Proceedings of ASME Turbo Expo 2011
GT2011
June 6-10, 2011, Vancouver, British Columbia, Canada

GT2011-45390

BREAKUP OF LIQUID JETS EMERGING FROM ELLIPTIC ORIFICES

Ghobad Amini
Mech. Eng. Department
Concordia University
Montreal, QC, Canada
ghobad@encs.concordia.ca

Ali Dolatabadi
Mech. Eng. Department
Concordia University
Montreal, QC, Canada
dolat@encs.concordia.ca

ABSTRACT

Passive control can result in increasing fuel efficiency and reducing combustion instabilities of gas turbine spray combustors. Through the use of geometric modifications of the conventional circular nozzles, this method potentially enhances mixing which is responsible for entraining the bulk air necessary for combustion. Several studies show that elliptic jets have higher mass entrainment and spreading rate compared to the equivalent circular jets [1]. The majority of these works have been limited to gaseous jets. The present study focuses on a liquid spray discharging into still ambient air from a single-hole injector with elliptic cross-section. The primary breakup is investigated using a theoretical approach. Characteristics of elliptic orifice jet are compared with circular orifice jet under different breakup regimes and various nozzle geometries.

INTRODUCTION

In plain-orifice atomizers which are being used in diesel engines and afterburners (reheat systems), passing a low viscosity liquid fuel through a small circular hole creates a jet that disintegrates into spray droplets. Jet disintegration can be enhanced by increasing fuel injection pressure which increases both the level of turbulence in the fuel jet and the aerodynamic forces exerted by the surrounding medium [2]. The dispersion of spray drops in the surrounding gas is important in order to bring about efficient heat and mass transfer between liquid and gas phases.

Spray properties are influenced by a large number of parameters, including the jet velocity profile, surface tension, nozzle geometry, turbulence at the nozzle exit, and the physical and thermodynamic states of both liquid and gas [3], [4], [5]. In the case of a round liquid jet injected into a stagnant gas, four main breakup regimes have been identified that correspond to different combinations of the above mentioned factors (Figure 1). These four regimes are the Rayleigh regime (a to b), the first wind-induced regime (b to c), the second wind-induced regime (c to d), and the atomization regime (beyond d). In the atomization mode, as the jet Weber number is increased more, it becomes difficult to define a precise breakup length, and probability density functions are found useful to quantify the breakup length [6].

Few studies have considered noncircular orifices, though there has been interest in them since the nineteenth century [7]. In many instances when a fluid exits an elliptic orifice, the free jet behavior is oscillatory. From elliptic the cross-section becomes circular down the jet, then again elliptic but with major axis perpendicular to that of the elliptic orifice, then circular, then elliptic with major axis in the original direction and so on. This phenomenon is known as axis-switching which can be used for the measurement of dynamic surface tension [8].

Elliptical orifices have been studied for possible practical applications such as liquid propellant rocket injectors [9]. Ho and Gutmark [1] showed that for fuel gas with an elliptical nozzle, the mass entrainment in combustion process is three to eight times higher than a circular jet. Mebarka *et al.* [10] investigated experimentally the mixing characteristics of an elliptic liquid jet in a co-flow current. Their results show that the elliptic jets even with large aspect ratios have more dilution than an equivalent round jet. Messina and Acharya [11] studied experimentally the velocity field and spreading rate of an acoustically modulated liquid spray issued from an air assisted elliptic nozzle. By active forcing of air stream, they manipulated the mixing and growth of the spray.

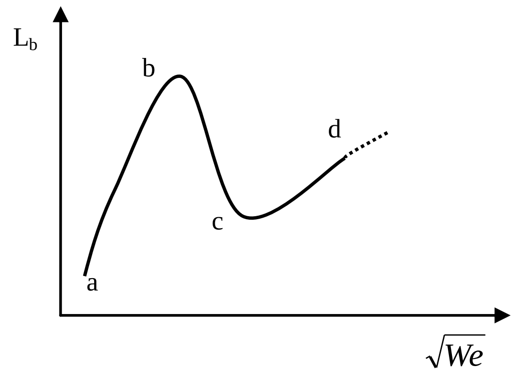

Figure 1. Jet stability curve; breakup length versus jet velocity.

Kasyap *et al.* [12] presented experimental results on the breakup of liquid jets issuing from elliptic nozzles. They described the visual observations on elliptic jets by the characterization of the axis-switching process and described the breakup curves of elliptic and circular jets (figure 2). Their work showed that a liquid jet emanating from an elliptic nozzle exhibits a faster breakup process than a corresponding circular liquid jet in a specific range of flow were axis switching was observed. Moreover, they found that increasing the aspect ratio of the elliptic nozzle in some ranges makes the elliptic liquid jet more unstable.

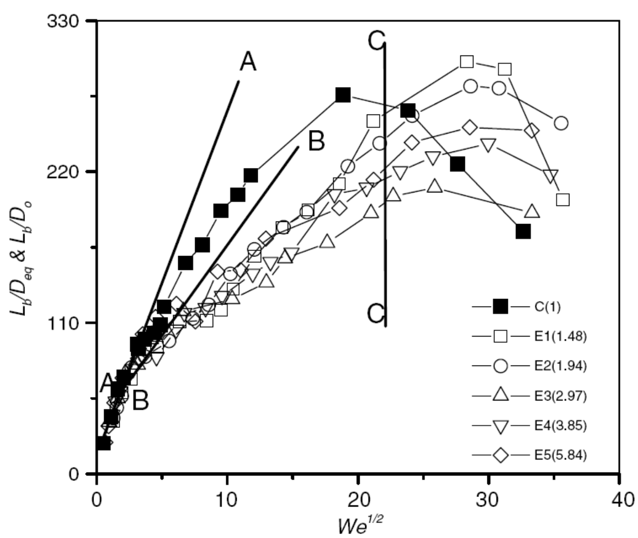

Figure 2. Variation of the dimensionless breakup length of elliptic jets against velocity ($\sqrt{We}$) for different ratios of major to minor axis (E), data from Ref. [12].

Regarding advantage of elliptic jets than conventional circular jets in terms of breakup and mixing characteristics, [10], [12], [13], this paper tries to investigate the effect of orifice eccentricity on instability of liquid jets issuing from elliptic orifices. Behavior of elliptic and circular liquid jets under the effect of various destabilizing forces will be compared with each other.

NOMENCLATURE

A	cross-sectional area of orifice (πab)
C_D	drag coefficient
D_e	equivalent diameter ($2(A/\pi)^{0.5}$)
D_h	hydraulic diameter ($4A/P$)
L	orifice length
L_b	breakup length of liquid jet
P	perimeter of ellipse
Δp_s	pressure difference across the fluid interface
Δp_l	Pressure loss in orifice due to friction
R_1, R_2	principle radii of curvature
Re	Reynolds number ($\rho_l U D_e \mu$)
R_e	equivalent radius (0.5 D_e)
S	contact surface between liquid jet and gas
U	average axial velocity
We	Weber number ($\rho_l U^2 D_e/\sigma$)
a	semi-major axis
b	semi-minor axis
e	ellipse aspect ratio (b/a)
f	Fanning friction factor
l_{fd}	fully developed length
l_r	relaxation length
u_z	local axial velocity
x, y	transverse coordinates
z	axial coordinate
μ	liquid dynamic viscosity
ρ_l	liquid density
ρ_g	gas density
σ	surface tension
ε	specific kinetic energy flux

METHODOLOGY

A convenient method for categorizing jet breakup regimes is to consider the length of the unbroken portion of the liquid jet, L_b, as a function of Weber number in terms of $\sqrt{We}$ (figure 1). Based on the trend of this curve, breakup behavior of elliptic and circular jets will be reviewed and their governing forces in each regime will be compared with each other. Considering ellipticity of liquid jet, main affecting parameters which are surface tension, aerodynamic forces, velocity relaxation and turbulence, will be discussed. Circular geometry will be considered as a special case of elliptic geometry when aspect ratio is one. A theoretical approach will be used to consider inside and outside effects of orifice.

Capillary force

Low jet velocities, small surface disturbances, and negligible aerodynamic effects characterize the Rayleigh breakup regime. The disturbances induced by surface tension forces are the dominant cause for jet breakup in this regime. Drops are pinched off from the end of the jet, with diameters comparable to that of the jet. Young–Laplace equation describes the capillary pressure difference sustained across the interface between two fluids due to the phenomenon of surface tension. It relates the pressure difference to the shape of the surface and is a statement of normal stress balance for static fluids meeting at an interface [14]. Young-Laplace equation is

$$\Delta p_s = \sigma\left(\frac{1}{R_1}+\frac{1}{R_2}\right) \quad (1)$$

where Δp_s is the pressure difference across the fluid interface, σ is the surface tension, and R_1 and R_2 are the principal radii of curvature, i.e., the radii of the two mutually perpendicular maximum circles which are tangent to the (two-dimensional) surface at the point of contact.

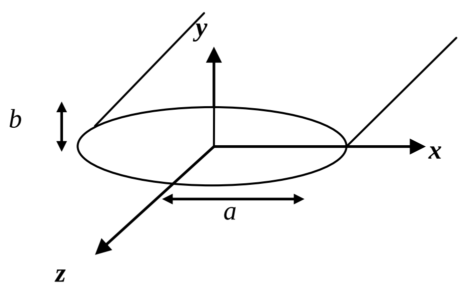

Figure 3. Cross section of elliptic orifice

For a circular jet, R_1 is infinity and R_2 is jet radius, R_e. For an elliptic jet R_1 is infinity too but R_2 is different at any point on ellipse; at the end of major axis is minimum, $R_2 = b^2/a$, and at the end of minor axis is maximum, $R_2 = a^2/b$ where a and b are semi-major and semi-minor axes respectively (figure 3). As a rough estimation, radius of curvature could be average of these maximum and minimum values which is equal to,

$$\frac{1}{2}\left(\frac{1}{a^2/b}+\frac{1}{b^2/a}\right)=\frac{1}{2}\left(\frac{1+e^3}{e^{1.5}}\right)\frac{1}{R_e} \quad (2)$$

where e is aspect ratio (ratio of minor to major axis) and R_e is the radius of a circular orifice which has the same cross-sectional area on an elliptic orifice. Average curvature in equation (2) is always greater than $1/R_e$ and by decreasing aspect ratio this difference becomes larger. Hence pressure difference due to surface tension in elliptic jet is greater than a circular jet. Moreover, interfacial area of this pressure difference is different in elliptic and circular configurations. Interfacial area is equal to perimeter multiplied by length of jet. For an ellipse with semi-major and semi-minor axes of a and b, perimeter is equal to (Ramanujan, first approximation)

$$P = \pi[3(a+b)-\sqrt{(a+3b)(3a+b)}] \quad (3)$$

and ratio of ellipse perimeter to circle perimeter, $2\pi R$, would be

$$\frac{3(1+e)-\sqrt{(1+3e)(3+e)}}{2\sqrt{e}} \quad (4)$$

For a unit length of jet, since perimeter of ellipse is always greater than a circle, i.e. ratio of (4) is always greater than one, interfacial area of elliptic jet is greater than circular jet. In conclusion, because of greater values of pressure difference and average curvature of an ellipse than that of a circle, capillary force in elliptic jets is larger than circular jets. Figure 4 shows their ratio as a function of aspect ratio. Since in Rayleigh and first-wind induced regime, surface tension is the dominant factor of breakup, one concludes that elliptic jets must be more unstable than circular jets and by increasing ellipticity, it becomes more unstable. However, in the second wind-induced and atomization regimes which are genuinely wind-induced, surface tension acts against the formation of small droplets generated by the interfacial pressure fluctuations [15].

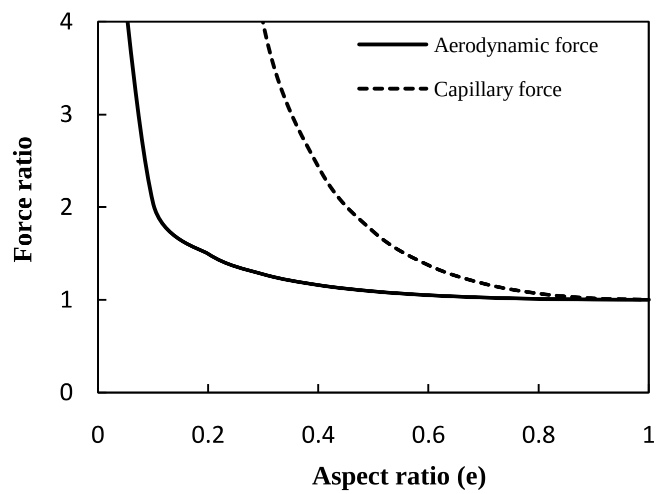

Figure 4. Ratio of aerodynamic and capillary forces on elliptic jet to those of circular jet as a function of aspect ratio.

Aerodynamic forces

The breakup of a low-speed jet is the result of the developing of the axisymmetric disturbance whose instability is produced by the surface tension. The breakup and atomization of high-speed jet results from the evolution of the asymmetric disturbance whose instability is caused by the aerodynamic force on the interface between the liquid and the gas because of relative velocity [4].

In the first wind-induced breakup regime, disturbances which are amplified by aerodynamic forces, disintegrates the jet into drops comparable to the size of the jet diameter. Nevertheless the capillary force remains dominant over the wind force. As the relative speed of gas-to-liquid increases (second-wind induced regime), the gas pressure fluctuation assists significantly the capillary force to break up the liquid jet. The breakup starts at some distance downstream of the nozzle exit, and a smooth unbroken section of the jet can be seen. The aerodynamic force is equal to

$$F = \frac{1}{2} C_D \rho_g U^2 S \tag{5}$$

where ρ_g is surrounding gas density, U is relative velocity of liquid to gas (assuming quiescent), and C_D is drag coefficient [14]. S is contact surface between liquid and gas which is equal to perimeter of jet, p, multiplied by the jet length. The aerodynamic force increases with an increase in the liquid gas relative velocity, air density and liquid-gas contact surface area. Among these parameters, liquid-gas contact surface of an elliptic jet is larger than a circular jet because of its larger perimeter which has been shown in equation (4). In conclusion, applied aerodynamic force on elliptic jet is larger than one applied on a circular jet. The ratio of this force in elliptic jet to circular jet is equal to equation (4). Figure 4 shows the ratio of aerodynamic force in elliptic jets to circular jets as a function of aspect ratio. This extra force assists to disintegrate the jet faster. It has to be noted that drag coefficient, C_D is a function of shape and is different for elliptic and circular jets. C_D is known for elliptic jets in cross-flow and is larger than that of circular jets; but to the best knowledge of authors, it has not been reported for elliptic shapes in co-flow jets. Furthermore, friction drag is comparable with wall friction inside pipes, which will be shown that in elliptic case is larger than that of circular case.

Velocity relaxation

The change in velocity profile that occurs downstream of the nozzle exit can have an important influence on the stability of the jet and on its subsequent breakup into drops [2], [16], [17], [18]. Once the constraint of the nozzle wall is removed at the nozzle exit, the process of velocity profile relaxation occurs by a mechanism of momentum transfer between transverse layers within the jet. This disruptive mechanism coordinates with other forces to destabilize the jet. The kinetic energy flux which depends on the character of the mean velocity profile generated by the nozzle geometry can be quantified by introducing the specific kinetic energy flux, ε. defined as

$$\varepsilon = \frac{\int u_z^3 dA}{U^3 A} \tag{6}$$

where u_z is the local axial velocity inside orifice and U is average axial velocity over cross sectional area, A. It can be shown easily that in circular tube, this parameter for plug flow is equal to 1, for fully developed laminar flow is 2 and for fully developed turbulent flow is between 1.1 and 1.2.

In order to minimize pressure losses, spray nozzles are of compact size and normally with short orifices. Many researchers have used long tubes as nozzles to ensure that the jet initially possesses either a fully developed laminar (parabolic) or a fully developed turbulent velocity profile. This attempt is to standardize the velocity profile in the emerging jet. Turbulent flow profiles are not significantly different from uniform profiles and are slightly susceptible to profile relaxation effects [2].

Poiseuille flow in a tube of elliptic cross section [14] has a velocity profile of

$$u_z(x, y) = 2U(1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}) \tag{7}$$

Integration of numerator of equation (6) using chain rule results

$$\int u_z^3 dA = 2\pi ab U^3 \tag{8}$$

which finally gives ε equal to 2. Since result does not depend on values of a and b, kinetic energy flux would be the same in fully developed flow in circular and elliptic case. On the other hand, it has been shown [19] that entrance length in elliptic pipes is smaller than that of circular pipes (figure 5). This means that in a short circular orifice, the velocity profile is close to plug flow, while in an elliptic orifice with the same length, velocity profile is close to parabolic flow. Consequently, in short orifices, specific energy flux, ε, is greater in elliptic orifices.

At the inlet of an orifice, the flow boundary condition changes from slip to no-slip while at the orifice exit, it changes from no-slip to slip condition. It has been suggested ([18], [20]) that relaxation length, the jet length required for the velocity profile to relax to a uniform velocity profile, is comparable with the entry length for laminar flow in a pipe. Brun and Lienhard [20] developed a relation for velocity profile relaxation as a function of downstream distance,

$$\frac{l_r}{D} = 0.025\,\mathrm{Re} \tag{9}$$

which is comparable with entrance length for laminar fully developed flow [21]:

$$\frac{l_{fd}}{D} = 0.057\,\mathrm{Re} \tag{10}$$

where D is diameter, l_r is relaxation length and l_{fd} is fully developed length of a circular orifice. To the best knowledge of authors, relaxation length of elliptic jet has not been reported; but since entrance length of elliptic pipes is smaller than that of circular pipes [19], it can be concluded that relaxation length of elliptic jets is also smaller than that of circular jets. In summary, the kinetic energy flux which transferred by velocity relaxation from an orifice, is greater and affects in a shorter length in an elliptic orifice than in a circular orifice. At the end it results increase of growth rate of instability in elliptic liquid jets and lead to breakup in a shorter distance than that of a circular orifice.

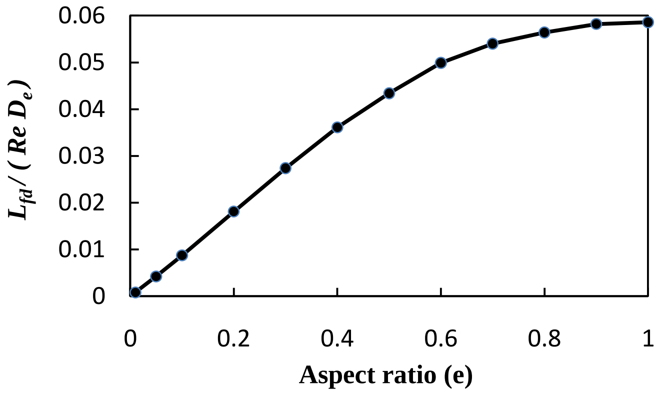

Figure 5. Hydrodynamic entrance lengths for elliptic pipes as a function of aspect ratio, data from Ref. [19]

Transition to turbulence

For high velocity jets, the action of the surrounding gas is the primary cause of atomization. In this case jet turbulence is a contributing factor because it ruffles the surface of the jet, making it more susceptible to aerodynamic effects. In this regime the jet consists of an unbroken inner liquid core in the vicinity of the nozzle exit, and droplets which are much smaller the jet diameter, are stripped from the core by the action of aerodynamic forces at the liquid-gas interface [5].

A turbulent flow is characterized by radial velocity profiles that tend to destabilize jet's surface and promote the breakup. In smooth tube with no disturbances, an initially laminar flow can remain laminar up to Reynolds numbers much higher than the critical value, but when Reynolds number exceeds the critical value, only a small disturbance is required to initiate a transition to turbulent flow [2]. The increase of turbulence intensity by nozzle wall friction promotes a higher level of initial perturbation.

Pressure loss due to wall friction in a pipe with a length of L, where average axial velocity is U and liquid density is ρ_l, is equal to

$$\Delta p_l = 2f\frac{L}{D_h}\rho_l U^2 \tag{11}$$

f is Fanning friction factor and D_h is hydraulic diameter where

$$D_h = \frac{4A}{P} \tag{12}$$

A is the area and P is the perimeter of pipe cross-section [14]. With a constant cross-sectional area, hydraulic diameter of an elliptic pipe is smaller than that of a circular pipe. On the other hand, it has been shown that Fanning friction factor of elliptic orifices is larger than that of equivalent circular orifices. Figure 6 shows variation of friction factor by dimensionless distance from orifice inlet, for different aspect ratios [19]. In entrance region, f does not change with aspect ratio, but in fully developed zone, by decreasing aspect ratio, friction factor in increased. Considering hydraulic diameter and friction factor, pressure loss and friction in an elliptic orifice are larger than that of an equivalent circular orifice. In conclusion, since friction promotes turbulence and turbulence increases instability, the exiting jet from an elliptic orifice could be more unstable than an equivalent circular orifice.

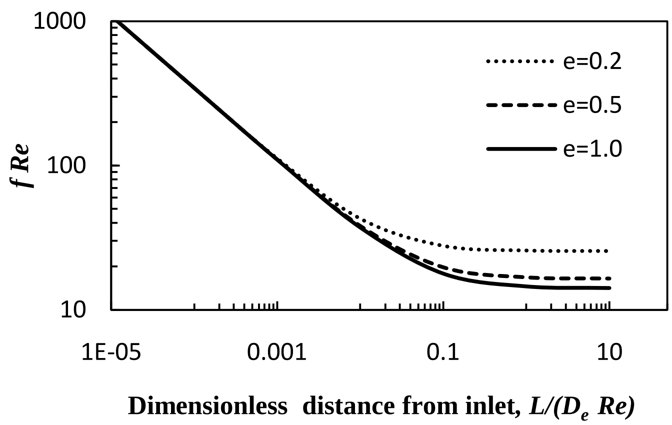

Figure 6. *f Re* for developing laminar flow in elliptic pipes, data from Ref. [19]

Figures 7, 8, 9 in the appendix show shadowgraph images of liquid jets emerging from one elliptic nozzle with aspect ratio of 0.5 and one circular nozzle. Length of both nozzles is 9.7 mm and their equivalent diameter is 0.7 mm. In each case, flowrate is the same which with the same cross-sectional area gives the same *Re* and *We* numbers. It can be seen that breakup length of elliptic jets is much shorter than that of circular jet. The higher pressure drop of elliptic nozzles in comparison with circular nozzles was observed during these experiments. By using short nozzles, this effect can be reduced considerably.

SUMMARY AND CONCLUSION

Regarding advantages of elliptic jets than conventional circular jets in terms of shorter breakup length and better mixing, this study has explained the physical reasons of these characteristics. Instability of liquid jets issuing from elliptic nozzles in small and moderate Reynolds numbers was investigated and its behavior was compared with circular jets.

Major affecting parameters are surface tension, aerodynamic forces, velocity profile relaxation and turbulence which have been compared between elliptic and circular jets. Due to the larger curvature and perimeter of elliptic jets than circular jets, capillary force becomes greater. Surface tension is the dominate parameter in Rayleigh and first-wind induced regimes and destabilizes the liquid jet while in higher speeds has the stabilizing effect. These different behaviors, explain decreasing and increasing of the breakup length of elliptic nozzles in comparison with circular nozzles (figure 2). Due to higher perimeter of ellipse than circle, exposed surface to drag force is larger which results a larger aerodynamic force than circular jet. Specific energy flux is larger for elliptic jets and also relaxation length for elliptic jet is shorter than circular jet which assists to break up the liquid jet in a shorter length. Finally, regarding more wall friction in elliptic orifice, intensity of turbulence can be promoted which helps to destabilize the elliptic liquid jet.

ACKNOWLEDGMENTS

The authors would like to thank the Natural Sciences and Engineering Research Council Canada (NSERC) for the financial support.

REFERENCES

[1] Ho C. and Gutmark E. J. 1987. “Vortex induction and mass entrainment in a small aspect ratio elliptic jet” J.Fluid Mech, **179**, pp. 383-405.

[2] Lefebvre A. H. 1989. “Atomization and sprays”, Hemisphere Pub. Corp.

[3] Chigier N. and Reitz R. D. 1996. “Regimes of jet breakup and breakup mechanisms (physical aspects). In recent advances in spray combustion: spray atomization and drop burning phenomena”, ed. K. K. Kuo, **1**, pp. 109–35.

[4] Lin S. P. and Reitz R. D. 1998. “Drop and spray formation from a liquid jet” Annu. Rev. Fluid Mech., **30**, pp. 85–105

[5] Reitz R. D. and Bracco F. V. 1982. “Mechanism of atomization of a liquid jet” Phys. Fluids, **25**, pp. 1730–42

[6] Leroux S., Dumouchel C. and Ledoux M. 1996. “ The stability curve of Newtonian liquid jets” Atomization and Sprays **6**, pp. 623–47

[7] Lord Rayleigh 1879a. ‘‘On the capillary phenomena of jets. Proceedings of the Royal Society of London’’, **29**, pp. 71-97.

[8] Bechtel S. E., M. G. Forest, D. D. Holm, and K. J. Lin 1988. “1-D closure models for 3-D incompressible viscoelastic free jets: Von Karman flow geometry and elliptical cross section” J. Fluid Mech., **196**, pp. 241-62.

[9] McHale R. M., Nurick W. H., and Clapp S. D. 1971. “Injector design criteria using noncircular orifice geometry. J. Spacecraft Rockets”, **8**, 4, pp. 408-10.

[10] Mebarka H., Abdelkarim H., and Abdellah G. 2007. “Exper-
imental investigation of elliptical jet in coflow”. International Journal of Applied Engineering Research, **2**, 1, pp. 31-43.

[11] Messina T. and Acharya S. 2006. “Characteristics of an acoustically modulated spray issued from circular and elliptical orifice nozzles” Atomization and Sprays, **16**, pp. 331-48.

[12] Kasyap T. V., Sivakumar D., and Raghunandan B. N. 2009. “Flow and breakup characteristics of elliptical liquid jets” International journal of multiphase flow, **35**, 1, pp. 8-19.

[13] Amini G. and Dolatabadi A. 2010. “Capillary instability of liquid jets issuing from elliptical nozzles” FEDSM ICNMM-2010-30549. Proceedings of ASME 2010

[14] White F. M. 2006. ‘‘Viscous fluid flow’’, McGraw-Hill, New York.

[15] Dumouchel C. 2008. ” On the experimental investigation on primary atomization of liquid streams” Exp. Fluids, **45**, pp. 371–422

[16] McCarthy M. J. and Molloy N. A. 1974. “Review of stability of liquid jets and the influence of nozzle design”. The Chemical Engineering Journal, 7, pp. 1-20.

[17] Sander W. and Weigand B. 2008. “Direct numerical simulation and analysis of instability enhancing parameters in liquid sheets at moderate Reynolds numbers”. Physics of Fluids, **20**, pp. 1-18.

[18] Sterling A. M. and Sleicher C. A. 1975. “The instability of capillary jets” J. Fluid Mech. **68**, pp. 477–95

[19] Muzychka Y. S. and Yovanovich M. M. 2009. “Pressure drop in laminar developing flow in noncircular ducts: a scaling and modeling approach” Journal of Fluid Engineering, **131**, pp. 111105-11

[20] Brun R. F. and Lienhard J. H. 1968. ‘‘Behaviour of free laminar jets leaving Poiseuille tubes’’, ASME Paper, 68-FE-44

[21] Langhaar H. L. 1942. “Steady flow in the transition length of a straight tube” *J. Appl. Mech.*, **9**, pp. A55–A58.

APPENDIX

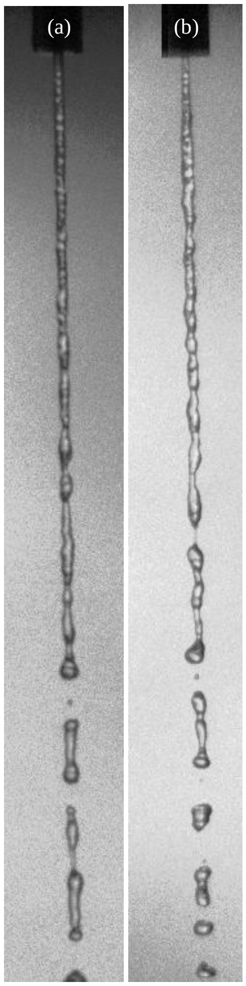

Figure 7. Breakup of liquid jets at Re= 6,400, We= 500 emerging from a) circular nozzle and b) elliptic nozzle with aspect ratio of 0.5.

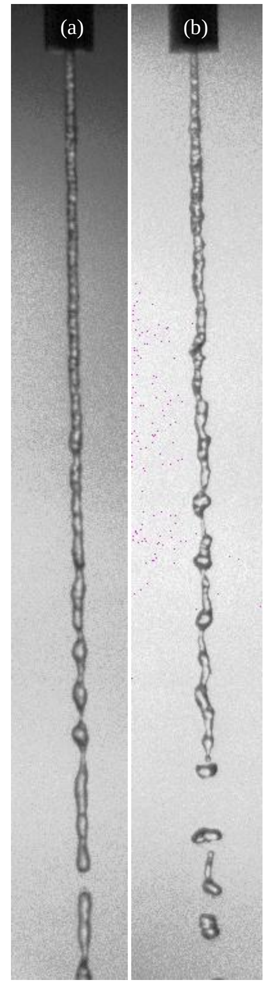

Figure 8. Breakup of liquid jets at Re= 8,500, We= 900 from a) circular nozzle and b) elliptic nozzle with aspect ratio of 0.5.

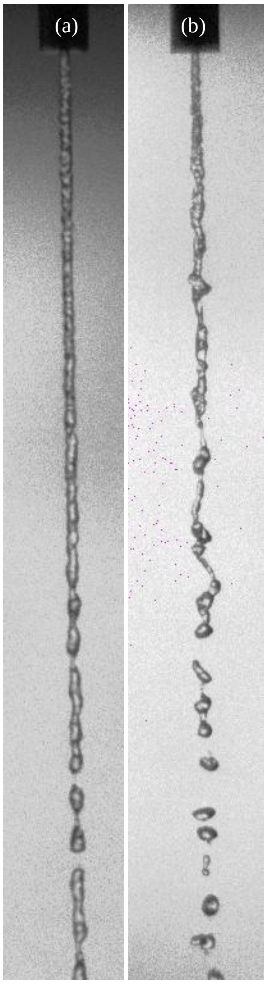

Figure 9. Breakup of liquid jets at *Re*= 10,600, *We*= 1,400 emerging from a) circular nozzle and b) elliptic nozzle with aspect ratio of 0.5.