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A COMPARATIVE ANALYSIS OF TURBINE ROTOR INLET TEMPERATURE MODELS

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ABSTRACT

Life usage algorithms constitute one of the principal components of gas turbine engines monitoring systems. These algorithms aim to determine the remaining useful life of gas turbines based on temperature and stress estimation in critical hot part elements. Knowing temperatures around these elements is therefore very important.

This paper deals with blades and disks of a high pressure turbine (HPT). In order to monitor their thermal state, it is necessary to set thermal boundary conditions. The main parameter to determine is the total gas temperature in relative motion at the inlet of HPT blades T_w^*. We propose to calculate this unmeasured temperature as a function of measured gas path variables using gas path thermodynamics.

Five models with different thermodynamic relations to calculate the temperature T_w^* are proposed and compared. All temperature models include some unmeasured parameters that are presented as polynomial functions of a measured power setting variable. A nonlinear thermodynamic model is used to calculate the unknown coefficients included in the polynomials and to validate the models considering the influence of engine deterioration and operating conditions. In the validation stage, the polynomial's inadequacy and the errors caused by the measurement inaccuracy are analyzed. Finally, the gas temperature models are compared using the criterion of total accuracy and the best model is selected.

INTRODUCTION

It is well known that the thermo-mechanical stresses in gas turbine hot part elements are extremely high, making turbine blades and disks one of the most critical engine elements.

In gas turbine useful life estimation algorithms, it is a common practice to assess the life by a number of remaining start-stop cycles, standard flights or missions. This is done through detailed calculations of critical element temperatures and stresses for a standard operational profile using material properties and failure models [1]. A limitation of existing stress and temperature calculations is that they are based on design temperatures and stress values, which may significantly differ from actual values because of the differences in real operational and engine health conditions. In order to significantly enhance the accuracy of the life prediction, the temperatures and stresses of the hot part elements need to be computed for actual conditions [2, 3].

The finite-element models are usually used to determine critical element stresses in the design stage. However, they are too complex for real-time analysis of maintenance data. For this reason some efforts are made to design simplified but precise enough models to estimate the temperature and stress in critical points of the elements [4-7]. All these models need temperature boundary conditions, which will determine the heat exchange between the hot part elements and external gas and cooling air. To estimate accurately the remaining life, it is important to know as precisely as possible these boundary conditions for real engine operation.

The thermal-stress condition of the turbine rotating elements depends on the temperature T_w^*. Reliable instruments for its direct measuring currently do not exist. Therefore, it is extremely important to create an accurate model for estimating T_w^* at any engine operating conditions. Such a model will determine the boundary conditions around the blades and discs and will allow calculating the temperature and stress distribution in these hot part elements. The model is intended for an onboard remaining life monitoring algorithm. As a consequence, the model must be kept as simple as possible in order to save computing time and memory.

Represent the unmeasured temperature T_w^* as a function of an engine operating mode for a healthy engine is possible, but such a representation will give significant errors in the case of a deteriorated engine. To overcome this problem, this article proposes that the temperature T_w^* be calculated as a function of measured gas path variables using the thermodynamic relations described, for example, in the textbook [8]. To take into account variations of ambient condition, all the variables are corrected to the standard atmosphere.

Some alternative temperature models are considered in the paper. The final choice is taken after their detailed comparative analysis. Accuracy of the obtained temperature T_w^* is employed as a criterion to choose the best model.

To verify the above ideas, an industrial aeroderivative turboshaft engine has been chosen as a test case. Its thermodynamic model allows generating all the data necessary to create and validate the proposed temperature models. This nonlinear model, in which each module is presented by its full manufacture performance map, demonstrates the option of a physical model. The capacity to reflect normal engine behavior is based on objective physical principles realized. Since the faults affect the module performances involving in the calculations, the thermodynamic model has special fault parameters for simulating gas turbine degradation. In the thermodynamic model, the gas path variables $\vec{Y}$ relate with the operating conditions $\vec{U}$ and the fault parameters $\vec{\Theta}$, i.e. present a vector function $\vec{Y}(\vec{U},\vec{\Theta})$.

This function is computed as a solution of an algebraic equations system reflecting the conditions of a gas turbine modules combined work. The software consists of approximately 60 subprograms, most of them are universal. The model was previously adapted to real engine data using the matching procedures described in [9,10].

NOMENCLATURE

c	**Absolute velocity**
Cp	**Specific heat at constant pressure**
F	**Area**
G	**Mass flow**
H_U	**Lower heating value**
k	**Isentropic factor**
L	**Specific work**
m	**Coefficient in the flow equation**
N	**Power**
n	**Rotor speed**
p	**Pressure**
p_0=101,3 kPa	**Standard atmospheric pressure**
R	**Gas constant**
$q(\lambda)$	**Flow function**
T	**Temperature**
T_0=288,16 K	**Standard atmospheric temperature**
w	**Relative velocity**
η	**Efficiency**
ν	**Air to gas flow ratio**
π	**Pressure ratio**
σ	**Pressure conservation factor; Mean square error**

Abbreviations, subscripts and superscripts

a	**Air**
C	**Compressor**
CC	**Combustion chamber**
c	**Absolute velocity**
cor	**Corrected variable**
F	**Fan**
f	**Fuel**
g	**Gas**
H	**Atmospheric parameters**
HP	**High pressure cascade**
HPT-LPT	**Duct between turbine cascades**
in	**Inlet**
LP	**Low pressure cascade**
m	**Mechanical**
MSE	**Mean square error**
NB	**Nozzle box**
T	**Turbine**
w	**Relative velocity**
*	**Stagnation parameter**

TEMPERATURE MODEL DEVELOPMENT

There is a common approach to calculate engine unknown (unmeasured) variables that is based on a linearized model and estimation procedures like Kalman filtering [11-13]. Non-linear matching procedures [14-16] may also be applied. The principal advantage of these methods is the use of a priori information about thermodynamic relations between measured and unmeasured variables given by the thermodynamic model. However, applicability of such methods significantly depends on the structure of a measurement system, which in turn determines the identifiability of the engine model.

There is an alternative approach to calculating the unmeasured variables on the basis of regression models. These models relate measured and unmeasured variables using, for example, exponential functions [17-20] or neural networks [21, 22]. Nevertheless, the robustness of such models to engine performance deterioration is questionable.

All previously mentioned approaches have no alternatives in the estimation of integral engine parameters such as thrust and specific fuel consumption. However, for some local gas path variables, another approach can be proposed. It is based on thermodynamic relations between the unknown variable to be estimated and the measured gas path variables. In the present paper such an approach is used to determine the gas temperature T_w^* as described below.

First, a general dependency of the temperature T_w^* from a commonly used variable, turbine inlet temperature T_g^*, is obtained. Then, five particular expressions that relate the temperature T_g^* with measured variables are formed. All gas path variables are corrected to standard atmospheric conditions in order to take into account the influence of ambient air parameters. Thus, we obtain five T_w^* models, each of them including the same dependency $T_w^* = f(T_g^*)$ and a particular expression for the temperature T_g^*. Finally, after the analysis of truncation and instrumental errors, some better candidates are selected from the five initial candidates. The choice of the best model to be recommended for practice is taken after validating the candidates on the data of deteriorated engines.

Dependency $T_w^*(T_g^*)$

The relation between T_w^* and T_g^* is based on the formula of turbine stage variables

$$Cp_g \cdot T_w^* + \frac{w_1^2}{2} = Cp_g \cdot T_g^* + \frac{c_1^2}{2},$$

which results in

$$T_{w.cor}^* = T_{g.cor}^* - \frac{c_{1.cor}^2 - w_{1.cor}^2}{2 \cdot Cp_g} = T_{g.cor}^* - C_6, \qquad (1)$$

where a coefficient C_6 is expressed by

$$C_6 = \frac{c_{1.cor}^2 - w_{1.cor}^2}{2 \cdot Cp_g}. \qquad (2)$$

Thus, the coefficient C_6 depends on $c_{1.cor}$ and $w_{1.cor}$ – absolute and relative corrected velocities at the turbine rotor inlet.

Models for the temperature T_w^*

Model 1

This model involves the equation of high pressure rotor energy balance

$$Cp_g \cdot \left(T_g^* - T_{HPT}^*\right) \cdot \eta_{m.HP} = Cp \cdot \left(T_C^* - T_{in}^*\right) \cdot \nu .$$

After some mathematical simplifications and the thermodynamic parameters correction, the model for $T_{w.cor}^*$ is written as:

$$T_{w.cor}^* = T_{HPT.cor}^* + C_1 \cdot \left(T_{C.cor}^* - T_0\right) - C_6, \qquad (3)$$

where

$$C_1 = \frac{Cp \cdot \nu}{Cp_g \cdot \eta_{m.HP}}. \qquad (4)$$

Model 2

This model is based on the equation of the turbine's work:

$$L_{HPT} = Cp_g \cdot \left(T_g^* - T_{HPT}^*\right) = Cp_g \cdot T_g^* \left(1 - \frac{1}{\pi_{HPT}^{\frac{kg-1}{kg}}}\right)\eta_{HPT}^*,$$

where

$$T_{w.cor}^* = \frac{T_{HPT.cor}^*}{1 - C_2} - C_6, \qquad (5)$$

$$C_2 = \left(1 - \frac{1}{\pi_{HPT}^{\frac{kg-1}{kg}}}\right)\eta_{HPT}^*. \qquad (6)$$

Model 3

From the combustion chamber energy balance equation

$$Cp \cdot T_C^* \cdot G_a + H_U \cdot \eta_{CC} \cdot G_f = Cp_g \cdot T_g^* \cdot G_g, \qquad (7)$$

it follows that model 3 can be written as

$$T_{w.cor}^* = T_{C.cor}^* \cdot C_{3.1} + G_{f.cor} \cdot C_{3.2} - C_6, \qquad (8)$$

$$C_{3.1} = \frac{Cp \cdot \nu}{Cp_g}; \; C_{3.2} = \frac{H_U \cdot \eta_{CC}}{Cp_g \cdot G_{g.cor}}. \qquad (9)$$

Model 4

This model is based on the equation (8). However, the fuel flow is now included in the coefficient C_4 through an air to gas flow ratio ν:

$$T_{w.cor}^* = T_{C.cor}^* \cdot C_{3.1} + C_4 - C_6, \qquad (10)$$

$$C_4 = \frac{H_U}{Cp_g}\eta_{CC}(1 - \nu). \qquad (11)$$

Model 5

From the mass conservation equation

$$m\frac{F_g \cdot q(\lambda_g) \cdot P_g^*}{\sqrt{T_g^*}} = m\frac{F_{HPT} \cdot q(\lambda_{HPT}) \cdot P_{HPT}^*}{\sqrt{T_{HPT}^*}}$$

applied to the HPT, the formula for model 5 is expressed as

$$T_{w.cor}^* = T_{HPT.cor}^* \cdot C_5 - C_6, \qquad (12)$$

where

$$C_5 = \left[\frac{F_g}{F_{HPT}} \cdot \frac{P_C^*}{P_{HPT}^*} \cdot \frac{q(\lambda_g)}{q(\lambda_{HPT})}\right]^2. \qquad (13)$$

The coefficients C_i included in the models described above contain some engine parameters which depend on an engine operating mode. To take it into consideration, all the coefficients are presented as polynomial functions:

$$C_i = \sum_{j=0}^{n} a_{i.j} \cdot x^j, \qquad (14)$$

where an engine power set variable x is a function argument, $a_{i.j}$ are constant values, and n means the order of polynomial. The constants depend on what engine variable is used as the argument. Different variables are analyzed and the best one is chosen at the model validation stage.

Errors of the temperature models

Figure 1 gives general structure of the algorithm to compute the temperature T_w^* and to determine five described temperature models. The figure helps to classify possible sources of temperature errors as follows:

I. Instrumental errors:
- errors of measured engine parameters;
- errors of measured atmospheric conditions;

II. Truncation errors:
- errors of the temperature model itself;
- errors of the polynomials to compute models' coefficients;
- errors of correction formulas.

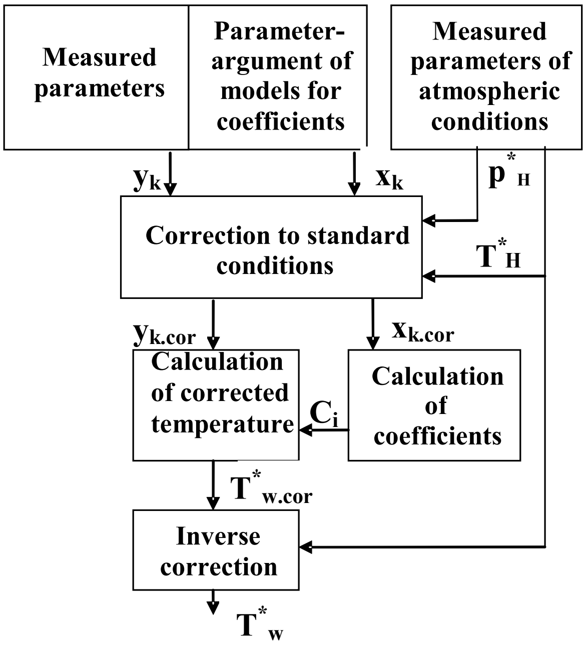

Fig. 1. Structure of the algorithm to compute the temperature T^*_w

DETERMINATION OF THE POLYNOMIAL FUNCTIONS

Healthy engine data approximation

The data set to determine unknown constants of the polynomials (14) was generated by the thermodynamic model of the analyzed engine. In this simulation, the engine was considered as healthy. The following values of the model inputs determine simulated engine modes:

T_H = 318, 308, 303, 298, 288, 278, 268, 253 and 243 K;
p_H = 101325, 90000 and 80000 kPa;
T^*_C = 680, 640, 612, 580 and 544 K;
n_{LP} = 6500, 5850, 5200, 4550, 3900 and 3250 rpm.

A total number of the modes (points of computation) was 245 and simulated parameters (coefficients *Cj* and different gas path variables used as a power set parameter *x*) from all the points were included in the data set.

In order to determine a proper structure of polynomials (14), the first step was to assign a proper polynomial order. Multiple calculations were performed for different coefficients *Cj* and with different arguments *x*. As a result, the second order has been chosen for all polynomials because a further order increase resulted in a negligible reduction of approximation errors (less than 5%).

The next step was to analyze what engine variable *x* is the best argument of the polynomial functions (14). The following engine variables were considered as candidates:

$$n_{HP.cor}, p^*_{C.cor}, T^*_{C.cor}, G_{f.cor}, T^*_{HPT.cor}, p^*_{HPT.cor}, T^*_{LPT.cor}.$$

Figure 2 illustrates the results of the experiments with different arguments on the example of the coefficient C_6. The polynomials are given here by curves and the thermodynamic model data are presented by points. It can be seen that the polynomial function accuracy depends on the chosen function argument.

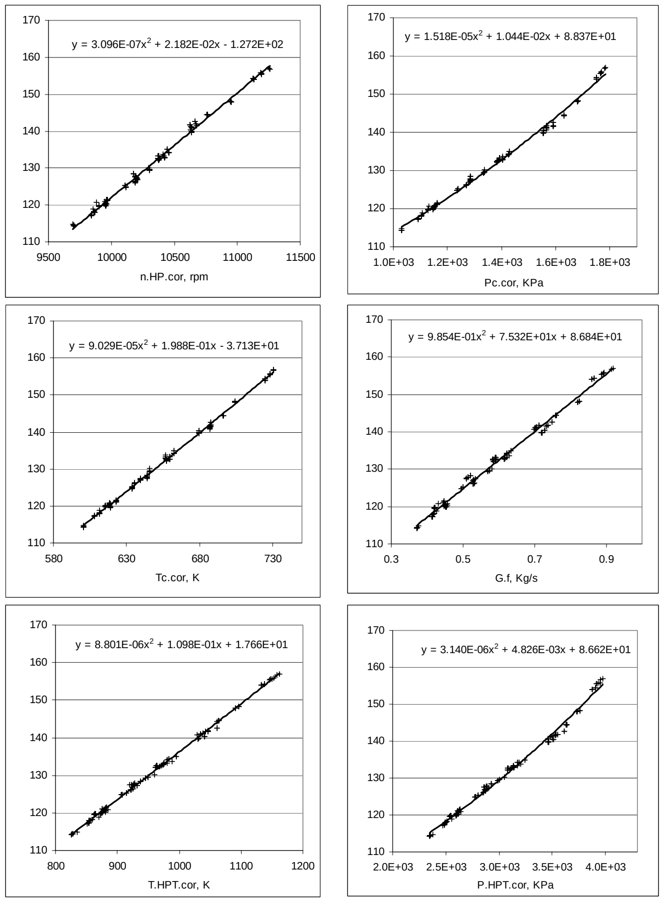

Fig. 2. Behavior of C_6 functions with different arguments

Table 1 contains in each cell the mean square error (MSE) obtained by the averaging errors at all simulated engine modes. At each mode the error is estimated according to the formula

$$Error(C_i) = \frac{C_{i.polynom} - C_i}{C_i} \cdot 100\%, \quad (15)$$

where $C_{i.polynom}$ - value obtained using formula (14);
C_i - value calculated according to formulas (4), (6), (9), (11) and (13) using the deteriorated engine data simulated by the thermodynamic model.

The presented data allow drawing some preliminary conclusions on the arguments. Coefficient C_6 is important because it is presented in all temperature models. As follows from Table 1, parameters $T^*_{C.cor}$ and $T^*_{HPT.cor}$ are preferable arguments of the polynomial function for C_6.

The argument $T^*_{LPT.cor}$ can be considered as the poorest because it gives by far the highest error on average as well as for the coefficients $C_{3.2}$, C_4, and C_6. The explanation may be that the corrected power turbine discharge temperature $T^*_{LPT.cor}$ depends on a power turbine rotation speed n_{LP} whereas gas generator variables that constitute the coefficients C_1- C_6 do not almost depend on the speed.

Table 1. MSE of the polynomial functions for a healthy engine

x	C_1	C_2	$C_{3.1}$	$C_{3.2}$	C_4	C_5	C_6	mean for x
$n_{HP.cor}$	1,05	1,39	1,05	0,99	1,24	0,49	0,77	1,00
$p^*_{C.cor}$	1,08	1,40	1,08	0,97	1,19	0,49	0,80	1,00
$T^*_{C.cor}$	1,15	1,43	1,15	1,70	1,60	0,50	0,58	1,16
$G_{f.cor}$	0,99	1,36	0,99	0,60	0,65	0,48	0,91	0,85
$T^*_{HPT.cor}$	1,05	1,38	1,05	1,24	0,41	0,48	0,61	0,89
$p^*_{HPT.cor}$	1,06	1,38	1,06	1,12	0,82	0,48	0,82	0,96
$T^*_{LPT.cor}$	1,05	1,31	1,05	3,98	4,28	0,47	3,07	2,17
mean	1,06	1,38	1,06	1,52	1,46	0,49	1,08	1,15

There is no sense to compare the errors for all arguments x and all coefficients C_j in the Table 1 because all the data correspond only to a healthy engine.

Validation on deteriorated engine data

The polynomial functions are validated against a healthy engine and against deteriorated engines. The deteriorated engines are simulated by 3% shifts in the following engine module performance maps: compressor efficiency $(\delta\eta_C)$, compressor air flow (δG_C), total pressure conservation in the combustion chamber $(\delta\sigma_{CC})$, HPT efficiency $(\delta\eta_{HPT})$, HPT nozzle box area $(\delta F_{NB.HPT})$, total pressure conservation between HPT-LPT $(\delta\sigma_{HPT-LPT})$, LPT efficiency $(\delta\eta_{LPT})$, LPT nozzle box area $(\delta F_{NB.LPT})$, air bleed for gas pumping station needs (δG_{st}), and combustion efficiency $(\delta\eta_{CC})$.

The differences have been estimated between the coefficients C_j of the deteriorated engines and the coefficients computed by the polynomials obtained on the healthy engine data. The differences are given in Appendix A in the same form of MSE like in Table 1. Values that differ by more than a factor of 1.5 from those obtained for a healthy engine (see Table 1) are displayed in bold. The analysis shows that the errors of the coefficients C_1, C_2 (with the arguments $n_{HP.cor}$, $p^*_{C.cor}$, $T^*_{C.cor}$, $G_{f.cor}$, $T^*_{HPT.cor}$, $p^*_{HPT.cor}$) and C_4 (with argument $T^*_{HPT.cor}$) have not been significantly increased. However, there are also coefficients with a visible error increase. To sum up, the impact of engine deterioration on the analyzed coefficients can be notable but it depends on particular coefficients and arguments. There is no necessity in choosing the best coefficients and arguments at this point because the influence of the coefficients on the required temperature T_w^* is different. The best temperature model (with its coefficients) and the best argument will be chosen below by the analyzing accuracy of this temperature.

TEMPERATURE MODEL VALIDATION

Truncation error analysis

A truncation error is defined as a relative difference between the temperature $T_{w.alg}$ calculated according to the algorithm in Fig. 1 and the temperature $T_{w.mod}$ computed by the thermodynamic model:

$$Error(T_w^*) = \frac{T^*_{w.alg} - T^*_{w.mod}}{T^*_{w.mod}} \cdot 100\% . \qquad (16)$$

The resulting MSE for a healthy engine are placed in Table 2. Analysis of these truncation errors shows that the fuel flow parameter $G_{f.cor}$ demonstrates the least mean error because all models except Model 4 give the best results with the parameter $G_{f.cor}$ as an argument of the polynomial functions. This can be explained by the fact that the required temperature is the most closely related to this measured parameter due to the gas turbine thermodynamics. For all arguments it can also be seen that Model 1 is the best temperature model and this model with the argument $G_{f.cor}$ provides the highest temperature accuracy (MSE=0,23%). Parameters $n_{HP.cor}$, $T^*_{HPT.cor}$, and $p^*_{HPT.cor}$ can be considered as alternative arguments for Model 1 because the corresponding errors of 0,28%-0,29% are not too large. Model 4 with the argument $T^*_{HPT.cor}$ can also be an alternative model because of its relatively small error of 0,28%. Model 3 is notably worse than Model 4 although both models are based on the same equation (8). A possible explanation is that the coefficient $C_{3.2}$ of Model 3 contains a significantly varying parameter − namely, gas flow.

Table 2. MSE of temperature models for a healthy engine, %

x	Model 1	Model 2	Model 3	Model 4	Model 5	mean for x
$n_{HP.cor}$	0,28	0,53	1,04	0,68	0,53	0,61
$p^*_{C.cor}$	0,31	0,55	1,11	0,53	0,55	0,61
$T^*_{C.cor}$	0,38	0,60	1,63	0,46	0,61	0,74
$G_{f.cor}$	0,23	0,47	0,66	0,62	0,47	0,49
$T^*_{HPT.cor}$	0,28	0,50	1,14	0,28	0,50	0,54
$p^*_{HPT.cor}$	0,29	0,51	1,11	0,41	0,51	0,57
$T^*_{LPT.cor}$	0,49	0,54	2,97	2,15	0,56	1,34
mean for model	0,32	0,53	1,38	0,73	0,53	0,70

Results for a deteriorated engine are shown in Appendix B. Like in Appendix A, the values that differ from those for a healthy engine by a factor of more than 1,5 are displayed here in bold. From the presented data it follows that Model 1 is the most accurate.

Table 3 illustrates the behavior of this model for different engine health conditions and different arguments. We can see that the argument $G_{f.cor}$ provides the best results and the arguments $T^*_{HPT.cor}$ is also good enough. As in the case of a healthy engine, only one model can compete with Model 1— Model 4 with argument $T^*_{HPT.cor}$. Errors of this model for different engine health conditions are given in Table 4.

Table 3. MSE of temperature Model 1 for different engine health conditions, %

x	Healthy	$\delta\eta c$ -0.03	δGc -0.03	$\delta\sigma cc$ +0.03	$\delta\eta_{HPT}$ -0.03	$\delta F_{NB.HPT}$ +0.03	$\delta\sigma_{HPT-.LPT}$ +0.03	$\delta\eta_{LPT}$- +0.03	$\delta F_{NB.LPT}$ +0.03	$\delta G_{\text{-st}}$ +0.03	$\delta\eta_{CC}$ -0.03	mean for x
$n_{HP.cor}$	0,28	0,59	0,33	0,45	0,70	0,36	0,41	0,29	0,42	0,28	0,30	0,43
$p^*_{C.cor}$	0,31	0,47	0,36	0,31	0,57	0,55	0,38	0,32	0,39	0,31	0,33	0,42
$T^*_{C.cor}$	0,38	0,38	0,41	0,35	0,62	0,56	0,45	0,39	0,45	0,38	0,40	0,47
$G_{f.cor}$	0,23	0,30	0,27	0,27	0,31	0,25	0,23	0,23	0,24	0,22	0,25	0,28
$T^*_{HPT.cor}$	0,28	0,29	0,31	0,51	0,32	0,25	0,32	0,29	0,32	0,28	0,30	0,34
$p^*_{HPT.cor}$	0,29	0,46	0,34	0,35	0,52	0,35	0,36	0,30	0,31	0,30	0,31	0,38
$T^*_{LPT.cor}$	0,49	0,56	0,57	1,12	0,70	0,59	0,63	0,58	0,55	0,59	0,59	0,69
mean for engine condition	0,32	0,44	0,37	0,48	0,53	0,42	0,40	0,34	0,38	0,34	0,35	

Table 4. MSE of temperature Model 4 with argument $T^*_{HPT.cor}$ for different engine conditions, %

x	Healthy	$\delta\eta c$ -0.03	δGc -0.03	$\delta\sigma cc$ +0.03	$\delta\eta_{HPT}$ -0.03	$\delta F_{NB.HPT}$ +0.03	$\delta\sigma_{HPT-.LPT}$ +0.03	$\delta\eta_{LPT}$- 0.03	$\delta F_{NB.LPT}$ +0.03	$\delta G_{\text{-st}}$ +0.03	$\delta\,\delta\eta_{CC}$ -0.03	mean for x
$T^*_{HPT.cor}$	0,28	0,29	0,45	0,52	0,33	0,25	0,30	0,29	0,31	0,29	0,29	0,33

Final choice of the coefficients and temperature models is to be done taking into account influence of measurement errors.

Instrumental error analysis

The technique of instrumental errors analysis was initially developed for a general case and then applied to the particular temperature models under analysis. For general case, according to the algorithm given in Fig. 1, the temperature T_w may be represented as a function:

$$T_w = T_{w.cor}\frac{T_H^*}{T_0} = f(C_{i.1}, C_{i.2}, C_6, y_{1.cor}, y_{2.cor}, x_{1.cor}, x_{2.cor}) = \\ = f(C_{i.1}, C_{i.2}, C_6, y_1, y_2, x_1, x_2, p_H^*, T_H^*) \quad (17)$$

where $y_{1.cor}$ and $y_{2.cor}$ are temperature model corrected parameters; $x_{1.cor}$ is a corrected parameter used for determination of coefficients $C_{i.1}$, $C_{i.2}$; $x_{2.cor}$ is a corrected parameter used for determination of coefficient C_6.
All of these parameters depend on the atmospheric conditions p_H^* and T_H^*. Therefore, final MSE of instrumental error should be determined as

$$\sigma_{T_w} = \sqrt{\sigma^2_{T_w}(y_1) + \sigma^2_{T_w}(y_2) + \sigma^2_{T_w}(x_1) + \sigma^2_{T_w}(x_2) + \sigma^2_{T_w}(p_H^*) + \sigma^2_{T_w}(T_H^*)} = \quad (18)$$

$$= \sqrt{\left(\frac{\partial T_w}{\partial y_1}\right)^2 \sigma^2_{y_1} + \left(\frac{\partial T_w}{\partial y_2}\right)^2 \sigma^2_{y_2} + \left(\frac{\partial T_w}{\partial x_1}\right)^2 \sigma^2_{x_1} + \left(\frac{\partial T_w}{\partial x_2}\right)^2 \sigma^2_{x_2} + \left(\frac{\partial T_w}{\partial p_H^*}\right)^2 \sigma^2_{p_H^*} + \left(\frac{\partial T_w}{\partial T_H^*}\right)^2 \sigma^2_{T_H^*}},$$

where

$$\frac{\partial T_w}{\partial y_1} = \frac{\partial T_{w.cor}}{\partial y_{1.cor}}\frac{\partial y_{1.cor}}{\partial y};\ \frac{\partial T_w}{\partial y_2} = \frac{\partial T_{w.cor}}{\partial y_{2.cor}}\frac{\partial y_{2.cor}}{\partial y};$$

$$\frac{\partial T_w}{\partial x_1} = \frac{\partial T_{w.cor}}{\partial x_{1.cor}}\frac{\partial x_{1.cor}}{\partial x_1};\ \frac{\partial T_w}{\partial x_1} = \frac{\partial T_{w.cor}}{\partial x_{2.cor}}\frac{\partial x_{2.cor}}{\partial x_2};$$

$$\frac{\partial T_w}{\partial p_H^*} = \Big(\frac{\partial T_{w.cor}}{\partial y_{1.cor}}\frac{\partial y_{1.cor}}{\partial p_H^*} + \frac{\partial T_{w.cor}}{\partial y_{1.cor}}\frac{\partial y_{1.cor}}{\partial p_H^*} + \\ + \frac{\partial T_{w.cor}}{\partial x_{1.cor}}\frac{\partial x_{1.cor}}{\partial p_H^*} + \frac{\partial T_{w.cor}}{\partial x_{2.cor}}\frac{\partial x_{2.cor}}{\partial p_H^*}\Big);$$

$$\frac{\partial T_w}{\partial T_H^*} = \Big(\frac{\partial T_{w.cor}}{\partial y_{1.cor}}\frac{\partial y_{1.cor}}{\partial T_H^*} + \frac{\partial T_{w.cor}}{\partial y_{2.cor}}\frac{\partial y_{2.cor}}{\partial T_H^*} + \\ + \frac{\partial T_{w.cor}}{\partial x_{1.cor}}\frac{\partial x_{1.cor}}{\partial T_H^*} + \frac{\partial T_{w.cor}}{\partial x_{2.cor}}\frac{\partial x_{2.cor}}{\partial T_H^*} + \frac{T_{w.cor}}{T_0}\Big). \quad (19)$$

To better specify measurement accuracy, we analyzed a lot of available information on errors of measured gas path parameters. The results are presented in Appendix C as mean square values of relative errors used by different authors in diagnostic analysis. Since some listed papers contain absolute error values, we transformed them to relative errors using common absolute values of engine parameters for a considered engine type. Information given in papers [16,24,25,32,34,37,39] is more optimistic and mainly concerns measurements in test conditions. Information about precision of atmospheric conditions given in [35] is too pessimistic and was ignored. As a result of the above analysis, Table 5 includes standard deviations of measurement errors accepted in this paper.

Table 5. Measurements uncertainties (σ,%)

p^*_H	T^*_H	n_{HP}	n_{PT}	G_f	p^*_C	T^*_C	p^*_{HPT}	T^*_{HPT}	T^*_{LPT}
0,03	0,2	0,05	0,1	0,5	0,2	0,2	0,3	0,25	0,2

Since the difference in truncation errors of some considered models is not too significant (see results for Model 1 with the arguments $n_{HP.cor}$, $p^*_{C.cor}$, $G_{f.cor}$, $T^*_{HPT.cor}$, $p^*_{HPT.cor}$ and Model 4 with the argument $T^*_{HPT.cor}$), analysis of instrumental errors may change the choice of the model.

Appendix D shows the instrumental temperature errors obtained by formulas (18) and (19). All possible combinations of arguments x_1 and x_2 are considered. Analysis of data in Appendix D shows that generally the best results correspond to cases when $T^*_{HPT.cor}$ is used as argument of model for coefficient C_6. These cases are shown in Table 6. The best results are demonstrated by Model 1 with all arguments of the coefficient C_1 and by Model 4 with the arguments $T^*_{HPT.cor}$, $p_{HPT.cor}$, and $T^*_{LPT.cor}$ of the coefficients $C_{3.1}$ and C_4.

Table 6. Instrumental errors of temperature estimation (σ,%)

x	Model 1	Model 2	Model 3	Model 4	Model 5	mean for x
$n_{HP.cor}$	0,19	0,24	0,78	0,36	0,24	0,36
$p^*_{C.cor}$	0,19	0,24	0,38	0,23	0,25	0,26
$T^*_{C.cor}$	0,18	0,25	0,51	0,31	0,25	0,30
$G_{f.cor}$	0,18	0,24	0,41	0,3	0,25	0,28
$T^*_{HPT.cor}$	0,18	0,24	0,41	0,18	0,24	0,25
$p^*_{HPT.cor}$	0,19	0,24	0,38	0,19	0,25	0,25
$T^*_{LPT.cor}$	0,19	0,24	0,58	0,18	0,24	0,28
mean for model	0,19	0,24	0,49	0,25	0,25	0,28

However, not all above models and arguments are good from the standpoint of truncation errors. For example, the argument $T^*_{LPT.\mathrm{cor}}$ demonstrates high truncation errors for both mentioned models (see Table 2). Therefore, a trade-off model providing the lowest total error of the desired temperature is to be found. Assuming that the truncations and instrumental error components are independent, the dispersion (σ^2) of a total error is obtained by summing the dispersions of these components. Table 7 shows σ values of the total error calculated on the data of Table 2 and Table 6. In accordance with Table 7, the minimal total errors correspond to Model 1 with the arguments $G_{f.cor}$ and $T^*_{HPT.cor}$ and Model 4 with the argument $T^*_{HPT.cor}$. Fortunately, these particular models demonstrate the best robustness to engine deterioration. Therefore they may be recommended for practical application.

Table 7. Total errors of temperature estimation (σ,%)

x	Model 1	Model 2	Model 3	Model 4	Model 5	mean for x
$n_{HP.cor}$	0,34	0,58	1,30	0,77	0,58	0,71
$p^*_{C.cor}$	0,36	0,60	1,17	0,58	0,60	0,66
$T^*_{C.cor}$	0,42	0,65	1,71	0,55	0,66	0,80
$G_{f.cor}$	0,29	0,53	0,78	0,69	0,53	0,56
$T^*_{HPT.cor}$	0,33	0,55	1,21	0,33	0,55	0,60
$p^*_{HPT.cor}$	0,35	0,56	1,17	0,45	0,57	0,62
$T^*_{LPT.cor}$	0,53	0,59	3,03	2,16	0,61	1,37
mean for model	0,37	0,58	1,48	0,79	0,59	0,76

CONCLUSIONS

A general algorithm for estimating a turbine rotor inlet temperature is presented in this paper. The algorithm is based on thermodynamic relations between unmeasured and measured gas turbine parameters. Five alternative temperature models have been formed and compared. Each model contains some coefficients composed of unmeasured gas path parameters, mainly engine module performances. Dependence of these coefficients on engine operational mode is approximated by polynomial functions. Different engine measured parameters were used in turn as an argument of these functions.

To find the optimal temperature model and to select the best function's argument, main accuracy performances of these models were selected and analyzed: truncation and instrumental errors as well as robustness to engine deterioration.

A thermodynamic model based on performances of the gas path modules was used to generate data for building and validation of the temperature models.

In conclusion, Models 1 and 4 showed the best performance. The parameters $G_{f.cor}$ and $T^*_{HPT.cor}$ for Model 1 and $T^*_{HPT.cor}$ for Model 4 were found to be the best arguments in the polynomial functions. These models have the lowest instrumental (0,18%) and truncation (0,23% and 0,28%) errors in the prediction of the temperature T^*_w for a healthy engine. They also demonstrate good robustness to the engine deterioration.

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APPENDIX A

Mean square errors of coefficient functions with different arguments and for different engine health conditions

x	C_1	C_2	$C_{3.1}$	$C_{3.2}$	C_4	C_5	C_6	mean for x
$\delta\eta_C = -0.03$								
$n_{HP.cor}$	1,03	1,21	1,02	0,88	**4,34**	0,42	**2,45**	1,62
$p^*_{C.cor}$	1,00	1,20	1,00	1,27	**3,47**	0,42	**1,81**	1,45
$T^*_{C.cor}$	1,00	1,21	1,00	**3,84**	1,25	0,42	0,55	1,32
$G_{f.cor}$	0,88	1,14	0,88	**2,16**	**2,17**	0,40	1,16	1,26
$T^*_{HPT.cor}$	0,91	1,15	0,91	**3,96**	0,33	0,40	0,61	1,18
$p^*_{HPT.cor}$	0,99	1,19	1,00	1,40	**3,44**	0,42	**1,85**	1,47
$T^*_{LPT.cor}$	0,92	1,08	0,92	5,31	3,86	0,38	2,90	2,19
mean	0,96	1,17	0,96	2,69	2,69	0,41	1,62	
$\delta G_C = -0.03$								
$n_{HP.cor}$	1,16	1,48	1,17	**3,02**	**2,56**	0,52	0,80	1,53
$p^*_{C.cor}$	1,17	1,50	1,17	1,08	1,46	0,53	1,45	1,19
$T^*_{C.cor}$	1,24	1,53	1,24	1,94	1,70	0,54	**1,01**	1,31
$G_{f.cor}$	1,07	1,46	1,07	0,87	0,87	0,51	1,41	1,04
$T^*_{HPT.cor}$	1,14	1,47	1,14	**1,71**	0,44	0,51	0,91	1,05
$p^*_{HPT.cor}$	1,15	1,48	1,15	1,36	1,03	0,52	1,38	1,15
$T^*_{LPT.cor}$	1,17	1,40	1,17	4,90	5,22	0,51	3,49	2,55
mean	1,16	1,48	1,16	2,13	1,90	0,52	1,49	
$\delta\sigma_{CC} = -0.03$								
$n_{HP.cor}$	1,07	**3,13**	1,06	**3,35**	**7,37**	**1,04**	**3,24**	2,89
$p^*_{C.cor}$	1,06	**3,01**	1,06	**4,33**	**4,84**	**1,03**	**1,78**	2,44
$T^*_{C.cor}$	1,21	**2,85**	1,22	**5,18**	**5,21**	**0,96**	**1,83**	2,64
$G_{f.cor}$	1,09	**3,03**	1,09	**6,19**	**3,54**	**1,03**	**1,32**	2,47
$T^*_{HPT.cor}$	1,41	**2,76**	1,42	**9,36**	0,64	**0,92**	**1,61**	2,59
$p^*_{HPT.cor}$	1,08	**3,05**	1,09	**4,04**	**5,02**	**1,03**	**2,07**	2,48
$T^*_{LPT.cor}$	**1,87**	**2,50**	**1,87**	**13,71**	**8,07**	**0,89**	**6,12**	5,00
mean	1,26	2,91	1,26	6,59	4,96	0,99	2,56	
$\delta\eta_{HPT} = -0.03$								
$n_{HP.cor}$	1,50	**4,46**	1,48	1,03	**7,71**	**1,47**	**3,30**	2,99
$p^*_{C.cor}$	1,40	**4,39**	1,41	**1,67**	**6,35**	**1,49**	**2,34**	2,72
$T^*_{C.cor}$	1,52	**4,42**	1,49	2,15	**6,38**	**1,48**	**2,30**	2,82
$G_{f.cor}$	1,19	**4,24**	1,19	**3,76**	**3,83**	**1,43**	**1,02**	2,38
$T^*_{HPT.cor}$	1,13	**4,04**	1,12	**7,00**	0,40	**1,33**	**1,96**	2,43
$p^*_{HPT.cor}$	1,37	**4,35**	1,40	**2,31**	**5,81**	**1,46**	**2,02**	2,67
$T^*_{LPT.cor}$	1,15	**3,84**	1,15	**9,31**	**5,67**	**1,37**	4,95	3,92
mean	1,32	4,25	1,32	3,89	5,17	1,44	2,55	
$\delta F_{NB.HPT} = +0.03$								
$n_{HP.cor}$	1,13	**2,88**	1,11	1,07	2,49	0,96	1,07	1,53
$p^*_{C.cor}$	1,28	**2,99**	1,29	**2,35**	4,37	1,04	2,26	2,23
$T^*_{C.cor}$	1,35	**3,00**	1,32	**2,57**	4,07	1,02	1,97	2,19
$G_{f.cor}$	1,02	**2,82**	1,02	1,16	1,22	0,97	0,69	1,27
$T^*_{HPT.cor}$	1,05	**2,77**	1,03	2,11	0,39	0,92	0,64	1,27
$p^*_{HPT.cor}$	1,12	**2,85**	1,14	1,17	1,93	0,98	0,80	1,43
$T^*_{LPT.cor}$	1,05	**2,68**	1,04	5,40	5,44	1,00	3,93	2,93
mean	1,14	2,86	1,14	2,26	2,84	0,98	1,62	
$\delta\sigma_{HPT-LPT} = +0.03$								
$n_{HP.cor}$	1,23	**2,59**	1,25	0,98	**4,52**	0,95	1,09	1,80
$p^*_{C.cor}$	1,22	**2,55**	1,22	1,15	**3,61**	0,90	0,70	1,62
$T^*_{C.cor}$	1,27	**2,57**	1,29	1,74	**4,16**	0,92	0,73	1,81
$G_{f.cor}$	1,07	**2,44**	1,07	**1,92**	**2,07**	0,87	0,96	1,48
$T^*_{HPT.cor}$	1,07	**2,40**	1,08	**3,91**	0,43	0,88	1,70	1,64
$p^*_{HPT.cor}$	1,19	**2,54**	1,18	1,28	**3,39**	0,91	0,69	1,60
$T^*_{LPT.cor}$	1,11	**2,27**	1,11	**7,20**	5,26	0,74	4,21	3,13
mean	1,17	2,48	1,17	2,60	3,35	0,88	1,44	
$\delta\eta_{LPT} = -0.03$								
$n_{HP.cor}$	1,07	1,45	1,07	0,92	1,31	0,50	0,78	1,01
$p^*_{C.cor}$	1,10	1,46	1,10	1,01	1,23	0,52	0,80	1,03
$T^*_{C.cor}$	1,18	1,49	1,18	1,77	1,68	0,52	0,73	1,22
$G_{f.cor}$	1,01	1,42	1,01	0,63	0,70	0,50	0,88	0,88
$T^*_{HPT.cor}$	1,08	1,43	1,08	1,39	0,43	0,50	0,56	0,92
$p^*_{HPT.cor}$	1,08	1,44	1,08	1,23	0,75	0,51	0,74	0,97
$T^*_{LPT.cor}$	1,14	1,35	1,14	4,79	5,31	0,50	3,48	2,53
mean	1,09	1,43	1,09	1,68	1,63	0,51	1,14	
$\delta F_{NB.LPT} = +0.03$								
$n_{HP.cor}$	1,29	**2,48**	1,31	0,96	**4,33**	0,91	1,04	1,76
$p^*_{C.cor}$	1,28	**2,45**	1,27	1,19	**3,37**	0,86	0,61	1,58
$T^*_{C.cor}$	1,33	**2,46**	1,35	1,85	**3,84**	0,88	0,61	1,76
$G_{f.cor}$	1,12	**2,34**	1,12	1,85	**1,95**	0,83	0,97	1,46
$T^*_{HPT.cor}$	1,12	**2,31**	1,13	**3,72**	0,45	0,85	1,64	1,60
$p^*_{HPT.cor}$	1,15	**2,34**	1,14	**3,01**	1,08	0,84	1,45	1,57
$T^*_{LPT.cor}$	1,15	**2,25**	1,16	**5,87**	4,85	0,74	3,56	2,80
mean	1,21	2,38	1,21	2,64	2,84	0,84	1,41	
$\delta G_{st} = +0.03$								
$n_{HP.cor}$	1,10	1,42	1,10	1,01	1,31	0,50	0,91	1,05
$p^*_{C.cor}$	1,13	1,43	1,13	1,02	1,26	0,50	0,87	1,05
$T^*_{C.cor}$	1,19	1,46	1,20	1,76	1,64	0,51	0,73	1,21
$G_{f.cor}$	1,04	1,39	1,04	0,66	0,73	0,49	0,96	0,90
$T^*_{HPT.cor}$	1,10	1,41	1,11	1,42	0,43	0,49	0,55	0,93
$p^*_{HPT.cor}$	1,11	1,41	1,10	1,24	0,81	0,50	0,82	1,00
$T^*_{LPT.cor}$	1,15	1,34	1,15	4,91	5,31	0,48	3,68	2,57
mean	1,12	1,41	1,12	1,72	1,64	0,50	1,22	
$\delta\eta_{CC} = -0.03$								
$n_{HP.cor}$	1,11	1,46	1,11	**3,31**	1,30	0,51	0,83	1,37
$p^*_{C.cor}$	1,14	1,47	1,14	**3,25**	1,25	0,52	0,81	1,37
$T^*_{C.cor}$	1,22	1,49	1,21	**3,57**	1,66	0,52	0,59	1,47
$G_{f.cor}$	1,05	1,42	1,05	**1,60**	**1,59**	0,50	0,96	1,16
$T^*_{HPT.cor}$	1,12	1,44	1,11	**3,14**	0,43	0,50	0,59	1,19
$p^*_{HPT.cor}$	1,12	1,45	1,12	**2,99**	0,79	0,51	0,77	1,25
$T^*_{LPT.cor}$	1,15	1,37	1,14	**5,92**	5,26	0,50	3,68	2,72
mean	1,13	1,44	1,13	3,40	1,75	0,51	1,17	

APPENDIX B

Mean square errors of temperature models with different arguments and for different engine health conditions

x	Model 1	Model 2	Model 3	Model 4	Model 5	mean for x
$\delta\eta_C$ = -0.03						
$n_{HP.cor}$	**0,59**	0,75	1,33	**2,79**	0,74	1,24
$p^*_{C.cor}$	**0,47**	0,67	1,16	**2,08**	0,69	1,01
$T^*_{C.cor}$	0,38	0,60	**3,21**	0,48	0,60	1,06
$G_{f.cor}$	0,30	0,53	1,50	**1,45**	0,54	0,87
$T^*_{HPT.cor}$	0,29	0,50	**3,28**	0,29	0,50	0,97
$p^*_{HPT.cor}$	0,46	0,64	1,20	**2,02**	0,65	0,99
$T^*_{LPT.cor}$	0,56	0,57	**5,03**	2,36	0,55	1,81
mean	0,44	0,61	2,39	1,64	0,61	
δG_C = -0.03						
$n_{HP.cor}$	0,33	0,56	**2,18**	**1,29**	0,56	0,99
$p^*_{C.cor}$	0,36	0,62	1,20	0,63	0,63	0,69
$T^*_{C.cor}$	0,41	0,66	1,76	0,49	0,66	0,80
$G_{f.cor}$	0,27	0,53	0,72	0,70	0,54	0,55
$T^*_{HPT.cor}$	0,31	0,54	1,35	0,30	0,54	0,61
$p^*_{HPT.cor}$	0,34	0,58	1,22	0,45	0,58	0,64
$T^*_{LPT.cor}$	0,57	0,61	3,57	2,56	0,63	1,59
mean	0,37	0,59	1,72	0,92	0,59	
$\delta\sigma_{CC}$ = -0.03						
$n_{HP.cor}$	0,45	**1,47**	**1,91**	**3,83**	**1,46**	1,82
$p^*_{C.cor}$	0,31	**1,27**	**2,63**	**2,69**	**1,28**	1,64
$T^*_{C.cor}$	0,35	**1,26**	**3,22**	**2,92**	**1,26**	1,80
$G_{f.cor}$	0,27	**1,17**	**3,68**	**2,19**	**1,18**	1,70
$T^*_{HPT.cor}$	0,51	**0,86**	**6,01**	0,52	**0,85**	1,75
$p^*_{HPT.cor}$	0,35	**1,27**	**2,40**	**2,74**	**1,28**	1,61
$T^*_{LPT.cor}$	**1,12**	0,60	**9,26**	**3,24**	0,63	2,97
mean	0,48	1,13	4,16	2,59	1,13	
$\delta\eta_{HPT}$ = -0.03						
$n_{HP.cor}$	**0,70**	**2,00**	**1,46**	**3,91**	**1,98**	2,01
$p^*_{C.cor}$	**0,57**	**1,87**	1,19	**3,22**	**1,90**	1,75
$T^*_{C.cor}$	**0,62**	**1,89**	1,74	**3,19**	**1,90**	1,87
$G_{f.cor}$	0,31	**1,62**	**2,04**	**2,06**	**1,64**	1,53
$T^*_{HPT.cor}$	0,32	**1,28**	**4,48**	0,33	**1,27**	1,53
$p^*_{HPT.cor}$	**0,52**	**1,81**	1,33	**2,94**	**1,83**	1,69
$T^*_{LPT.cor}$	**0,70**	**1,14**	**6,37**	**2,80**	**1,24**	2,45
mean	0,53	1,66	2,66	2,64	1,68	
$\delta F_{NB.HPT}$ = +0.03						
$n_{HP.cor}$	0,36	**1,17**	1,16	**1,15**	**1,16**	1,00
$p^*_{C.cor}$	0,55	**1,36**	**2,25**	**1,98**	**1,39**	1,51
$T^*_{C.cor}$	0,56	**1,35**	**2,39**	**1,76**	**1,35**	1,48
$G_{f.cor}$	0,25	**1,04**	0,72	0,68	**1,06**	0,75
$T^*_{HPT.cor}$	0,25	**0,97**	1,40	0,25	**0,96**	0,76
$p^*_{HPT.cor}$	0,35	**1,12**	1,07	**0,79**	**1,14**	0,89
$T^*_{LPT.cor}$	0,59	**0,97**	**3,92**	2,72	**1,07**	1,85
mean	0,42	1,14	1,84	1,33	1,16	
$\delta\sigma_{HPT-LPT}$ = +0.03						
$n_{HP.cor}$	0,41	**1,12**	1,15	**2,18**	**1,14**	1,20
$p^*_{C.cor}$	0,38	**1,06**	1,11	**1,72**	**1,03**	1,06
$T^*_{C.cor}$	0,45	**1,12**	1,62	**1,89**	**1,11**	1,24
$G_{f.cor}$	0,23	**0,90**	1,13	**1,18**	**0,88**	0,86
$T^*_{HPT.cor}$	0,32	0,79	**2,53**	0,30	0,80	0,95
$p^*_{HPT.cor}$	0,36	**1,04**	1,14	**1,65**	**1,02**	1,04
$T^*_{LPT.cor}$	0,63	0,77	**4,92**	2,50	0,71	1,91
mean	0,40	0,97	1,94	1,63	0,96	
$\delta\eta_{LPT}$ = -0.03						
$n_{HP.cor}$	0,29	0,56	1,03	0,71	0,56	0,63
$p^*_{C.cor}$	0,32	0,58	1,14	0,57	0,59	0,64
$T^*_{C.cor}$	0,39	0,64	1,69	0,50	0,64	0,77
$G_{f.cor}$	0,23	0,50	0,65	0,66	0,51	0,51
$T^*_{HPT.cor}$	0,29	0,52	1,22	0,29	0,52	0,57
$p^*_{HPT.cor}$	0,30	0,54	1,16	0,39	0,55	0,59
$T^*_{LPT.cor}$	0,58	0,59	3,55	2,58	0,60	1,58
mean	0,34	0,56	1,49	0,81	0,57	
$\delta F_{NB.LPT}$ = +0.03						
$n_{HP.cor}$	0,42	**1,07**	1,17	**2,06**	**1,08**	1,16
$p^*_{C.cor}$	0,39	**1,01**	1,16	**1,58**	**0,98**	1,02
$T^*_{C.cor}$	0,45	**1,06**	1,69	**1,69**	**1,05**	1,19
$G_{f.cor}$	0,24	0,85	**1,09**	**1,12**	**0,83**	0,83
$T^*_{HPT.cor}$	0,32	0,77	**2,41**	0,31	0,78	0,92
$p^*_{HPT.cor}$	0,31	**0,81**	**2,00**	0,57	0,79	0,89
$T^*_{LPT.cor}$	0,55	**0,82**	**4,01**	2,34	0,75	1,69
mean	0,38	0,91	1,93	1,38	0,90	
δG_{st} = +0.03						
$n_{HP.cor}$	0,28	0,55	1,06	0,75	0,54	0,64
$p^*_{C.cor}$	0,31	0,56	1,15	0,61	0,57	0,64
$T^*_{C.cor}$	0,38	0,62	1,68	0,51	0,62	0,76
$G_{f.cor}$	0,22	0,48	0,69	0,68	0,49	0,51
$T^*_{HPT.cor}$	0,28	0,51	1,26	0,29	0,51	0,57
$p^*_{HPT.cor}$	0,30	0,53	1,19	0,44	0,53	0,60
$T^*_{LPT.cor}$	0,58	0,59	3,13	2,43	0,59	1,47
mean	0,34	0,54	1,34	0,77	0,54	
$\delta\eta_{cc}$ = -0.03						
$n_{HP.cor}$	0,30	0,56	**2,26**	0,72	0,56	0,88
$p^*_{C.cor}$	0,33	0,58	**2,28**	0,57	0,58	0,87
$T^*_{C.cor}$	0,40	0,63	**2,60**	0,49	0,63	0,95
$G_{f.cor}$	0,25	0,48	1,04	**1,02**	0,48	0,65
$T^*_{HPT.cor}$	0,30	0,52	**2,17**	0,29	0,52	0,76
$p^*_{HPT.cor}$	0,31	0,54	**2,11**	0,41	0,54	0,78
$T^*_{LPT.cor}$	0,59	0,60	**4,21**	2,60	0,63	1,73
mean	0,35	0,56	**2,38**	0,87	0,56	

APPENDIX C

Mean squares of engine measured parameters uncertainties used in different papers on gas turbine diagnosis, %

Literature	Measured parameter																	
	p_H^*	p_H	T_H^*	M_f	p_F^*	T_F^*	$p^*_{HPC.in}$	$T^*_{HPC.in}$	n_{HP}	n_{LP}	G_f	p_C	T_C^*	p^*_{HPT}	T^*_{HPT}	p^*_{LPT}	T^*_{LPT}	N
[11]					0,5	0,5	0,5		0,25	0,25						0,5		
[12]							0,5	0,75	0,25	0,25		0,5	0,75				0,75	
[16]	0,01	0,03	0,15		0,07	0,1	0,1	0,15	0.1	0,1	0,45	0,25	0,25		0,25	0,05	0,2	
[20]											1,5	2	0.33		0,53			
[21]				0,67					0,08	0,08		0,133	0,167	0,133	0,2		0,2	0,17
[23]							0,46				0,5	0,24	0,23		0,52		0,52	0,5
[24]	0,033	0,033	0,23		0,1	0,2			0,02	0,05	0,12	0,1	0,11				0,1	
[25]	0,027	0,153	0,101		0,164	0,23	0,094	0,162	0,034	0,051	0,161						0,097	
[26]							0,33	0,267			0,67						0,33	
[27]	0,33		0,23	0,33			0,66		0,042	0,14	0,5	0,18					0,04	
[28]		0,033					0,133		0,08	0,08	0,67							0,17
[29]	0,17	0,17	0,33						0,33	0,33		0,17	0,33	0,33	0,48	0,33	0,48	
[30]		0,17	0,25				0,5	0,75			0,67	0,17			0,5	1,67	0,67	0,33
[31]					0,5				0,25	0,25		0,5	0,75		0,75			
[32]									0,033	0,033	0,133	0,067	0,133				0,133	
[33]		0,2	0,035							0,11	0,65	0,5	0,34				0,44	0,27
[34]					0,056	0,2				0,033		0,11	0,12				0,12	
[35]	4	4	1				0,25	0,75	0,25	0,25		0,5	0,75		0,75			
[36]	0,033		0,23							0,5		0,02	0,11			0.033	0,011	
[37]					0,16		0,15	0,2	0,031	0,043	0,17	0,14	0,09				0,1	
[38]					0,5	0,75		0,5	0,25	0,25	1	0,5	0,75	1	0,75	0,5	0,75	
[39]							0,164	0,23	0.034	0.051		0,162	0,094		0,1		0,17	
[40]		0,1	0,35	0,8					0,2						0,15			
[41]	0,22		0,19				0,34					0,54			0,01	1,5		

APPENDIX D

Mean squares of instrumental errors for different models

Model 1

Argument of C_6	Argument of C_1						
	$n_{HP.cor}$	$p^*_{C.cor}$	$T^*_{C.cor}$	$G_{f.cor}$	$T^*_{HPT.cor}$	$p^*_{HPT.cor}$	$T^*_{LPT.cor}$
$n_{HP.cor}$	0,22	0,28	0,29	0,28	0,29	0,29	0,29
$p^*_{C.cor}$	0,24	0,24	0,24	0,24	0,24	0,25	0,22
$T^*_{C.cor}$	0,20	0,21	0,20	0,20	0,21	0,21	0,21
$G_{f.cor}$	0,23	0,23	0,22	0,22	0,23	0,23	0,23
$T^*_{HPT.cor}$	0,19	0,19	0,19	0,18	0,19	0,19	0,19
$p^*_{HPT.cor}$	0,22	0,21	0,21	0,21	0,22	0,22	0,22
$T^*_{LPT.cor}$	0,22	0,21	0,21	0,21	0,22	0,21	0,22

Model 2

Argument of C_6	Argument of C_2						
	$n_{HP.cor}$	$p^*_{C.cor}$	$T^*_{C.cor}$	$G_{f.cor}$	$T^*_{HPT.cor}$	$p^*_{HPT.cor}$	$T^*_{LPT.cor}$
$n_{HP.cor}$	0,28	0,33	0,33	0,33	0,32	0,33	0,33
$p^*_{C.cor}$	0,30	0,28	0,30	0,30	0,30	0,30	0,30
$T^*_{C.cor}$	0,31	0,31	0,28	0,31	0,31	0,31	0,31
$G_{f.cor}$	0,28	0,28	0,28	0,28	0,28	0,28	0,28
$T^*_{HPT.cor}$	0,24	0,25	0,25	0,24	0,24	0,25	0,24
$p^*_{HPT.cor}$	0,27	0,27	0,27	0,27	0,27	0,28	0,27
$T^*_{LPT.cor}$	0,27	0,27	0,27	0,27	0,27	0,27	0,26

Model 3

Argument of C_6	Argument of $C_{3.1}$, $C_{3.2}$						
	$n_{HP.cor}$	$p^*_{C.cor}$	$T^*_{C.cor}$	$G_{f.cor}$	$T^*_{HPT.cor}$	$p^*_{HPT.cor}$	$T^*_{LPT.cor}$
$n_{HP.cor}$	0,86	0,40	0,52	0,37	0,42	0,40	0,55
$p^*_{C.cor}$	0,76	0,38	0,50	0,43	0,39	0,40	0,56
$T^*_{C.cor}$	0,80	0,38	0,56	0,41	0,40	0,38	0,61
$G_{f.cor}$	0,81	0,34	0,52	0,36	0,38	0,33	0,60
$T^*_{HPT.cor}$	0,78	0,38	0,51	0,41	0,41	0,38	0,58
$p^*_{HPT.cor}$	0,78	0,38	0,51	0,41	0,38	0,38	0,58
$T^*_{LPT.cor}$	0,80	0,38	0,50	0,41	0,38	0,38	0,65

Model 4

Argument of C_6	Argument of $C_{3.1}$, C_4						
	$n_{HP.cor}$	$p^*_{C.cor}$	$T^*_{C.cor}$	$G_{f.cor}$	$T^*_{HPT.cor}$	$p^*_{HPT.cor}$	$T^*_{LPT.cor}$
$n_{HP.cor}$	0,31	0,27	0,37	0,31	0,28	0,25	0,33
$p^*_{C.cor}$	0,37	0,20	0,32	0,34	0,24	0,22	0,30
$T^*_{C.cor}$	0,35	0,24	0,27	0,31	0,19	0,19	0,32
$G_{f.cor}$	0,36	0,24	0,31	0,26	0,22	0,20	0,32
$T^*_{HPT.cor}$	0,36	0,23	0,31	0,30	0,18	0,19	0,33
$p^*_{HPT.cor}$	0,36	0,23	0,31	0,30	0,21	0,16	0,33
$T^*_{LPT.cor}$	0,36	0,23	0,31	0,30	0,21	0,18	0,27

Model 5

Argument of C_6	Argument of $C_{3.1}$, C_4						
	$n_{HP.cor}$	$p^*_{C.cor}$	$T^*_{C.cor}$	$G_{f.cor}$	$T^*_{HPT.cor}$	$p^*_{HPT.cor}$	$T^*_{LPT.cor}$
$n_{HP.cor}$	0,28	0,33	0,33	0,33	0,32	0,33	0,33
$p^*_{C.cor}$	0,30	0,28	0,30	0,30	0,30	0,30	0,30
$T^*_{C.cor}$	0,31	0,31	0,28	0,31	0,31	0,31	0,31
$G_{f.cor}$	0,28	0,28	0,28	0,28	0,28	0,28	0,28
$T^*_{HPT.cor}$	0,24	0,25	0,25	0,24	0,24	0,25	0,24
$p^*_{HPT.cor}$	0,27	0,27	0,27	0,27	0,27	0,28	0,27
$T^*_{LPT.cor}$	0,27	0,27	0,27	0,27	0,27	0,27	0,26