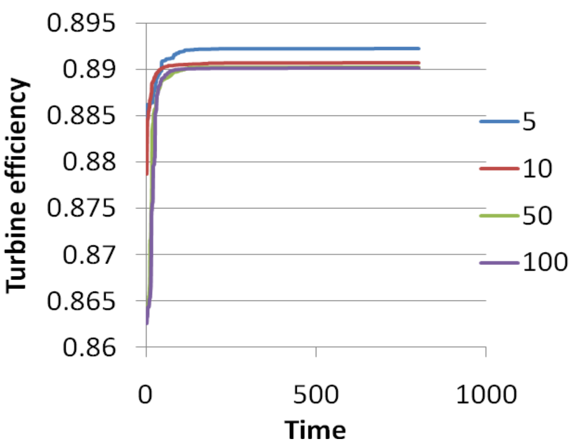

FIGURE 5. TIME EVOLUTION OF THE MEAN EFFICIENCY FOR for *NPOP* = 10000 and *SIG* = 5, 10, 50, 100

Generally, for a fixed population size, the maximum efficiency found by the GA is larger for the runs with greater variability in the initial population (smaller *SIG* parameter). Minor deviations from this rule for some runs are most likely due to flukes in the random numerical generator output. The reason is that the more variability exists in the initial population, the more opportunities arise for better fitted candidate solutions to appear (similarly to real life genetics). Also, the runs with larger variability also converge faster to the final solution, since in most cases, a solution close enough to the final one will likely be found in the initial population if enough variability exists. Finally, the final solution of the GA runs improves as the population size increases. Again, this relates to the chromosome variability not only in the initial population, but also during the run cycle.

Thus, the best results are achieved for a population size of 10,000, with a variance factor *SIG* of 5. The best run of this case was carried out for an additional 1,200 generations, to a total of 2,000. The total computational time was 49.25 single CPU hours on an 8 processors (type Intel Core2 Quad) machine. The run provided a maximum efficiency of 0.89388, at a turbine power of 1.091 MW. The optimization parameters yielding the maximum efficiency are given in Table 4, and the shape of the resulting turbine stage is presented in Fig. 6.

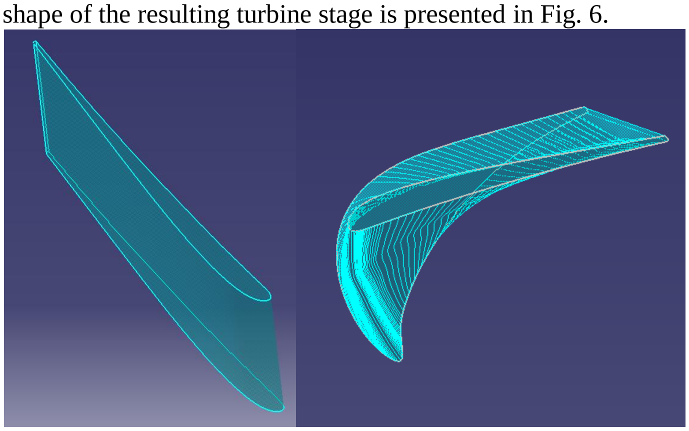

FIGURE 6. THE SHAPE OF THE RESULTING TURBINE STAGE (STATOR AND ROTOR BLADES)

TABLE 4 OPTIMIZATION PARAMETERS FOR BEST EFFICIENCY

Name	Units	Value
Shroud radius for the stator inlet	[m]	0.14864
Axial length of the stator	[m]	0.004961
Axial length of the rotor	[m]	0.003051
Channel divergence angle at the inner diameter of stator row	[°]	0.7265
Channel divergence angle at the inner diameter of rotor row	[°]	5.5712
Channel divergence angle at the outer diameter of stator row	[°]	6.5528
Channel divergence angle at the outer diameter of rotor row	[°]	0.0
Tip clearance for the rotor blade	[mm]	0.1
Number of stator blades	[-]	43
Number of rotor blades	[-]	62
Trailing edge radius for the stator blades	[mm]	1.0152
Trailing edge radius for the rotor blades	[mm]	1.2004
Stator blade chord	[mm]	272.032
Shroud rotor blade chord	[mm]	208.7488
Tip rotor blade chord	[mm]	218.9888
The positioning of the maximum thickness, for the stator blades	[% chord]	33.3495
The positioning of the maximum thickness, for the rotor blades	[% chord]	42.328
The admissible variation of the positioning of the maximum thickness, for the stator blades	[% chord]	1.0
The admissible variation of the positioning of the maximum thickness, for the rotor blades	[% chord]	1.0
Stator blades maximum thickness	[% chord]	5.8932
Shroud rotor blades maximum thickness	[% chord]	18.6192
Mean radius rotor blades maximum thickness	[% chord]	7.8272
Tip rotor blades maximum thickness	[% chord]	5.0
The abscissa of the gravity center of the profile for the stator blades	[mm]	-3.94
The ordinate of the gravity center of the profile for the stator blades	[mm]	1.544
The abscissa of the gravity center of the profile for the rotor blades	[mm]	3.914
The ordinate of the gravity center of the profile for the rotor blades	[mm]	1.816

DESIGN ALGORITHM VALIDATION THROUGH CFD

The approach presented in this paper postpones the use of CFD tools in the axial turbine design process until the optimization is completed. The CFD tools role of predicting the performances of an axial gas turbine is taken over by the Ainley and Mathieson algorithm [6].

Therefore, once the optimal turbine stage geometry is known, a CFD study was conducted in order to validate the turbine design algorithm presented earlier. The validation was achieved by comparing the results provided by the two methods at the same operating conditions. The comparison was conducted both at design conditions as well as at off-design conditions.

The full geometry of a turbine consists of at least one stator and one rotor row, as in the case considered in this paper, each row having a few dozens of blades. In order to avoid the complexity of this geometry, the CFD model was reduced to a single stator and a single rotor blade passage, using periodic boundaries. The mesh resulted contains over 1.5 millions of elements. The CFD numerical simulation was carried on using the ANSYS CFX solver.

Using the known sector geometry, the SST (Shear Stress Transport) turbulence model and the steady frozen rotor simulation type, and with known thermodynamic parameters for the turbine's inlet, the same as those used in the GA performance evaluation computations, part of the turbine's universal characteristic was determined by varying the rotational speed of the rotor as well as the mass flow rate.

By definition, the universal characteristic of an axial turbine represents a set of curves which contain the variations of the gas expansion ratio in relation with the similarity parameters of mass flow rate and rotational speed [35]. The turbine characteristic is required to establish the gas turbine's work line in correlation with the other gas turbine components.

The similarity parameters are those parameters that provide the turbine characteristic its universality, a single point on the universal characteristic representing a set of operating regimes under the Mach similarity criterion [36.].

Applying this criterion, the similarity parameters for the mass flow rate, enthalpy and rotational speed are defined by ratios of their effects and, therefore, they are further called the reduced parameters:

$$\Delta h^*_{red} = \Delta h^* / T_1^* \tag{20}$$

$$\dot{M}_{g\,red} = \dot{M}_g \cdot \sqrt{T_1^*} / p_1^* \tag{21}$$

$$n_{red} = n / \sqrt{T_1^*} \tag{22}$$

For the presented case, an inlet total temperature of 1124 K and an inlet total pressure of 527,573 Pa were considered.

The universal characteristic for the considered turbine, computed through CFD analysis (black) and using the turbine design algorithm described earlier and based on the Ainley and Mathieson method [6] (red), is shown in Fig. 7. The figure presents 4 constant reduced mass flow rate lines, having the values presented in the legend, the top value corresponding to the curve having the smallest slope. This figure shows a very small difference in results obtained using the two methods proving the validity of the design and optimization approach proposed here.

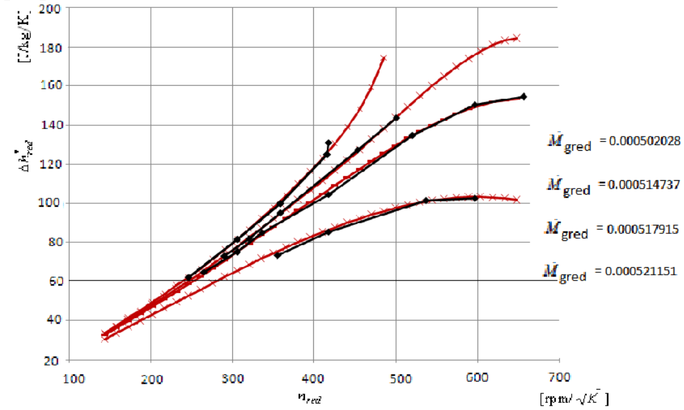

FIGURE 7. UNIVERSAL CHARACTERISTIC OF AN AXIAL TURBINE

To allow a better quantitative evaluation of this difference, a numerical example is given below. For a mass flow rate of 8.15 kg/s and a rotational speed of 14000 rpm, by taking into account the considered values for the total inlet pressure and temperature, the reduced rotational speed is of 417.6 rpm/ $\sqrt{K}$ on the first curve from the chart, counted from top to bottom. That leads to a calculated value of 126.7 J/kg/K for the reduced total specific enthalpy variation for the first method and 131.1 J/kg/K for the CFD one. In terms of power output, this is equivalent to 1.616 MW, respectively to 1.201 MW. The relative error between these two values is of approximately 3.5 %, and represents the maximum difference between the two methods, corresponding to the most distant two points, as it can be seen in Fig. 7.

Fig. 8 presents the velocity streamlines and the temperature field on the blades obtained with CFD,which is usefull in the blade material choosing process.

From an aerodynamic standpoint, the results show a vortex free flow, both in the stator and in the rotor. Also, secondary flow patterns cannot be observed in the CFD data. Both these results confirm that a minimal aerodynamic losses configuration has been obtained.

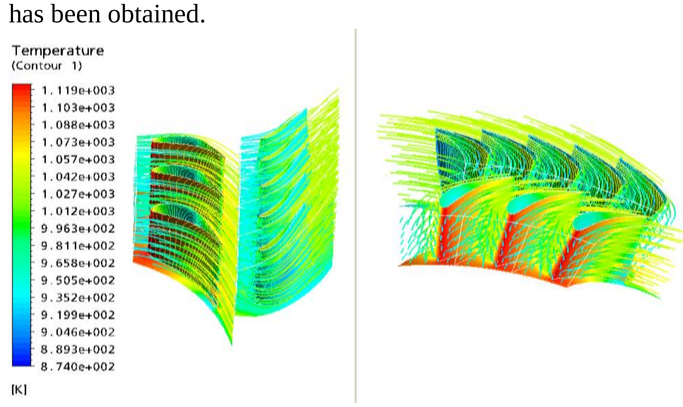

FIGURE 7. VELOCITY STREAMLINES AND TEMPERATURE CONTOURS

CONCLUSIONS AND FUTURE WORK

A novel genetic algorithm for the optimization of turbine stage geometry has been developed and used for the optimization of an axial turbine. The GA uses the turbine stage efficiency as fitness function and verifies the turbine power to remain above 1 MW. The turbine performance evaluation required by the GA is performed using a design algorithm based on the methodology developed by Ainley and Mathieson [6].

A parametric study of the influence of population size and initial population distribution on the GA performance shows that the algorithm provides better results if the population chromosome variability is sufficiently high throughout the GA run. The population variability is limited, however, geometrical constraints imposed on the optimization parameters in order to avoid non-physical, or unrealistic solutions.

As previously mentioned, even though a GA is guaranteed to eventually produce a global maximum [34], the computational time required to do so may be very large. To verify that a global maximum has been reached for the given initial conditions, a restarting technique has been used, by running the algorithm for several times using different random number generator seeds. The final result presented in the paper has been recorded in all the five instances the numerical procedure has been carried out, indicating a high probability that it is, indeed, a global optimum. Theoretically, two parameters may be considered important for the time required to find the global optimum: the mutation rate and the reproduction probability. If the reproduction probability is controlled only by the turbine profile efficiency, in an attempt to improve the odds to reproduce of the best fitted “individuals”, the effect of the mutation rate, set at 0.5% in this paper, will be analyzed in a future study.

The GA provides an optimal solution yielding an efficiency of 0.89388, at a turbine power of 1.091 MW. The solution geometry is tested by a CFD simulation both at the design point, and for off-design regimes. The maximum differences for the tested geometry are below 4 %.

Future developments of the method will aim at improving the empirical coefficients of the design algorithm in order to achieve a better match with the CFD simulation results, parallelize the GA to reduce the computational time and find a method for the a priori detection of the geometrically ill defined candidate solutions such that their performance evaluation to be completely avoided, as well as a better definition of the possible solution space that provides the GA initial population.

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