THE USE OF MATHCAD FUNCTIONS IN TEACHING AN UNDERGRADUATE AEROSPACE PROPULSION COURSE

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ABSTRACT

This paper investigates the educational efficacy of using a set of Mathcad functions in teaching a typical undergraduate course in Aerospace Propulsion. This approach runs counter to two other methods often utilized in this type of course. The first, a traditional, time-consuming technique, requires students to solve problems by hand using tabular property data. The second is to supply students with a computer algorithm which calculates engine performance in a "black-box" fashion. The approach in this study attempted to bridge the divide between these two methods, incorporating the best features of each. The inclusion of Mathcad functions reduced the time required in performing mindless interpolations while still requiring the students to master the basic Thermodynamic principles required when modeling modern gas turbine engines, as verified by in-class testing. Feedback from students is presented, as well as some unexpected lessons learned from the study. Modifications to the functions are also provided.

NOMENCLATURE

- h Enthaply, kJ/kg or BTU/lbm.
- k specific heat ratio.
- ke specific kinetic energy, kJ/kg or BTU/lbm.
- P Pressure (absolute), kPa or psia.
- Pr Relative Pressure.
- R Universal Gas Constant
- q Specific heat transfer, kJ/kg or BTU/lbm.
- T Temperature (absolute), K or R.
- w specific work output, kJ/kg or BTU/lbm.

INTRODUCTION

This paper describes a pedagogical approach used in a senior level undergraduate course in Aerospace Propulsion. In

this elective class, the primary focus is on presenting the fundamental principles underlying modern gas turbine technology. The course material covers topics ranging from Thermodynamics to Turbomachinery and Heat Transfer. As with most engineering courses, a large number of homework problems are assigned in order to reinforce, and apply, the concepts discussed in class. The quantity and length of these homework assignments can lead to considerable frustration on the part of the students. This is particularly true when they are required to look up, and often interpolate, large quantities of values in the pertinent gas tables.

In order to minimize the amount of time students spend in interpolating tabular values, an alternative approach using Mathcad functions was made available to students who used them extensively. While the experience was generally positive, some surprising issues were encountered. This paper discusses the rationale behind the use of these functions, explores the pros and cons of this approach, and discusses improvements made to the Mathcad functions.

RATIONALE FOR USE OF MATHCAD SUPPLIED FUNCTIONS

When studying prime movers for aerospace purposes, a common assumption is to assume ideal gas behavior for the working fluid. This assumption leads to several possible methods for handling Conservation of Energy. For example, the applicable form of the First Law of Thermodynamics for a steady flow device (with negligible changes in potential energy and other simplifying assumptions) is:

$$_{1}q_{2} - _{1}w_{2} = (h_{2} - h_{1}) + \Delta(ke)$$
 (1)

with three approaches generally considered for evaluating the change in enthalpy term. The first two of these involve the assumption of constant specific heats, which sacrifices numerical accuracy for the sake of expediency. A more rigorous approach utilizes the use of gas tables where enthalpy values are tabulated against absolute temperature.

A similar arrangement is seen when computing temperature ratios for a given pressure ratio (which is a common occurrence when dealing with compressors or turbines). For an isentropic process where a constant value of specific heat is assumed, the temperature ratio can be computed from the relation:

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\left(\frac{k-1}{k}\right)}$$
(2)

However, as before, this assumption of constant specific heats limits the accuracy of the computed temperatures, particularly given the relatively large changes in specific heat that occur across typical compressors and turbines. In contrast, an approach involving the variability of specific heat values takes on the form:

$$\frac{\mathrm{Pr}_2}{\mathrm{Pr}_1} = \frac{P_2}{P_1} \tag{3}$$

where Pr is the relative pressure, and is typically tabulated in most Thermodynamics books as a function of absolute temperature.

The approach taken in this course was to account for the variation in specific heats when computing engine parameters throughout the propulsion system. This decision was based upon the following logic:

- This technique provides for more accurate solutions than those obtained by assuming constant specific heats,
- This is the approach generally used by the course textbook [1], and
- All aerospace propulsion companies will assume variable specific heats when computing engine values. Since many of these seniors will soon be entering the workplace and possibly be working in the gas turbine industry, they should become comfortable with this approach.

In spite of these advantages for the assumption of variable specific heats when dealing with air, there are a number of disadvantages to this approach. Namely, because the values of enthalpy and relative pressure are tabulated, a large number of interpolations are required in solving typical homework problems in this class. Since this course is a technical elective usually taken by senior level mechanical engineering students, and has a prerequisite of Thermodynamics, they have previously been exposed to the use of property tables and interpolation. However, the time required to do these multiple interpolations has led to general frustration and complaints from the students taking the course. Since the purpose of the class is not learning to interpolate in tables, the primary author sought ways to mitigate the time requirements without compromising on the fundamental application of thermodynamics necessary to solve these problems.

One solution considered was to use some sort of gas turbine simulation software. A widely used example of this approach can be found bundled with a popular Gas Turbine textbook [2]. The software package is extremely versatile in allowing the user to select many types of engine configurations (single spool or twin spool turbojet, turbofan, turboprop, etc.), flight conditions, and component parameters (such as polytropic efficiencies and pressure ratios). It also allows students to perform parametric studies in order to investigate the effects of changing a single variable.

However, the primary author's classroom experience has shown that students who rely on this type of software often fail to grasp the underlying thermodynamic principles and simply treat the software as a "black box." This has been reflected in poor exam performance and in the inability of students to answer fundamental in-class queries. A better approach was found from McClain [3], which presented a set of Mathcad functions that can be easily incorporated into a worksheet. He has reported general success in using these functions in an Internal Combustion Engines course. Some background on this follows.

Background of Mathcad Functions

Mathcad is a computational software package that also serves as an engineering document or calculation report generator [4]. Mathcad has capabilities similar to Matlab [5], EES [6], and many other engineering analysis software packages. Mathcad's strength compared to the other engineering calculation packages is manifested in Mathcad's ability to produce a "what you see is what you get" mathematical calculation appearance and in its ability to perform calculations with automatic unit handling and conversion. For example, if a function is needed to evaluate the specific heat of a gas as a function of temperature with the appropriate units, the function may be entered as

$$c_{p}(T) := \left\lfloor 0.905 + 0.00000135 \left(\frac{T}{K}\right)^{2} \right\rfloor \cdot \frac{kJ}{kg \cdot K}$$

The construction above allows the function to be called with temperature specified in any units. While the function is created in units of $kJ/(kg\cdot K)$, the results of the function may be easily converted to BTU/(lb·°R) as shown below.

$$c_{p}(500 \cdot K) = 1.243 \frac{kJ}{kg \cdot K}$$
 $c_{p}(600 \cdot R) = 0.252 \frac{BTU}{lb \cdot R}$

Further, the following construction would be used to determine the change in the standard state reference-entropy between 500 K and 300 K.

$$\Delta s_0 := \int_{300 \cdot K}^{500 \cdot K} \frac{c_p(T)}{T} \, dT = 0.57 \frac{kJ}{kg \cdot K}$$

The constructions above demonstrate how Mathcad is used to create engineering documents in a mathematical report for mat where the calculations appear as they would be written using pencil and paper.

The original ideal gas functions for Mathcad were based on data from "The Chemkin Thermodynamic Data Base" as reported by Turns [7]. Turns reports fourteen constants used to determine thermodynamic data for twelve species (CO, CO₂, H₂, H, OH, H₂O, N₂, N, NO, NO₂, O, O₂) of the carbonhydrogen-oxygen-nitrogen (CHON) system as a function of temperature. The first seven constants for each species are used to determine thermodynamic properties in the temperature range of 300 K to 1000 K. The second seven constants for each species are valid between 1000 K and 5000 K.

Based on the success of the initial ideal gas property functions in the internal combustion engines course, Mathcad functions were created to evaluate the thermodynamic properties of working fluids commonly studied in the twocourse undergraduate experience in thermodynamics [8]. In addition to the ideal-gas CHON functions, ideal gas functions for air were included, and equation-of-state based functions were generated to evaluate the thermodynamic properties of water, R-134a, R-22, propane, and ammonia. All of the function sets are available to the educational public free of charge (see Acknowledgements and Disclaimer section for details).

While developed for a two-course sequence in undergraduate thermodynamics, the Mathcad function sets have proven useful for many other engineering courses and for engineering research. The Mathcad functions have been found to be very useful in courses such as combustion, compressible flow, power generation, thermo-fluid experimental methods, and heating ventilating and air conditioning [9-12].

Advantages of Mathcad Functions in an Aerospace Propulsion Course

In contrast to the approach taken by Mattingly [2] as discussed above, the advantages of using a Mathcad function approach can be summarized as:

• Students must understand, and apply, the fundamental thermodynamic principles underlying the propulsion system under consideration,

- Students are generally already familiar with the Mathcad software from previous coursework, so minimal supplemental material must be provided,
- In solving a typical problem, students would still find it necessary to set up the problem as if they were solving it by hand,
- Once a basic problem was set up in Mathcad (such as a simple turbojet engine with isentropic components), a more complicated model could be built based upon the previous case (such as a turbojet or turbofan engine model with component efficiencies),
- Once a model was successfully generated, a parametric study could easily be performed. For example, the engine performance could be compared over a range of Mach numbers or flight altitudes. Similarly, the effect of compressor efficiency could be studied with minimal additional effort.
- If desired, homework could be submitted electronically, and
- A significant amount of non-productive time would be saved by not having to interpolate values in the appendices.

These functions can be used to compute relative pressure and enthalpy for a given temperature, or can compute the temperature given another parameter (such as Pr). A few examples of the notation used in finding these values for standard air follow:

- h_T_{air}(T) Returns the enthalpy of dry air as a function of temperature.
- pr_T_{air}(T) Returns the relative pressure of dry air as a function of temperature.
- T_h_{air}(h) Returns the temperature of dry air as a function of enthalpy.

Similar functions exist for going the other way (for example, finding enthalpy given the value of relative pressure). The format of this is:

• h_pr_{air}(pr) Returns the enthalpy of dry air as a function of relative pressure.

CLASSROOM EXPERIENCES FROM USING THE MATHCAD SUPPLIED FUNCTIONS

The results from the use of the Mathcad functions were somewhat mixed. On the positive note, the students gave the software high marks for ease of use and ability to save time. Although the Mathcad functions themselves performed flawlessly, a number of unforeseen problems with their use were found.

The first issue was in-class tests. Once the students became accustomed to the software, they found it difficult to use tabular values to solve test questions. Though not employed in this course, one possible way to mitigate this problem would be to allow the students to use a computer during the test. However, this would require several modifications to testing procedures. First, each student would need access to their own computer having Mathcad installed. Secondly, these computers should be purged of all old worksheet files to preclude the possibility of accessing an old homework problem and copying the pertinent relations (except for the file containing the user supplied functional relations). Lastly, network access should probably be curtailed so that internet pages or network drives could not be read. The logistics of implementing these changes in order to allow for computer usage during exams was deemed to be too great.

One item that cannot be ignored is that of sharing of files among students when working on homework assignments. Though this is difficult to police, this possibility is probably the same as that of sharing written homework solutions. Thus, the use of Mathcad solutions should not increase the frequency of circulating homework solutions.

One surprising finding from this class involved the use of units. Since Mathcad automatically handles unit conversions, students apparently began to rely on the software exclusively to handle their unit conversions. This was found to be a significant problem on tests where access to the software was not available. Surprisingly, errors were commonly seen when dealing with SI units, where students are typically very comfortable. This was typically prevalent in dealing with inlets and exhaust nozzles, where the factor of 1000 m^2/s^2 per 1 kJ/kg was often neglected. However, this problem was most common when students were working with problems involving the British Gravitational System (BGS). The author frequently saw values of 778 or 550 randomly dropped into quizzes or exam solutions in an attempt to handle a unit conversion that a student knew must be present, but apparently was not sure exactly where to place.

COMMENTS FROM STUDENTS FROM THE FORMAL CLASS EVALUATION

Standard procedure at John Brown University (JBU) is to formally evaluate every course during the final two weeks of the semester. The course instructor may add extra questions to the instrument to solicit specific information not available from the generic questionnaire. As such, two questions pertinent to the Mathcad functions were added to the course assessment tool when the course was last offered in the spring of 2009 with an enrollment of eleven students. (Note that the course was next offered in the spring of 2011, but results were not available to include in this paper). All eleven students chose to respond to the question "Please comment on the usefulness of the Mathcad software that was provided. Overall, was it a help or a hindrance?" All of the responses expressed appreciation of the Mathcad functions. Some representative responses reflecting their general tenor are given below.

- I found Mathcad to be helpful. Having a background in Mathcad helped me to excel in understanding the course work. It helped me check my units and change my answer if need be. Using the lookup command helped save time from interpolating and looking up values in the tables.
- Without the Mathcad software, I would have spent several more hours accomplishing the same task.
- It was a great help! The first assignment with the program was a little more time-consuming because I wasn't as familiar with it. But overall it was a tremendous help in those lengthy problems.
- I think it was bad for me, because I would learn my way into a problem using the software but at the time of the test I would get all confused (dealing with the units, the data tables and other details). And it is a great tool, but I guess it is dangerous if the tests will not use it.
- Overall, I believe that it helped in understanding how some things worked and was wonderful at converting units. At the same time, it was really easy to use the same file and change numbers, which I think hurt the understanding of later homeworks.

A second question was also placed on the course assessment tool, asking "Do you feel that your exam performance was hindered by using the provided Mathcad worksheet for homework assignments?" Ten students responded to this question. In general, the overall opinion was that their performance was hindered to a small degree. A sample of responses is given below:

- A little. It was hard trying to do the test by hand. With practice and knowledge of where the tables are and how to use them, solving the test was not hard.
- Yes I do. I usually prepare for tests through homework, but since home work was done in Mathcad, during the test I got confused.
- The only hindrance I can think of is that when it came to test time, I didn't know where to find the tables in the book for certain values. Finding them took time. Also, I had gotten used to the notation that I used in Mathcad and the book notation was unfamiliar to me, causing me to use up more valuable test time.
- I think it was only because I began to rely on Mathcad rather than what I had learned. I don't think it was directly a result of using Mathcad, but rather because I

used Mathcad to help me get through some homework assignments.

• Maybe slightly, but not really. One just had to remember to keep solution units consistent, and no problems ensued.

MODIFICATIONS MADE TO THE MATHCAD SUPPLIED FUNCTIONS

Some improvements have recently been made to the original Mathcad functions as developed by McClain. NASA has published a revised set of functional property relations utilizing a 9-constant form [13-14]. The functions originally used by McClain were of the 7-constant form. These are represented in the following forms:

$$C_{p}^{o}(T)/R = a_{1}T^{-2} + a_{2}T^{-1} + a_{3} + a_{4}T + a_{5}T^{2} + a_{6}T^{3} + a_{7}T^{4}$$
(4)

$$H^{o}(T)/RT = -a_{1}T^{-2} + a_{2}(\ln T)/T + a_{3} + a_{4}T/2 + a_{5}T^{2}/3 + a_{6}T^{3}/4 + a_{7}T^{4}/5 + b_{1}/T$$
(5)

$$S^{o}(T) / R = -a_{1}T^{-2} / 2 - a_{2}T^{-1} + a_{3}(\ln T) + a_{4}T + a_{5}T^{2} / 2 + a_{6}T^{3} / 3 + a_{7}T^{4} / 4 + b_{2}$$
(6)

Since these relationships were considered to be an improvement over the previous form, the values obtained were generally closer to the tabulated values given in the textbook. A revised worksheet with these newer functions will be made available to the students when the course is next offered for their evaluation.

CONCLUSIONS

In conclusion, the use of the Mathcad add-in functions achieved many of its objectives. Primarily, repetitive and mindless calculations on homework problems were, in large measure, avoided. Simple models were easily converted into more complex models as the course progressed. And students were still expected to master the underlying thermodynamic principles of the various aerospace propulsion systems.

However, a number of concerns were found – namely, how to handle in-class tests, and the reliance of students on Mathcad for handling conversions between units. A number of ways to address these issues will be implemented the next time this course is offered (in the spring of 2011). First, an increased number of quizzes will be given during class, emphasizing the proper conversion of units. These quizzes will also require the use of tabular data from the textbook appendices. Another approach will be to require a few homework assignments to be completed by hand. A more drastic option may be to prohibit the use of units in all Mathcad solutions, requiring the students to check all units by hand. Lastly, students will be reminded repeatedly about the potential pitfalls of relying on Mathcad.

ACKNOWLEDGMENTS AND DISCLAIMER

The authors have no known financial interest in Mathcad or its parent company Parametric Technology Corporation (PTC). The opinions expressed herein are theirs and theirs alone. All of the functions presented are available to the educational public free of charge. The authors only request that if the functions are requested by an instructor and used for instruction, the instructor agrees to have his or her students complete a survey form at the end of the course and return the competed surveys to Dr. McClain.

The basic function sets may be obtained by emailing Dr. McClain at stephen_mcclain@baylor.edu. Please include the subject line "MathCAD: Thermo Function Set Request" in your email. The reader should note that all functions presented in this paper are for educational use only, and that the user assumes all responsibility for calculations performed using the functions if used for an industrial application. More specifically, the authors, Baylor University, John Brown University, and The University of Alabama at Birmingham are not liable for engineering decisions made while using the functions for an industrial application.

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