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VALIDATION OF A 3D LINEARIZED METHOD FOR TURBOMACHINERY TONE NOISE ANALYSIS

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ABSTRACT

A fully three-dimensional noise propagation analysis method accounting for 3D radial modes was developed in order to predict the tone noise emissions of a multistage low-pressure turbine. This propagation procedure employs a time-linearized aeroacoustic solver with 3D non-reflecting boundary conditions to compute the acoustic response of each single row and allows the acoustic waves to propagate across adjacent rows up to the turbine exit, where such perturbations radiate to the far field. This method follows the evolution of the acoustic modes within the machine, taking into account both circumferential and radial mode orders. The overall turbine exit sound power level, due to specific rotor-stator interactions, may thus be evaluated.

Some numerical results are compared with experimental data measured in an acoustic two-stage test rig which is representative of the last stages of a low-pressure turbine in terms of row geometry, blade loading and flow deflection.

NOMENCLATURE

c_{ax}	axial chord
$\vec{I}_a$	acoustic intensity
m	circumferential order
N, B, V	number of blades
n, k	harmonic order or scattering index
p	pressure
s	specific entropy
T	temperature
$\vec{w}$	velocity
W_a	sound power
x	axial coordinate
Greek:	
μ	radial order
ρ	density
Σ	section of the annular duct
σ	inter-blade phase angle (IBPA)
Ω	rotational speed
ω	radian frequency
Subscripts:	
$_\delta$	perturbation of quantities
$_{\mathrm{ref}}$	reference quantities
Acronyms:	
BPF	Blade Passing Frequency
LPT	Low-Pressure Turbine
PWL	sound PoWer Level
RANS	Reynolds Averaged Navier-Stokes
SPL	Sound Pressure Level
TRF	Turbine Rear Frame

INTRODUCTION

The last decades have seen an impressive evolution of the aeronautical sector with a constant increase in civil air traffic.

During this period airplane noise has become a real concern, as the zones surrounding airports are often densely populated. For these reasons regulations on noise emissions from aircraft are becoming increasingly stringent and demanding. As a result, aeroacoustics is now a major concern for the aeronautical industry and low noise criteria are normally employed in aircraft design.

It is well-known that significant progress in noise reduction has been achieved thanks to the development of high bypass ratio turbofans which have increased overall engine performance and also radically cut jet noise. Unfortunately, the reduction in individually emitted noise has been compensated by the constant increase in civil air traffic. Low noise design methods should therefore keep pace with growing air traffic in order to continue reducing the overall acoustic emissions of modern aircraft.

Usually, acoustic emissions by civil aircraft are classified as either external noise, due to the flow around the aircraft, or internal engine noise radiating outside. The latter component of the overall emitted noise is mainly generated by the rotor-stator interaction in the fan, compressor and turbine. Such interactions produce both tone and broadband noise. Tone noise emissions can be easily identified in a noise spectrum measured at the machine exit as they are represented by a succession of high-peak discrete frequency components. Each tone has a well defined frequency (engine order or BPF) depending on the shaft rotational speed and the number of blades.

Quieter fan design and acoustic treatment in the by-pass duct have led designers to pay attention to low-pressure turbine tone noise as well. These sources may indeed have an important role in the overall acoustic emissions, especially at approach conditions where LPT tones are more relevant.

Therefore reducing emitted noise is crucial for airplane certification and a number of methods have been developed in order to numerically predict the noise radiated into the far field. Some of these methods, such as the one presented in this paper, are aimed at evaluating the tone noise of a low-pressure turbine (LPT). Early methods were based on analytical models derived from flat plate theory [1,2] where the sound emission mainly depends on the difference between the wave angle and the stagger angle of the flat plate. Starting from the flat plate theory, more accurate analytical methods were developed in order to account for real turbomachine geometries. One of these approaches was described by Heinig [3]: this method may be applied to turbomachines with non-uniform swirling flows.

Since the 1990s, thanks to the growth in computer technology, "time-linearized" methods have been developed, capable of accounting for real blade geometries and non-uniform mean flows [4,5,6,7,8]. Such methods were first employed to evaluate the acoustic response of a single blade row, but can be also efficiently used to predict the tone noise emission at the turbine exit by integrating the time-linearized code with a proper propagation procedure [9].

During the ongoing research project on turbomachinery Computational Aeroelasticity (CA) and Computational Aeroacoustics (CAA) carried out at the "Sergio Stecco" Department of Energy Engineering (University of Florence) in collaboration with Avio S.p.A., a fully 3D procedure was implemented to analyze the generation and propagation of tone noise through a turbine. This strategy is based on the time-linearized acoustic solver *Lars* [10,11,12], which was designed to work together with the *Traf* steady/unsteady aerodynamic solver [13,14,15]. The *Lars* code is able to deal with incoming perturbations due to unsteady interactions with upstream/downstream rows (tone noise) which are treated with a non-reflecting boundary condition approach based on radial mode decomposition [16].

For the numerical evaluation of the tone noise reaching the turbine exit, both generation and propagation phenomena have to be properly simulated. Moreover, to have a better understanding of the physical mechanisms which govern certain behaviors, the propagation analysis has necessarily to account for the 3D physics of waves in annular ducts. Hence it is important that the propagation strategy follow the evolution of each acoustic mode from its source up to the engine exit, taking into account the radial shape of the traveling perturbations. The developed procedure may be applied to both downstream and upstream running waves, and is suitable for turbine as well as compressor/fan applications.

The need for an accurate numerical tool for tone noise prediction, which can be used in the engine design process, drove the development of this technique.

The validation of this procedure was performed by analyzing the acoustic emission of a cold-flow test rig located at the Avio facility in Turin, for which acoustic measurements are available at various operating points [17]. This annular two-stage rig is representative of high lift low-pressure turbines and generates acoustic tones in a common range of turbine noise frequencies. The results show a good agreement in terms of total acoustic power for all the studied interactions at the turbine exhaust. The SPL and PWL of the radial components of each interaction were also compared to the measurements. The method validation is currently ongoing, testing different operating conditions and turbine setups.

NUMERICAL METHOD

The tone noise analysis method consists of a multi-step procedure. Firstly, the rotor-stator interactions which could generate the tone noise peaks in the noise spectrum, have to be identified. This allows us to find where such perturbations originate and whether scattering phenomena occur during the propagation.

Generation of the sound is the following step. In the present case sound is generated by the non-linear interaction phenomena due to wakes and potential field in the axial gaps between rows. These interactions can be simulated by a CFD steady state computation. From the CFD solution, acoustic and convected waves

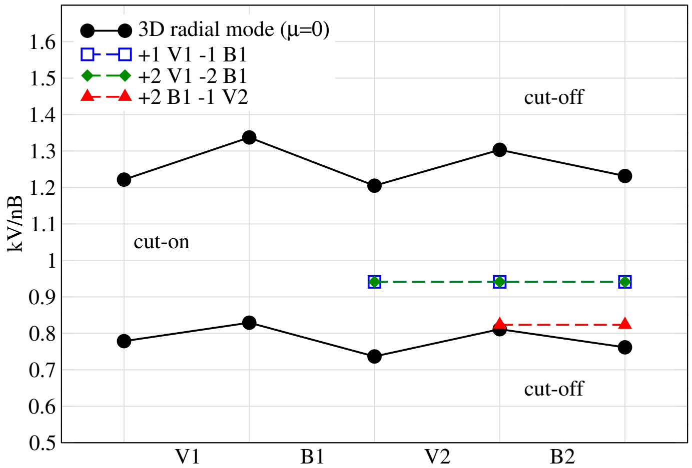

FIGURE 1. **Cut-on ranges and studied interactions**

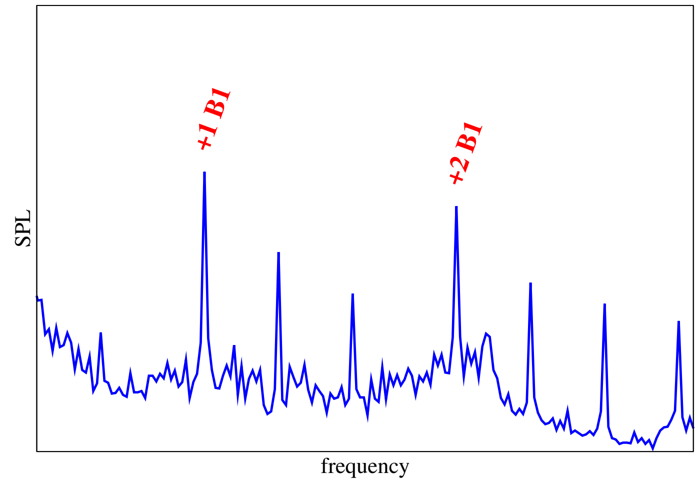

FIGURE 2. **Measured noise spectrum and tone identification**

are extracted by means of a wave splitting method based on radial modes (see, for instance, [18]). These waves are imposed as incoming perturbations for the propagation procedure.

The propagation method consists of a sequence of linearized aeroacoustic computations, which transmit the most relevant waves in the propagation direction. This process is iterated until the analyzed waves reach the turbine exhaust.

Identification of acoustic tones

The propagation procedure starts with the tone noise identification, so that the potentially most relevant modes traveling through the machine can be found. Once the blade counts are defined, the circumferential order for all possible waves can be obtained from the following formula:

$$m_n = k_{n-1}N_{n-1} - k_nN_n \qquad k_{n-1}, k_n \in \mathbb{Z} \tag{1}$$

Each spinning mode has its radian frequency given by:

$$\omega_n = k_{n-1}N_{n-1}(\Omega_{n-1} - \Omega_n) \tag{2}$$

and can be represented as a superposition of radial mode shapes. In light of the physics of acoustic waves inside a duct, each mode within the perturbation has its own axial wave-number. The latter can be a real quantity (the corresponding mode is then cut-on) or a complex value (the mode is then cut-off).

Cut-off modes decay as they propagate, whereas cut-on waves keep their amplitude unchanged. It is clear that the latter must be avoided as much as possible in order to lower the overall acoustic emissions. Indeed modern aircraft engines are designed so that most of the modes generated from the rotor-stator interactions will be cut-off under typical operating conditions.

In order to have an a priori identification of the wave behavior, it might be interesting to find out the cut-on ranges. Finding the cut-on range in terms of kV/nB ratio [19] in the axial gaps between rows allows us to identify the behavior of each traveling wave and, most importantly, its evolution along the machine (see fig. 1). The cut-on range limits can be found by numerically solving the eigenvalue problem which governs radial mode shapes on sheared swirling flows [16]. The most critical interactions can be identified as the ones which are inside the cut-on range, especially at the turbine exhaust. The corresponding frequencies may be also localized as peaks in a measured noise spectrum (see fig. 2).

Tone noise generation

It is common practice to evaluate the sound generation by analyzing the steady state aerodynamic field. In this approach the circumferential nonuniformities due to steady wakes can be seen as a rotating perturbation with respect to the successive row. Afterward such disturbances are split up into acoustic and convected waves by using a wave splitting method.

Wave splitting The 3D wave splitting strategy is closely linked to radial mode decomposition [16]. By using acoustic modes as a modal base, the rotating steady state field can be decomposed into acoustic and non-acoustic components. The waves obtained will be imposed at the receiving row, starting the acoustic propagation procedure. This highlights the importance of properly computing the radial components of the acoustic disturbances. Figure 3 shows an example of the acoustic radial modes.

Linearized aeroacoustic solver

The implemented method is based on a time-linearized code. Incoming perturbations from upstream or downstream rows are split into harmonic waves, in both time and circumferential coordinate. Each selected mode is characterized by the values of the radian frequency ω and the inter-blade phase angle σ, as well as by its amplitude and phase; this mode is considered as a small perturbation to be imposed at the inlet or outlet boundary.

In the governing equations for the fluid motion the conservative variables are written as a sum of their mean values and their small time-harmonic perturbations. By splitting finite terms from infinitesimal ones, two sets of governing equations are obtained: one for the mean conservative variables, and one for their perturbations. The former equations are identical to those for the steady fluid motion: the mean solution is computed by a CFD solver, such as *Traf* [13, 14, 15]. The latter are linearized and solved by an appropriate frequency domain solver, such as *Lars* (time-Linearized Aeroelastic Response Solver). The *Lars* code [10, 20, 11] was derived from *Traf* and shares basically the same numerical scheme. As explained above, *Lars* computes the solution perturbation by solving fluid motion equations, that have been time-linearized around a steady solution (computed by *Traf*). Although *Lars* was initially dedicated to flutter analysis, the code was later modified to allow the imposition of incoming waves from upstream/downstream rows at inlet/outlet boundaries in order to perform aeroacoustic tone noise analyses [12].

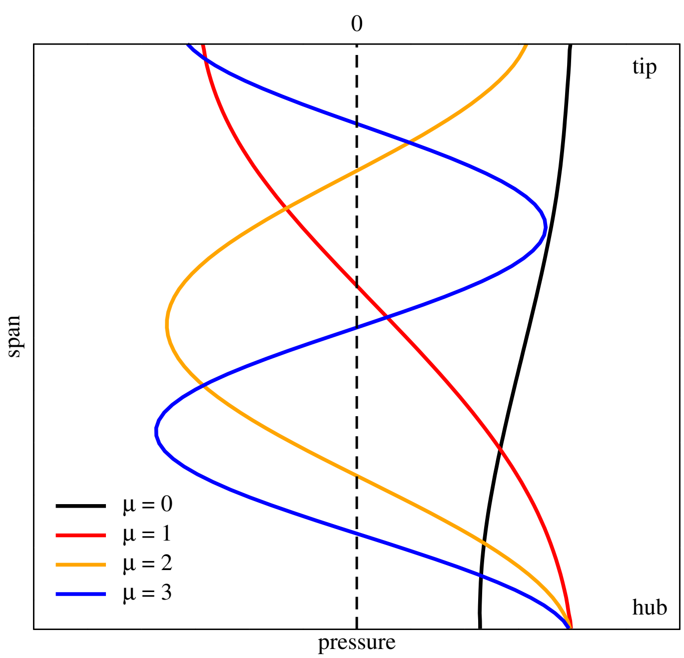

FIGURE 3. Radial mode shapes (in terms of pressure) at B2 exit for +2B1-1V2 mode

Two types of non-reflecting boundary conditions may be employed for aeroacoustic analysis with Lars: two-dimensional conditions and three-dimensional radial mode based conditions.

3D radial mode based conditions 3D non-reflecting boundary conditions are based on radial mode decomposition in sheared swirling flows [21,22]. Essentially acoustic radial modes are obtained as a solution of the eigenvalue problem governing the physics of waves in an annular duct. Since in general the mean flow is sheared and swirling, the problem cannot be solved analytically and must be processed numerically. The determined eigenvectors represent convected and acoustic modes and must be properly classified and selected. These modes are employed to characterize the boundary-crossing waves, so that outgoing waves are allowed to pass, while incoming ones are suppressed or assigned as imposed modes. The radial mode decomposition technique and the 3D non-reflecting boundary conditions used in this work are described in detail in [16].

Acoustic propagation strategy

The principal aim of this work is to compute noise emissions at the turbine exhaust, so that outgoing disturbances can be evaluated in terms of SPL and PWL. For this reason the sound propagation technique plays an important role in the overall analysis procedure [23].

Essentially this procedure allows the acoustic waves to propagate between adjacent rows by changing the radian frequency ω and the inter-blade phase angle σ. This change in frame of reference must take into account the relative rotational speed and blade numbers of the two neighboring rows:

$$\omega_n = -\omega_{n-1} + m_{n-1}(\Omega_n - \Omega_{n-1}) \qquad \sigma_n = \frac{2\pi}{N_n} m_n \tag{3}$$

Practically speaking, the acoustic response at the outlet (or inlet) of one row is employed as the incoming wave at the inlet (or outlet) of the successive row depending on the propagation direction. From now on, only downstream propagation will be considered, even though the developed procedure may deal with upstream propagation in case it is applied to compressors or fans. While crossing grid boundaries, the acoustic modes keep their circumferential and radial shape unchanged. Afterward, when such waves impinge on the successive rows, mode scattering occurs. This phenomenon generates other acoustic perturbations with a circumferential periodicity which is a function of the number of blades of the scattering row.

Sound Power

The sound power is a useful quantity to describe the transport of energy associated with acoustic disturbances. It allows the comparison of the overall emitted noise coming from different interactions, especially when strong scattered waves are present at the same time. Therefore, the sound power is computed by integrating the acoustic intensity over a section of the annular duct:

$$W_a = \int_{\Sigma} \vec{I}_a \cdot \vec{n} d\Sigma \tag{4}$$

and by taking the time average of this integral quantity. The acoustic intensity can be written as the following expression provided by Myers [24]

$$\vec{I}_a = \left(\frac{p_\delta}{\rho} + \vec{w} \cdot \vec{w}_\delta\right)(\rho \vec{w}_\delta + \rho_\delta \vec{w}) + \rho \vec{w} T_\delta s_\delta \tag{5}$$

which is valid for an arbitrary steady background flow in absence of body forces and neglecting the thermo-viscous terms. In the following analysis, both the overall and the single mode PWL will be used for comparison with the acoustic measurements.

LOW PRESSURE TURBINE ANALYSIS

The methodology validation was started by computing the acoustic emission of the two-stage cold-flow test rig located at the Avio facility. This rig was designed so that some of the acoustic modes generated by the main rotor-stator interactions are always cut-on, while other ones can be cut-on or cut-off depending on the operating conditions. Acoustic measurements are available for various operating points: approach conditions, where LPT tones are more relevant, were chosen to make numerical/experimental comparisons.

Acoustic test rig

The Avio annular rig (fig. 4) is aimed at investigating tone noise generation and propagation [17], through a modular design which will allow easy replacement of rows, variation of row axial spacing, addition of rows (inlet or outlet guide vanes, single stage to two-stage setups). In setting up the model turbine, engine-to-rig similitude criteria have been followed. Endwall dimensions have been chosen such that the turbine may be operated at conditions representative of noise certification points, still allowing operation at aerodynamic design point. A constant radius endwall was chosen, which is representative of last turbine stages, and allows an easy axial repositioning of rows (intended to investigate spacing effects). Blade dimensions and numbers are selected to obtain high aspect ratio and low solidity, while providing aeroacoustic similitude in terms of non-dimensional frequency. Row geometry, blade loading and flow deflection are representative of high lift turbine design. Current setup consists of two stages with no inlet or outlet guide vanes (see fig. 5).

FIGURE 4. Cold-flow rig

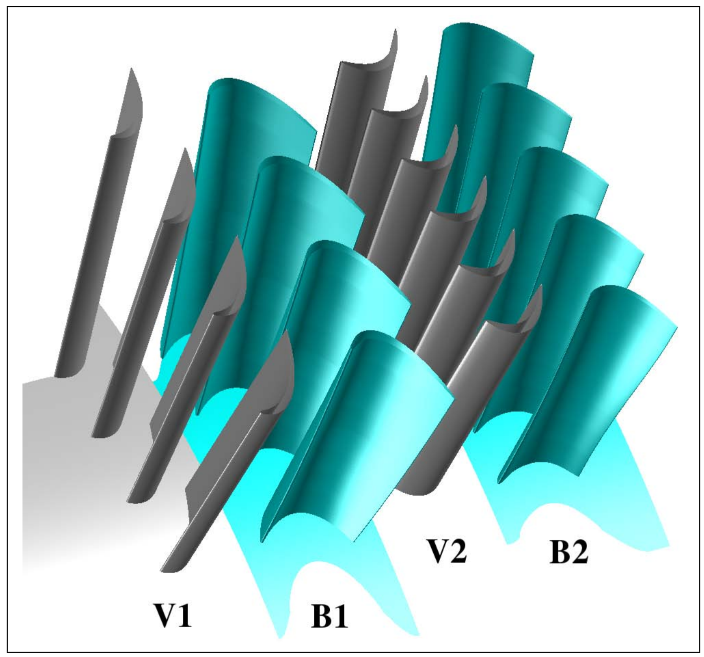

FIGURE 5. Cold-flow rig: geometry of the blades

Results

Tone identification Before choosing the acoustic modes to be computed, they were identified in the noise spectrum measured at the turbine exhaust. As can be seen in fig. 2, the cold-flow spectrum shows the typical peaks due to the rotor-stator interactions. All the peaks were detected by the procedure and each of them was connected with its frequency source. The two most significant tones (with +1B1 and +2B1 frequency) were chosen for the analysis.

On the basis of the cut-on range analysis along the turbine (see fig. 1), it was determined that these two peaks were caused by the interactions listed in table 1.

TABLE 1. **Studied interactions**

frequency	interaction
+1 B1	+1 V1 -1 B1
+2 B1	+2 V1 -2 B1
+2 B1	+2 B1 -1 V2

Steady state computation A steady CFD simulation must be done before performing the linearized acoustic simulations, since the steady state flow field is the frozen background for the linearized computations.

The whole LP turbine steady flow solution was calculated using a multistage mixing plane technique at the approach operating point, taking the needed boundary conditions from the experimental measurements. The computational grids (one single block elliptic non-periodic H–mesh per blade) were chosen in order to ensure a good compromise between the aerodynamic and aeroacoustic requirements (see grid dimensions in table 2). In order to have an appropriate resolution of the blade leading and trailing edges, the meshes are conveniently re-clustered around these areas.

Since the used turbulence model considerably affects the prediction of the blade wake, which is the main acoustic source, a number of turbulence models were tested before performing acoustic calculations. From this study it was determined that the $k-\omega$ model in high-Reynolds formulation gives a better prediction of the wake spreading. Wall spacing for boundary layer was chosen so that the y^+ values of the mesh nodes closest to the wall were below unity. Steady state non-reflecting boundary conditions were employed, following Giles' formulation [25]. Steady results at three different spans of the last stator vane show a really good agreement in terms of static pressure (see fig. 6). Since the other stator vane shows a similarly good agreement (not shown here), the steady solution is a suitable starting point for the acoustic simulations.

TABLE 2. **H-type grid dimensions (cells)**

row	dimensions
V1	$236 \times 88 \times 88$
B1	$164 \times 88 \times 88$
V2	$164 \times 88 \times 88$
B2	$216 \times 88 \times 88$

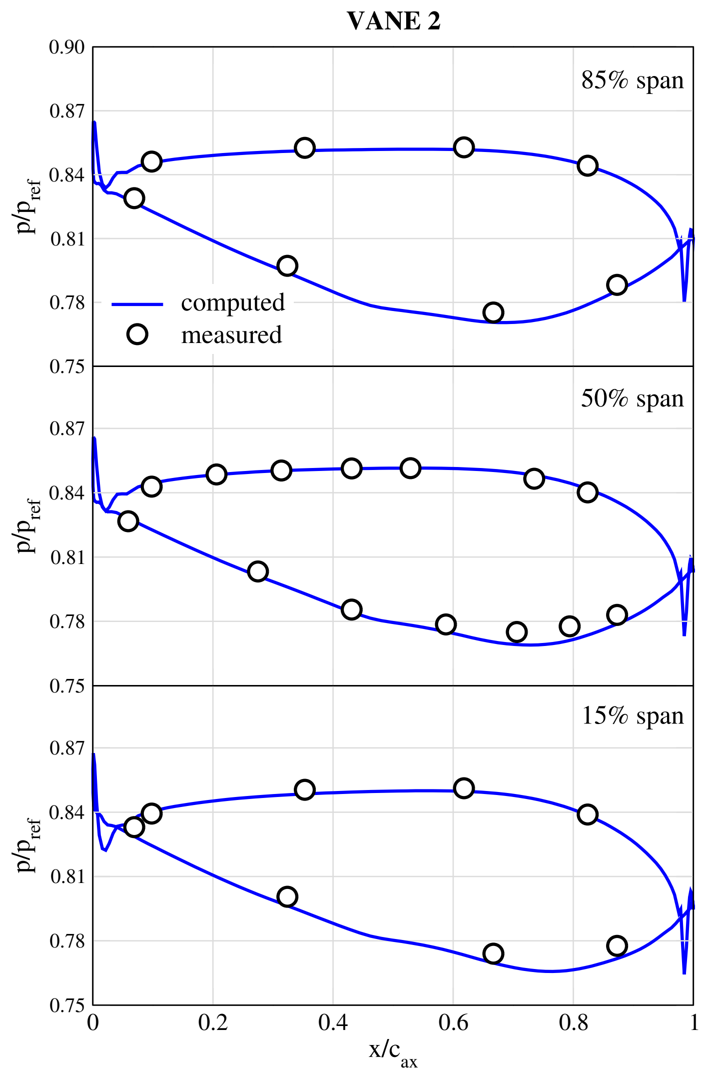

FIGURE 6. **Steady state results**

Tone noise computations The aim of this propagation study is to evaluate the acoustic emissions for the modes listed in table 1 at the turbine exit, where the noise measurements were made. To do this, three acoustic propagation analyses were performed at the approach operating point. It should be noted that before starting the propagation, radial mode shapes need to be computed at the grid boundaries for each single row since 3D non-reflecting boundary conditions are based on these modes. Each propagation analysis starts with the imposition of the acoustic and convected wave to the inlet of the receiving row. Then, only the acoustic modes are propagated across the successive rows up to the turbine exit. For all the analyzed interactions, a good accuracy in wave resolution is guaranteed by the adopted grids: even considering scattered waves, the number of grid points per wavelength is always above 30 for convected waves and 70 for acoustic waves [26,27]. Interactions +1V1-1B1 and +2B1-1V2 produce two cut-on modes ($\mu = 0, 1$) at the turbine exit, whereas four cut-on radial modes were found for the +2V1-2B1 interaction. Only cut-on modes will be compared in the following results, as they turn out to be the dominant ones at the measurement location.

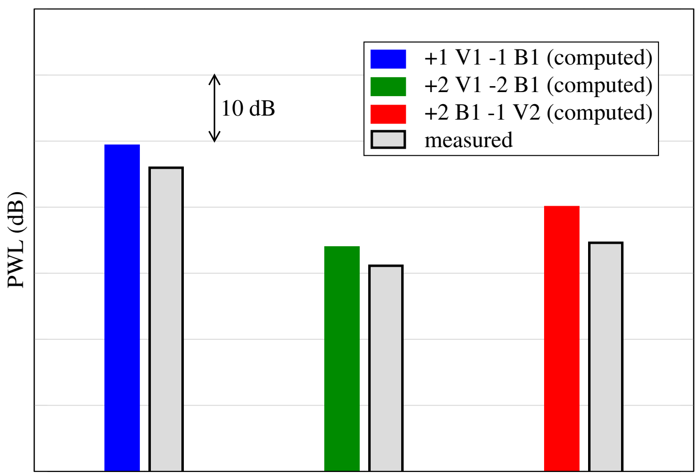

FIGURE 7. **Total PWL at B2 exit**

As far as the overall emissions of the interactions are concerned, fig. 7 shows the comparison with experimental data in terms of total PWL at the turbine exit (experimental sound power was computed using eqs. 4–5 from measured acoustic quantities). As can be seen, the agreement is fairly good. The total values are always slightly overestimated, but the trend is properly predicted. These slight discrepancies are most likely caused by an overestimation of the wake defect from the CFD computation.

A further confirmation of the accuracy of the result can be obtained by looking at the acoustic emissions split up into radial components. The histograms in figs. 8 and 9 represent the relative relevance of the radial components for the +1V1-1B1 interaction in terms of PWL and SPL (the latter is computed for given circumferential and radial orders at the radial location where the maximum sound pressure is found). These results illustrate the capabilities of the developed procedure in handling the radial

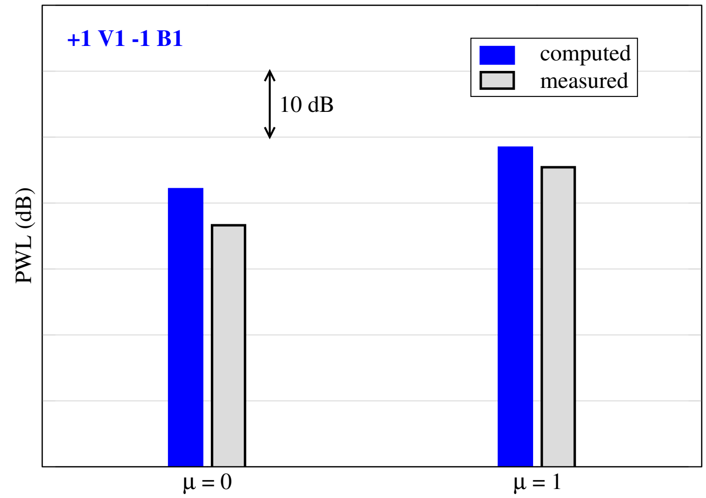

FIGURE 8. **+1V1-1B1 interaction (PWL)**

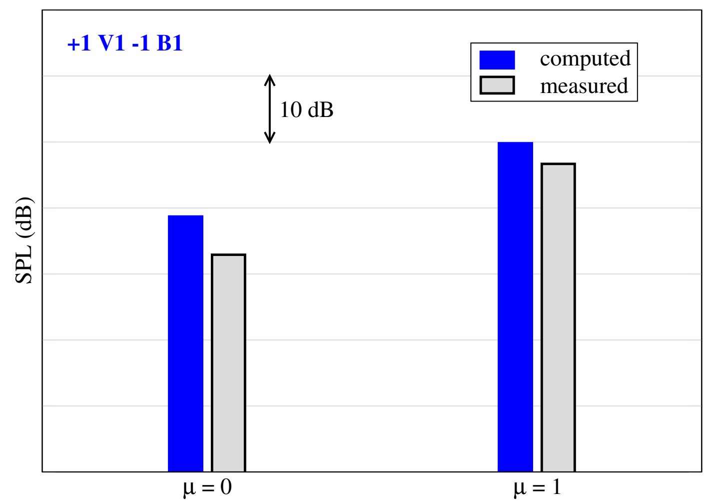

FIGURE 9. **+1V1-1B1 interaction (SPL)**

shape of the perturbation. As far as the experimental accuracy of SPL and PWL is concerned, it was highlighted during the calibration of sensors [28] that the total accuracy is little influenced by the resolution provided by the microphones. Therefore the variance of acoustic measurements is mainly due to the variance of test conditions (for instance, the SPL uncertainty for the two cut-on radial modes generated by +1V1-1B1 interaction at the turbine exhaust is about 1.5 dB). Further details concerning the experimental measurements can be found in [28, 17].

It is also possible to visualize the evolution of each single mode during the propagation along the machine. This can be made by extracting the propagating waves at various sections inside the inter-row gaps. Figure 10 shows the axial evolution of

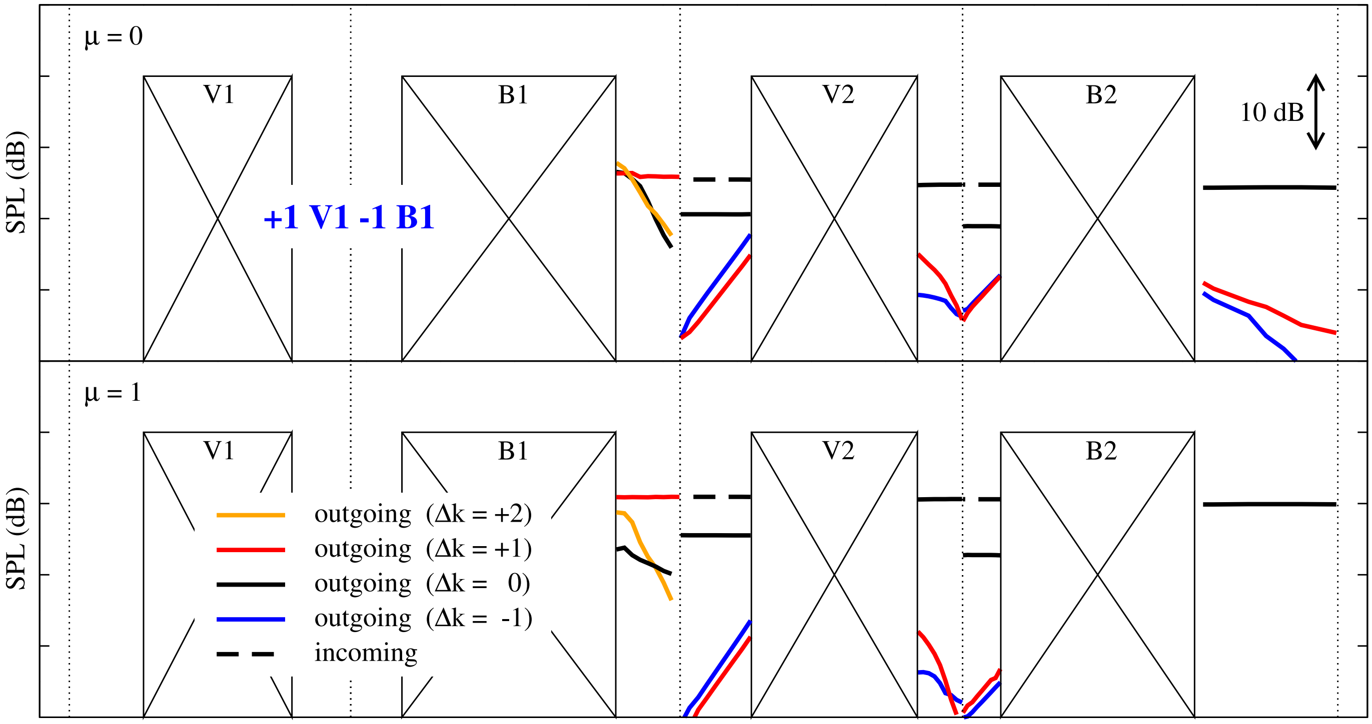

FIGURE 10. Evolution of the +1V1-1B1 interaction

the two cut-on modes in terms of SPL from their source up to the turbine exit. It can be seen how the acoustic perturbations perfectly match at the grid interfaces: there are no spurious reflections at the mesh boundaries which could have affected the acoustic solution. As a matter of fact all the incoming waves, but the imposed ones, are well below the outgoing waves and are not shown on the plot.

In addition fig. 11 gives an idea of the 3D acoustic solution: it visualizes the acoustic field at the turbine exit generated by the +1V1-1B1 interaction. The circumferential and radial shapes of the acoustic disturbance can be identified.

Additional investigations, which are currently ongoing, consider different operating conditions and turbine setups.

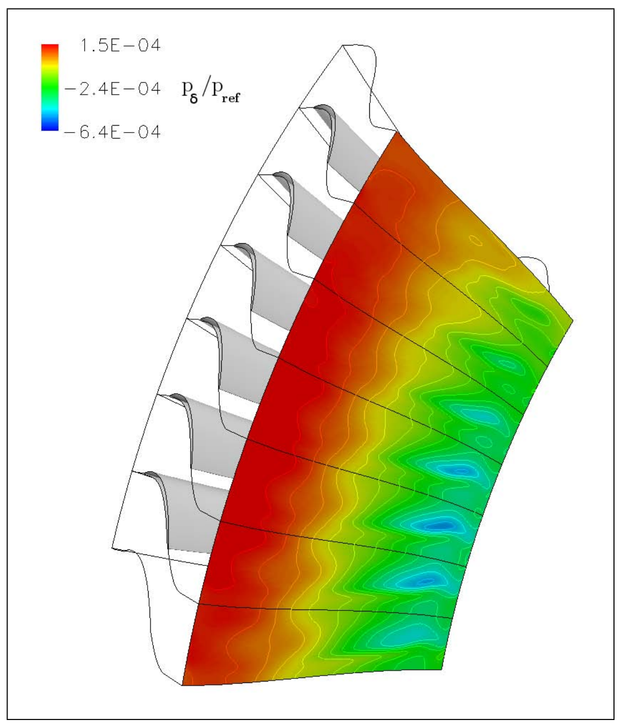

FIGURE 11. Acoustic pressure perturbation at turbine exit (+1V1-1B1 interaction)

CONCLUSIONS AND FUTURE WORK

A fully 3D numerical method was developed in order to evaluate the tone noise emissions at the exit of a low-pressure turbine. It uses a single row linearized aeroacoustic solver integrated in a proper sound propagation procedure to numerically solve the wave evolution along the machine. After a tone noise identification, sound generation is evaluated using a 3D wave splitting method based on the radial modes. Then, the relevant acoustic modes coming from each single row are propagated up to the turbine exhaust, where the overall acoustic power can be evaluated. The method was applied to evaluate the acoustic emissions of a cold-flow test rig located at Avio facility for which acoustic measurements are available at different operating points.

The results show a good agreement with experimental data in terms of total PWL. The total values are slightly overestimated, probably because of an overestimation of the wake defect from the CFD computation. Further investigations concerning the wake structure, carried out with methods which are able to resolve the physical details, such as large eddy simulation (LES) or direct numerical simulation (DNS), could suggest additional ideas to enhance RANS turbulence closures. This could consequently improve the matching with the experiments in terms of tone noise emissions at the turbine exhaust.

Since a TRF is foreseen to be mounted in the cold-flow rig to better represent low-pressure turbine exhaust, future work will be focused on the effects of TRF geometry and of possible multiple reflections between the last rotor and the TRF itself.

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