A UNIFIED CORRECTION METHOD FOR REYNOLDS NUMBER, SIZE AND ROUGHNESS EFFECTS ON THE PERFORMANCE OF COMPRESSORS

M. V. Casey* and C. J. Robinson

PCA Engineers, Lincoln, England
*Also Institute of Thermal Turbomachinery (ITSM), University of Stuttgart, Germany

ABSTRACT
An equation is derived that relates the changes in turbomachinery efficiency with Reynolds number to the changes in the friction factor of an equivalent flat-plate. This equation takes into account the different Reynolds number and roughness dependencies of the individual components, and can be used for whole stages and multistage machines. The new method is sufficiently general to correct for changes in Reynolds number due to changes in fluid properties or speed, changes in machine size, or changes in the surface roughness of components for all types of turbomachinery, but is calibrated here for use on axial and radial compressors. The method uses friction factor equations for a flat plate which include fully rough behaviour above an upper critical Reynolds number, a transition region depending on roughness and a region with laminar flow below the lower critical Reynolds number.

The correction equation for efficiency includes a single empirical factor. Based on a simple loss analysis and a calibration with over 30 sets of experimental test data covering a wide range of machine types, a suggestion for the variation of this factor with specific speed has been made. Additional correction equations are derived for the shift in flow and the change in pressure rise with Reynolds number and these are also calibrated against the same data.

NOMENCLATURE

- $a$: Reynolds independent loss fraction (-)
- $A, B, C, D$: Coefficients in correction equations (-)
- $b$: Channel width or height (m)
- $c$: Chord length (m)
- $C_L, C_D$: Lift and drag coefficients (-)
- $D_2$: Tip diameter (m)
- $f$: Friction factor (-)
- $h$: Specific enthalpy (J/Kg)
- $j$: Dissipation loss (-)
- $k$: Coefficient in equations 3 and 4 (-)
- $k_s$: Sand roughness (m)
- $n$: Exponent (-)
- $Ra$: Centre-line average roughness (m)
- $Re$: Chord Reynolds number (-)
- $T$: Temperature (K)
- $u$: Blade speed (m/s)
- $w$: Relative velocity (m/s)
- $V$: Volume flow rate (m$^3$/s)
- $w_{1/2}$: Specific shaft work (J/kg)
- $\alpha_m$: Mean stagger angle
- $\delta$: Displacement thickness (m)
- $\epsilon$: Blockage factor (-)
- $\zeta$: Entropy loss coefficient (-)
- $\eta$: Efficiency (-)
- $\phi$: Flow coefficient (-)
- $\chi$: Coefficient in equation 6 (-)
- $\psi$: Pressure coefficient (-)
- $\omega$: Angular velocity (radians/sec)
- $\omega_s$: Specific speed (-)

Subscripts
- $1$: condition 1, or inlet
- $2$: condition 2, or outlet
- $cr$: critical
- $l$: lower
- $p$: polytropic
- $ref$: at reference conditions
- $s$: isentropic
- $u$: upper
INTRODUCTION

As part of the EU-Project MOET, PCA Engineers has developed a new empirical method for predicting the effect of Reynolds number on the performance of compressors. The new method is a development of that published for radial compressors by Casey (1). A review of recent literature (see below) showed that methods similar to this, which use an equivalent friction factor as the basis for the change in loss generation with Reynolds number, are now recommended by most authors in this area. No agreement can be found in recent literature, however, on the structure of the most suitable correction equations. This issue is tackled in this paper by deriving a new form of correction equation for the changes in efficiency from first principles. The new equation allows a unified correction method for all types of turbomachinery to be developed which takes into account the different Reynolds number and roughness dependencies of the individual components, whole stages and multistage machines. The method is used here for axial and radial compressors.

The paper is organised as follows. The first section discusses the applications and difficulties of the general correction procedure, which is followed by the main results of a literature review. An analytical section then describes the derivation of a new correction equation for efficiency which is applicable to all types of compressors (and could also be used for turbines and pumps). Some simplifications of this equation are discussed as are the significance of the terms in the equation and their expected size. The equation for the change in efficiency includes a single empirical factor, but also naturally leads to a clear process for the determination of this factor using experimental data. Suggestions are also made here for correction methods for the shift in flow and the change in pressure rise. A subsequent section describes the calibration of the empirical coefficients using a data set comprising of over 30 different compressors and one pump, mostly from open literature sources. The paper ends with a discussion of the results and some conclusions.

BACKGROUND TO A GENERAL PROCEDURE

There are many applications for the equations derived in this paper. The changes in compressor performance due to changes in Reynolds number involve a change in efficiency, a shift in flow and a change in the pressure rise. Although these are quite small in some cases, they are nevertheless important. For example, in a multistage aero-engine axial compressor the variation in Reynolds number due to density reduction with altitude provides typically an efficiency loss of around 1 to 2 % points for a halving of the Reynolds number. This certainly needs to be taken into account in performance calculations, Schaeffler (2), and may bring a significant change in mismatch of stages with consequences on the operability. In other applications the changes may be much larger than this. Casey (1) includes test data of a low specific speed radial compressor where the polytropic efficiency increases by more than 15 % points in efficiency due to changes in Reynolds number through changes in inlet pressure from 0.3 to 3.0 bar. Even larger changes can be expected when such impellers are used in high pressure barrel compressors with an inlet pressure above 100 bar. Similar large changes in efficiency are given for an axial tunnel ventilator at low speeds by Hess and Pelz (3). Large changes in efficiency can arise in pumps designed for water but operating on a fluid of high viscosity such as oil, leading to a laminar flow on the blade surfaces, Gülich (4).

Another related effect is the change in performance due to changes in the absolute surface roughness. A common issue in the cost optimisation of compressors is whether is it worthwhile to improve the surface finish of a particular component of a stage in order to improve the performance. For example, does the associated performance improvement make it worthwhile to improve the surface finish of the wetted flow surfaces of a cast compressor outlet volute in a turbocharger? Roberts et al. (5) show that 0.5 % points of efficiency improvement can be obtained from the super-polishing of a transonic fan rotor blade; but how can this be related to the volute situation? The equations developed here can be applied to both of these problems. A similar effect is the question of degradation in performance due to erosion or wear during service, for example what would be the performance loss to be expected due to surface erosion from particles in the flow, Bons (6).
In many turbomachinery applications (pumps, ventilators, industrial compressors, turbochargers) manufacturers reduce costs by scaling a suitable and well-tried design to give a machine with a higher or lower flow capacity so that prediction of the effect of a change in size is important. In some applications there are even international standards for the effect of scaling from a sub-scale test stand to a full-scale prototype (for example, ASME and ISO). Recent interest and research on micro-compressors shows that a reduction in size can also lead to quite large performance decrements. Provided we are considering scaling of machines with geometric similarity the equations given here can also be applied to this problem.

The change in performance in all these cases is related to a change in the Reynolds number and in the relative roughness of the surfaces, as shown schematically in figure 1. This diagram is based on the well-known effects of Reynolds number on flows in pipes or on flat-plates and the transfer of this knowledge to our understanding of compressors. At low chord Reynolds number below the so-called lower critical Reynolds number, say a chord Reynolds number below 200,000, the flow can be expected to have laminar behaviour and the performance then deteriorates strongly with a further decrease in Reynolds number. In this region there is generally no effect of roughness as the laminar boundary layers are much thicker than the roughness elements on the surface. As the Reynolds number increases the boundary layers have a turbulent behaviour and the rate of increase in performance with Reynolds number decreases. In this region the performance is also affected by the roughness as the largest roughness elements may extend above the thin laminar sub-layer of the turbulent boundary layers. If the roughness is smaller than the laminar sub-layer this leads to so-called hydraulically smooth behaviour. At a much higher Reynolds number the laminar sub-layer of the boundary layer becomes thinner than the roughness elements and these then completely dominate the structure of the turbulent flow; the flow becomes hydraulically rough and above this upper critical Reynolds number there is no further change in performance with Reynolds number.

![Figure 1: Schematic variation of compressor efficiency with a change in Reynolds number, size and roughness](image)

There are a number of general difficulties related to the development of a unified method for prediction of the effects of Reynolds number, size and surface roughness which need to be mentioned here. One problem is that when the changes in performance are very small, this can lead to large uncertainties in correlations if poor quality test data are used to calibrate the coefficients, so high quality test data of low error is imperative. The test data that has been assembled in this paper all has an excellent pedigree and provides accurate test data. Details of the experimental error for each case cannot be given here due to lack of space, but examination of the references given for the test data shows that the estimated error in efficiency measurements of the original authors are usually better than ± 1%, except at extreme low speeds and Reynolds number.

The second major difficulty is the quantification of the effect of surface roughness on the flow. In general the manufactured surface roughness is quantified by the measured centre line average
roughness, \( Ra \), and the hydraulic effects are quantified in terms of an equivalent sand grain roughness, \( k_s \), which is not directly measurable. This concept assumes that the surface is covered with closely-packed spheres of radius \( k_s \) which result in the same influence on the boundary layer as the real surface roughness. There is no fully reliable method of relating the hydraulic effects of a particular machined surface to an equivalent sand roughness, as the effects are not only related to the roughness level, but also to the manufacturing technique with changes in orientation, spacing, and texture of the roughness elements on the surface, Bons (6). In this paper this issue has not been examined in detail. The range of conversion factors given in the literature varies from \( k_s = 1Ra \) to \( k_s = 10Ra \), with values of 8.9 in Schaeffler (2), 6.2 in Koch and Smith (7), 5.2 in Hummel et al. (8), 2.3 in Güllich (4) and 2 in Casey (1), Simon et al. (9) and Strub (10). Here \( k_s = 5Ra \) is used, which is in the middle of this range. Other values have been examined but do not improve the correlation unless specific values are used for specific cases. This may be related to the different surface structures of individual cases but would defeat the object of the unified method presented here.

A third problem is that geometric similarity is difficult to maintain with a change in size. Test data involving a change in size may include changes to the machine design and casing construction leading to changes in relative wall thicknesses and flexibility such that relative clearance levels are a function of size. There may also be changes in fillet radii and even modification of blade thickness with size. These latter effects need to be considered separately and are not part of the present correlation which is based on geometric similarity. Changes in manufacturing methods at different sizes may give a change in the surface finish but this is included in the method.

EARLIER CORRECTION METHODS

The extensive literature on this subject has been reviewed many times. Equations similar to equation 1 have often been suggested for the variation in peak efficiency with Reynolds number

\[
\frac{1-\eta}{1-\eta_{ref}} = a + (1-a) \left( \frac{Re_{ref}}{Re} \right)^n
\]

The term \( a \) is the fraction of losses that are independent of the Reynolds number (typically between 0 and 0.5) and \( n \) is the Reynolds ratio exponent, with a value between 0.16 and 0.5. The identification of suitable values for \( n \) and \( a \) has caused substantial difficulties and no general agreement has been reached for different machine types in the literature. This approach has most recently been used by Hess and Pelz (3), where they recognize that \( a \) is not a constant and is a function of the operating point. The first significant improvement in such methods became possible when the physical variation of the exponent \( n \) was related to the variation in friction factor expected in an equivalent pipe flow; see Simon and Bülskämper (9) and Strub et al. (10) and other references quoted in these papers, leading to an equation of the form

\[
\frac{1-\eta}{1-\eta_{ref}} = a + (1-a) \left( \frac{f}{f_{ref}} \right)
\]

where \( f \) is the friction factor of an equivalent fully turbulent pipe flow and \( f_{ref} \) is the friction factor for a hydraulically rough pipe flow of the same relative roughness, as determined from the Moody diagram or appropriate equations.

Casey (1) argues that the fundamental problem with equations of this type is that they are incorrectly formulated if the value of the Reynolds independent loss fraction \( a \) is considered to be a constant and describes an alternative method based on a simplified loss model for radial stages. In this approach the problem of estimating \( a \) is side-stepped by deriving an equation for the additive change in efficiency with a change in the equivalent friction factor in the following form:

\[
\eta - \eta_{ref} = -k(f - f_{ref}) \quad \text{and} \quad k = f(b/D_2, \Delta h/u^2)
\]

In these equations the value of the coefficient \( k \) was determined from a loss model and calibrated empirically by comparison with test data. Its value increased for low flow coefficient stages.
The basic idea behind these methods has recently been improved in the work of Gülich (4), where the equations for the skin friction of a flat plate are used, including a laminar flow region. Gülich still uses a Reynolds number based on tip-speed and impeller tip-diameter rather than a chord-based Reynolds number. In this work he uses a loss analysis to determine a value of a correction factor for the Reynolds number effects, which is in fact a multiplicative change in the efficiency so the structure of his correction method is

$$\eta = k\eta_{ref} \quad k = f(Re, Ra/D)$$  \hspace{1cm} (4)

The importance of these approaches is that, although published as correction methods for the effect of Reynolds number they include the effect of size and roughness in the relative roughness term in the equation for the friction factor. Some evidence for their satisfactory use to predict roughness effects in this way is given by Gülich (4), Simon and Bülskämper (9), Childs and Noronha (11) and Benra et al. (12), who use such a method to identify the penalties in performance with a rough surface finish at different Reynolds numbers.

A second situation for the use of a Reynolds number correction is the calculation of performance using correlations. Here the procedure is to calculate the frictional losses for some known reference Reynolds number and to adapt them for a change in Reynolds number, as follows:

$$\zeta = \zeta_{ref} \left( Re_{ref} / Re \right)^n$$  \hspace{1cm} (5)

where $n$ is the Reynolds number exponent which is different for laminar flow ($n = 0.5$) and turbulent flow ($n = 0.2$). The correlation methods of Kacker and Okapuu (13) use this procedure for turbines and Wright and Miller (14) suggest a similar procedure for compressors. An alternative procedure is suggested by Traupel (15), and by Koch and Smith (7), in which they adopt a similar approach but include the effect of roughness in a scaling factor

$$\zeta = \zeta_{ref} \chi$$  \hspace{1cm} (6)

The scaling factor $\chi$ includes both the effects of roughness and Reynolds number and needs to be determined from a diagram including both an upper and a lower critical Reynolds number which is essentially similar to a Moody diagram. The equations given here can also be used for this purpose.

Some, but not all, earlier methods also include corrections for the other effects of an increase in Reynolds number. The key additional effects are an increase in pressure rise related to the increase in efficiency, and a shift in volume flow related to changes in flow blockage. In general there is no large change in operational range of individual stages due to this effect, and the whole characteristic seems to shift to a slightly higher flow, but retain the same range, as the Reynolds number increases, see the examples provided by Simon and Bülskämper (9) and Hess and Pelz (3).

**DERIVATION OF A UNIFIED CORRECTION EQUATION**

The review given above demonstrates that although there is general agreement on the friction factor approach in the most recent literature, there is no agreement on the most suitable structure of the equations for the correction methods. To deal with this the analysis given below develops a new correction equation from first principles. Key features of the method are:

- The chord Reynolds number based on the locally relevant inlet flow velocity and flow-path length (chord) of each component is used rather than a Reynolds number based on tip diameter, Gülich (4), or on hydraulic diameter, Casey (1). In internal flows where the boundary layers on the blade surfaces are thin and do not become fully developed as in a pipe flow then the most appropriate Reynolds number is based on the length of the flow path (chord length), and the local mean velocity of the flow (inlet relative velocity at mid-span).
- The effect of the frictional resistance in the flow is modeled using equations for the friction factor of a flat plate rather than equations for fully developed turbulent pipe flow. In pumps and compressors, the blockage due to the boundary layers may become as high as 10%, but fully developed flow with merged boundary layers from both sides of the flow channels does not usually occur at the design point. An analogy between the flow in a compressor and the flow on
a flat plate is then a more reasonable approximation than an analogy with the fully developed flow in a pipe. In some very low specific speed devices with low blade span, the end-wall boundary layers may merge, see discussion in Casey (1), but this is certainly not the case for higher specific speed radial, mixed-flow and axial machines. In early developments of the method described here within the project MOET, Casey (16), the friction factor for a pipe flow was used for the low specific speed cases, but when it was discovered that the flat plate analogy gave better results this feature was not pursued further.

- The flat plate friction factor equations used in this method are described in Appendix 1 and include most features relevant to the physics, including an upper and lower critical Reynolds number, and regions of laminar, transition, fully rough and hydraulically smooth flow.
- The new analysis splits the loss sources in different components so that these can have different Reynolds numbers, roughness and the fluid properties can change through the machine.
- The new method provides a clear process for determination of empirical coefficients through analysis of test data.
- The method is derived at the best point of the characteristic but can be used to predict the change in the whole characteristic by applying the best point corrections to efficiency flow and pressure rise to each point on the whole characteristic.

**Derivation of equation for the change in efficiency**

The most rational efficiency definition for a compression process is the polytropic efficiency based on the dissipation losses or entropy rise, as defined by Casey (17), as

\[ \eta_p = 1 - \frac{j_{12}}{w_{t,12}} = 1 - \frac{\int T ds}{w_{t,12}} \]  

(7)

In this equation \( j_{12} \) are the dissipation losses related to the entropy increase, and \( w_{t,12} \) is the shaft work. If we now consider separately the losses in each blade row of a multistage adiabatic machine we can write an equation for inefficiency of the whole machine as

\[ 1 - \eta_p = \frac{\sum_j j_i}{\sum_i \Delta h_{t,i}} \]  

(8)

where \( j \) is the dissipation loss in each individual blade row, \( i \), and \( \Delta h_i \) is the total enthalpy change of the rotor blade rows (the total enthalpy change is zero for stators). The dissipation loss in each blade row can be written in terms of a non-dimensional entropy loss coefficient and the local blade inlet relative velocity as

\[ j_i = (1/2) w_i^2 \zeta_i \]  

(9)

For incompressible flow this loss coefficient is identical to the usual total pressure loss coefficient and the efficiency is then also identical to the isentropic efficiency. We consider the losses to be made up from many individual independent loss sources, only some of which depend of the Reynolds number, so we can write that the loss coefficient for any component is the sum of a Reynolds dependent loss and a loss which has no dependence on the Reynolds number:

\[ \zeta_i = \zeta_{Re} + \zeta_{non_Re} \]  

(10)

In combination with equation 8 and 9 we obtain

\[ 1 - \eta_p = \frac{\sum_i \frac{1}{2} w_i^2 (\zeta_{non_Re})}{\sum_i \Delta h_{t,i}} + \frac{\sum_i \frac{1}{2} w_i^2 (\zeta_{Re})}{\sum_i \Delta h_{t,i}} = A + B \]  

(11)
In this equation the first term, $A$, represents the losses (actually the inefficiency in terms of $1 - \eta$) that are independent of the Reynolds number. The term $B$ represents the losses (again as $1 - \eta$) that are Reynolds dependent, being mainly the profile losses and endwall friction losses, but may include also disc friction losses where these are significant. Both terms are not physically the losses but are inefficiencies, that is they are the losses divided by the work input. For a given design we may assume that $A$ is a constant, unless some aspect of the design, such as the clearance level, is changed, whereas $B$ changes with the Reynolds number and relative roughness. The expected variation in $B$ with the Reynolds number for a given roughness is shown in figure 2. $B$ is not a constant but decreases with increasing Reynolds number until the upper critical Reynolds number is reached. The value of $B$ at high Reynolds number is then dependent on the roughness. To avoid this, suitable reference conditions for $B$ need to be defined, see below. Note that this variation of the losses with Reynolds number makes equations such as equation 1 and 2 inadequate as a Reynolds correction method as the fraction of losses that is dependent on the Reynolds number clearly changes with the Reynolds number.

We now assume that the changes in the Reynolds dependent loss coefficient with the Reynolds number is similar to the effects of the Reynolds number or roughness on the friction factor of a representative flat plate flow, and with the introduction of reference conditions, we may write that

$$\zeta_{Re} = \zeta_{ref} \frac{f}{f_{ref}}$$  \hspace{1cm} (12)

This does not imply that the losses can be predicted as if the flow was that of a flat plate, as the actual compressor losses might be considerably larger than the flat plate losses. It simply states that the Reynolds dependent dissipation losses scale proportionally to the change in the friction factor of the representative flat plate flow. Substituting this into equation 11 leads to the following equation:

$$B = \sum_i \frac{1}{2} w_i^2 (\zeta_{Re})_i \frac{f}{f_{ref}} = \sum_i \frac{1}{2} w_i^2 (\zeta_{ref} \frac{f}{f_{ref}}) = \sum_i \Delta h_{i,j}$$  \hspace{1cm} (13)

Equation 13 can be used for a multistage machine in that each individual blade row could have a different friction factor and if sufficient information were available on the individual loss coefficients it could be used in this way to assess the Reynolds effects on a component by component basis. In a blade-row by blade-row stacking method this would be how the change in losses is calculated and in this way it is also possible to use this equation to estimate the sensitivity of roughness or Reynolds number on an individual component. An approach like this is, however, probably too complex for practical use as a general correction method of the global efficiency and a further simplification is made that each blade row has the same fractional change in friction factor, or alternatively that the change in friction factor in the first rotor row is representative for all blade rows. This leads to an equation of the form given below which can then be used for individual blade rows, for stages or for multistage machines:

$$B = \frac{\sum_i \frac{1}{2} w_i^2 (\zeta_{ref})_i f}{\sum_i \Delta h_{i,j} f_{ref}} = B_{ref} \frac{f}{f_{ref}}$$  \hspace{1cm} (14)

where $B_{ref}$ is the value of the Reynolds dependent inefficiency ($1 - \eta$) at a certain reference value of the friction factor, which is defined below.

On this basis the effect on the inefficiency of a change in the friction factor due to a change in the Reynolds number or relative roughness can be calculated for a typical component or stage as:

$$1 - \eta_p = A + B_{ref} \frac{f}{f_{ref}}$$  \hspace{1cm} (16)
The first term in this equation represents the Reynolds independent inefficiency, and the second represents the Reynolds dependent inefficiencies. Note that $A$ can be expected to remain constant, but $B_{\text{ref}}$ depends on the reference conditions selected.

We consider a condition $I$ at a known Reynolds number, where the optimum efficiency is known. If we then change the Reynolds number, roughness or size with a change in the associated friction factor to condition 2, then we obtain from this an expression for the change in efficiency at the optimum point due to the change in Reynolds number as:

$$
\Delta \eta = \eta_2 - \eta_1 = -B_{\text{ref}} \frac{f_2 - f_1}{f_{\text{ref}}} = -B_{\text{ref}} \frac{\Delta f}{f_{\text{ref}}} \tag{17}
$$

This equation, which is shown schematically in figure 3, has the same fundamental structure as the method of Casey (1) in that the change in efficiency is proportional to a change in friction factor of a representative flow and the correction is additive. It has the major advantage, however, that it is not directly coupled to any model for the losses, in that the coefficient $B_{\text{ref}}$ is simply the inefficiency due to friction losses at the reference conditions, whereas in the earlier model an analysis of the dissipation losses in radial compressors was used to derive its value. It has an additional advantage that the method is symmetrical; the increase in efficiency from point 1 to 2 is the same as the decrease in efficiency from point 2 to point 1, which was not the case with the earlier method.

**Figure 3:** Variation of efficiency with friction factor, following equation 16

**Figure 4:** Calibration procedure for efficiency coefficient $B_{\text{ref}}$ based on test data

It can be shown, Casey (1), that correction methods similar to equation 1 and 2 can be derived from an additive correction equation of this type, so this gives us some additional confidence in the physics of the derivation. Equation 17 has the considerable advantage over these methods that it does not include the terms $a$ or $A$ which represent the Reynolds independent losses, so these are not needed in the calculation procedure and in the calculation of the change in performance the Reynolds independent losses play no part.

**Expected value of $B_{\text{ref}}$ in the efficiency correction equation**

In the work which follows, we take the reference value of the friction factor to be 0.012, which with the equations for the friction factor used here, is close to that of the skin friction factor for a hydraulically smooth surface at an extremely high chord Reynolds number of 10'000'000. This represents an extreme condition where the efficiency is expected to be very high. Further improvements in efficiency would not be expected at even higher Reynolds numbers as the upper critical Reynolds number is likely to be lower than this. The value of $B_{\text{ref}}$ is then the loss in efficiency due to frictional losses at this very high Reynolds number for a hydraulically smooth case.

If we consider an axial compressor cascade with no losses other than profile losses (so that all losses depend on the Reynolds number and $A = 0$), Horlock (18) shows that the losses in this case can be modeled as
\[ \eta = 1 - \frac{2}{\sin 2\alpha_m} \frac{C_D}{C_L}, \quad \text{or} \quad B_{\text{ref}} = \frac{2}{\sin 2\alpha_m} \frac{C_D}{C_L} \] (18)

For a good axial compressor cascade at high Reynolds number with a stagger angle of \( \alpha_m = 45^\circ \) and a typical lift to drag ratio at high Reynolds number of 40 this leads to an expected value of \( B_{\text{ref}} = 0.05 \). \( B_{\text{ref}} \) depends on the lift to drag ratio and on the stagger angle, and this suggests that different design philosophies related to degree of reaction, loading and blade number will lead to slightly different values. For radial compressors higher values of \( B_{\text{ref}} \) would also be expected as the frictional losses are higher due to the narrower flow channels with a higher wetted surface area from the end walls. This suggests that \( B_{\text{ref}} \) can be expected to increase as the hub/tip ratio increases, or with a decrease in specific speed, essentially similar to the factor \( k \) in equation 3 in the method of Casey (1). To generalize this it is postulated that \( B_{\text{ref}} \) will change with machine type as characterized by specific speed but also depend slightly on the blade loading, blade number and degree of reaction. It should be possible to make a detailed loss analysis to try to identify the expected value of \( B_{\text{ref}} \) for each machine type, but in the correlation given in this paper the actual value of this factor is determined from test data on a range of compressor types with no further theoretical analysis, see below.

**Correction for the shift in volume flow**

An increase in the friction factor (related to a decrease in Reynolds number) also causes an increase in the boundary layer displacement thickness and a change in the blockage, as follows:

\[ \frac{\delta^*}{c} = kf, \quad \Delta \delta^* = k \Delta f, \quad \Delta \varepsilon = \frac{\delta^*}{b}, \quad \Delta \varepsilon = k \frac{c}{b} \Delta f \] (19)

The change in flow coefficient associated with an increase in blockage is then given by

\[ \frac{\Delta \phi}{\phi} = -\Delta \varepsilon = -k \frac{c}{b} \Delta f = -C_{\text{ref}} \frac{\Delta f}{f_{\text{ref}}} \] (20)

The coefficient \( C_{\text{ref}} \) is the sensitivity in volume flow to a change in friction factor. Again the theoretical analysis provides some guidance on this, as it shows that \( C_{\text{ref}} \) is proportional to aspect ratio. It can be expected then that it will increase as the specific speed decreases. No further detailed theoretical analysis has been made and in this paper the sensitivity is determined from test data.

**Correction for the change in pressure rise**

A change in the blockage factor and a change in the efficiency (related to a change in Reynolds number) could also be expected to change the flow deviation from the blades so a change in work input coefficient is likely to occur. In the radial compressor methods of Strub (10), Casey (1) and Simon and Bulskämper (9) roughly 50% of the efficiency increase was considered to cause an increase in pressure rise and the remaining 50% was taken to be related to a reduction of the work input. In the current method this is generalized into an equation of the form:

\[ \frac{\Delta \psi}{\psi_{\text{ref}}} = D_{\text{ref}} \frac{\Delta \eta}{\eta_{\text{ref}}} \] (21)

The change in pressure rise coefficient of the stage is taken to be related to the change in efficiency. Note that the value of the pressure rise coefficient is taken at the optimum efficiency point so that stages with higher work input (such as radial stages) with a flatter work input characteristic can be expected to have different behaviour to axial stages with low work input. The value of \( D_{\text{ref}} \) is again assumed to be a function of the type of machine as characterized by the specific speed and is determined from test data. A value of \( D_{\text{ref}} \) of unity would imply no change in work input at the optimum point such that the efficiency change appears entirely as an increase in pressure rise.
CALIBRATION OF THE COEFFICIENTS

A particularly elegant feature of the new equation set for the change in performance is that the
equations naturally lead to a consistent approach to the calibration of the coefficients from test data.
Equations 17 and 20 suggest that the changes in efficiency and in flow coefficient are linearly
proportional to the change in the equivalent friction factor. Based on this linear relationship,
experimental values of the efficiency, or flow coefficient, at the best point can be plotted as a
function of the equivalent flat plate friction factor to determine the slope of the linear relationship
and this slope then defines the value of the coefficients, see figure 4. Changes in friction factor from
tests with variation of Reynolds number, roughness or size can be included and the coefficients $B_{ref}$,
$C_{ref}$ and $D_{ref}$ can then be determined from the best straight line fit through the test data. This
approach is similar to that used by Casey (1) in the determination of the coefficient $k$ in equation 3.

This procedure has been adopted to calibrate the value of the coefficients against specific speed
for different compressors. Data from over 30 compressors and a single pump with a variation in
Reynolds number, roughness or size have been collected from the open literature and through
contacts with leading turbomachinery manufacturers, many of whom have provided unpublished
commercially sensitive data on their performance levels to the first author. A full list of companies
and organisations that have supported this endeavour is provided in the acknowledgements. It is
only possible in this short paper to provide a few examples of the many cases examined.

The first example is given in figure 5 and is the change in efficiency of a low flow coefficient
impeller for tests with variation of gas and inlet pressure, as published by Casey et al. (19). The test
data for this case has been kindly provided by Man Turbo and Diesel (formerly Sulzer Turbo). The
experimental procedure and the closed loop test rig used are also described in some detail by
Dalbert et al. (20) and the error band on efficiency is considered to be better than 1%. The linear
relationship is strongly supported by the test data and at this very low specific speed a value of $B_{ref}$
of 0.261 results from the analysis. Extensive radial compressor test cases from the same source, as
included in the paper of Casey (1), have also kindly been provided by MAN. Other radial
compressor tests cases used include those of Teerman (21), Simon and Bülskämper (9) from
Siemens and unpublished cases from ABB Turbosystems for the effect of size.

A second example is given in figure 6. This shows tests on a centrifugal pump carried out by
Rotzoll (22) in which the Reynolds number was changed by variations in speed, variations in fluid
(water and oil) and variation in fluid temperature. In this case the friction factor is very high as the
tests were mostly in the laminar region and a remarkably good agreement with the linear
relationship expected from equation 16 is found. This example suggests that the same procedure
can be used for centrifugal pumps but more calibration with experimental data is needed to confirm this.

**Figure 5:** Variation of efficiency with friction factor for low flow coefficient impeller tested
in air and R134a, data from Casey et al. (19)

**Figure 6:** Variation of efficiency with friction factor for centrifugal pump tested in oil and water of
different temperatures, data from Rotzoll (22)
A third example, figure 7, shows an axial compressor test case from the open literature, Bammert and Woelk (23), in which emery grade roughness has been applied to the blade surfaces of an axial compressor and the deterioration in performance for different roughness levels has been measured. The figure shows test data for a smooth blade and three different roughness levels and the linear relationship is again confirmed by these measurements. For this and other axial compressor cases, the resulting value of $B_{ref}$ is remarkably similar to the value of 0.05 suggested by the Horlock analysis for an axial compressor given above. Other axial compressor and fan test cases used include those of Roberts (5), Schaeffler (2), Suder (24), Wisler (25) and Hess and Pelz (3).

**FINAL CORRECTION EQUATIONS**

The final coefficients derived by this calibration procedure for the parameters $B_{ref}$, $C_{ref}$ and $D_{ref}$ for all test cases are shown in figures 8, 9 and 10. The efficiency parameter $B_{ref}$ varies smoothly from 0.05 at high specific speeds to a value 6 times this at very low specific speeds. The figure includes test cases in which the Reynolds number, roughness and size has been varied. The ten cases above a specific speed of 1.5 are axial and the others are radial machines. Most test data lies within a narrow band of ±25% from the correlation equation given in Appendix 2.

The data for the flow coefficient parameter $C_{ref}$ given in figure 9 shows much more scatter than that for the efficiency, but taking into account the variety of stages and the range of Mach numbers including choked transonic machines this is probably unavoidable. Similar scatter is found with the pressure rise sensitivity to the efficiency, parameter $D_{ref}$, but there is some evidence of a trend to increasing values with higher specific speed. Although the agreement is better than any other correlation known to the authors, the scatter indicates that more effort is still needed in these areas to find improved correlations. If more data were available the parameters $C_{ref}$ and $D_{ref}$ might be better correlated by separating the different types of machines (subsonic, transonic, single stage, multistage, etc.) Note that fewer points are shown in figures 9 and 10 than in figure 8, as not all publications give details of the flow shift and change in pressure rise.

Correlation equations for the lines shown in figure 8, 9 and 10 are given in Appendix 2.
DISCUSSION

The level of agreement of the final correlation for the measured change in efficiency with the different test cases is generally within ±25%, such that a change of 1% point in efficiency is calculated to within 1 ± 0.25% points. Given that the parameter $B_{ref}$ varies by over 600% over the whole range of compressors considered this scatter is actually quite small. This is really a lot better than might be expected with such a model when applied to a wide range of compressor types; including axial and radial, single stage and multistage, incompressible and compressible, subsonic and transonic, and highly loaded and weakly loaded machines.

The theory given above suggests that the coefficient $B_{ref}$ should be a function of the machine type, the blade loading, blade number and reaction and equation 14 can be rearranged to give

$$B_{ref} = \frac{\sum \frac{1}{2} (w/u)^2 (\zeta_{ref})_i}{\sum_i (\Delta h/ u^2)_{i,t}}$$

where $u$ is the blade speed. This shows that stages with a higher loading in terms of the work input may be expected to have lower values of $B_{ref}$ and stages with high losses or high inlet velocities might have higher values. Some of the scatter may thus be related to the differences in loading, blade number or reaction of the different cases, which has not been considered here. More detailed analysis of machines with different loadings for a certain specific speed might identify this effect.

The value of the coefficient $B_{ref}$ determined from the test data for axial compressors of high specific speed agrees closely with the value suggested by the analysis of the profile losses given by Horlock, equation 18. Of particular interest, however, is the way in which the coefficient increases as the specific speed is decreased. A radial compressor close to the so-called optimum specific speed (between 0.6 and 0.8) is roughly twice as sensitive to changes in the chord Reynolds number as an axial machine. A very low specific speed impeller, where the wetted surface on the end-walls is high compared to the profile surface, is up to six times as sensitive as an axial compressor. Equations 1 and 2 do not show this important effect in such a clear way. A consequence of this is that correction methods based on equations 1 or 2 will always need a different value of the Reynolds independent loss fraction, $a$, to model Reynolds number effects for different machine types and this clearly demonstrates the advantage of the procedure and equation structure described here.

The scatter for $C_{ref}$ and $D_{ref}$ is also quite large and these parameters might be better correlated by separating the different types of machines (subsonic, transonic, etc.) if more data were available.
The real physical effects related to the change in Reynolds number and roughness are only included in this model to the extent that they are similar to the changes in the flow structure and dissipation of a flat plate with zero pressure gradient. Clearly the approach does not include other detailed effects known to affect the frictional losses in compressors, such as the extent of the laminar flow near the leading edge, the changes in the location and structure of transition and laminar separation bubbles due to the effects of turbulence and upstream wakes, the effects of pressure gradients, or any effects due to 2D or 3D turbulent separations and secondary flows. These effects may be different for different cases and may also account for some of the scatter.

Another substantial cause of the level of scatter is the extreme difficulties of properly taking account the effect of machined surface roughness on the flow. Any improvements in the procedure described here will need to be much more precise in this respect rather than to use the simple equation \( k_s = 5Ra \) based on an equivalent sand roughness. In specific cases with particular surface types a different relationship might be more appropriate and reduce the scatter.

For any specific stage where measurements are available to calibrate the empirical coefficients in this method then these should be used for these cases to give the best prediction of the effects. The calibration method described allows this to be done. For specific compressor designs, as used by individual companies over a smaller range of machine types with specific reaction levels, loading levels and roughness textures, it may then be possible to produce an improved correlation with less scatter for these cases. In the absence of detailed data, the correlation here is recommended as the best guess, and predicts the change in efficiency within ±25% for all machines.

CONCLUSIONS

The following main conclusions can be derived from this work:

- A unified method for the correction of the effects of Reynolds number, size and roughness on the performance of all types of compressors and has been developed, based on a flat plate analogy for the variation of the skin friction coefficient.
- The method can be applied to any system of correlations for losses in turbomachinery or as a correction equation for the performance of a whole machine, a stage or an individual component due to Reynolds number, roughness or size effects.
- The method includes corrections for the change in efficiency, flow and pressure rise at the optimum point and these corrections can be applied across the whole characteristic.
- The method identifies that the sensitivity of different types of stages to these effects is mainly determined by the type of machine, as characterised by the specific speed, and that the loading and reaction may also play a role.
- The method provides a logical and consistent approach for the calibration of coefficients and to assess the sensitivity of the effects to different components.
- The method has been calibrated with success for many axial and radial compressors and a global correlation for the empirical coefficients has been provided. This correlation predicts the change in efficiency to within 25%, that is a 1% change is predicted to be between 0.75 and 1.25%.
- The theoretical method developed should also be suitable for axial and radial turbines and for pumps, but a similar extensive calibration of coefficients will be needed.

ACKNOWLEDGEMENTS

PCA Engineers would like to thank the EU project MOET (More Open Electrical Technologies, www.eurtd.com/moet/) for financial support. Many industrial companies provided access to commercially sensitive test data on these effects and thanks are due to Beat Ribi and Hannes Bennetschik of Man Diesel and Turbo, to Janpeter Kuehnel, Armin Reichl and Daniel Rusch, of ABB Turbosystems, to Prof. Peter Pelz and Michael Hess of the TU Darmstadt, to the German R&D consortium FLT, to John Bolger of Rolls Royce and to Werner Jonen of Siemens. Thanks are given to Johann Göllich for clarification of his friction factor equations for a flat plate.
REFERENCES
APPENDIX 1: EQUATIONS FOR THE FRICTION FACTOR OF A FLAT PLATE

Following Gülich (4), we write the laminar and turbulent equations for the skin friction of a flat plate using the chord Reynolds number as

\[
c_{f,\text{lam}} = \frac{k_1}{Re^{0.5}}, \quad c_{f,\text{turb}} = \frac{0.136}{\log_{10}\left(\frac{0.2k_1c}{Re} + \frac{j2.5}{Re}\right)^{2.15}}, \quad k_2 = 5Ra, \quad Re = \frac{w_1c}{\nu}
\]

We combine these to determine the friction factor for a laminar or turbulent flow as

\[
f = 4c_f, \quad c_f = Pc_{f,\text{lam}} + (1 - P)c_{f,\text{turb}}, \quad P = \frac{1}{1 + e^{-t}}, \quad t = k_2\left(\frac{c_{f,\text{lam}}}{c_{f,\text{turb}}} - 1\right)
\]

\(P\) is a blending function that varies between 0 for turbulent and 1 for laminar flow and blends the laminar and turbulent regions together. The constant \(k_2\) determines the speed of the blending and a value of 5 gives a fairly abrupt transition, as in the friction factor diagram of Traupel (15) and Koch and Smith (7). The constant \(k_1\) is related to the lower critical Reynolds number and a value of 2.656 leads to a lower critical Reynolds number of 200,000 as recommended by Schaeffler (2).

APPENDIX 2: CORRELATIONS FOR FACTORS IN CORRECTION EQUATIONS

\[
\phi = \frac{\dot{V}}{u_2D_2^2}, \quad \psi = \frac{\Delta h_y}{u_2}, \quad \omega_s = \omega \frac{(\dot{V})^{1/2}}{(\Delta h_y)^{3/4}} = 2\frac{\phi^{1/2}}{\psi^{3/4}}, \quad \omega_{s,\text{ref}} = 2\frac{\phi_{\text{ref}}^{1/2}}{\psi_{\text{ref}}^{3/4}}, \\
B_{\text{ref}} = 0.05 + \frac{0.025}{(\omega_{s,\text{ref}} + 0.2)^2}, \quad C_{\text{ref}} = 0.04 + \frac{0.06}{(\omega_{s,\text{ref}} + 0.35)^2}, \quad D_{\text{ref}} = 0.8 - \frac{0.5}{(\omega_{s,\text{ref}} + 1)^2}
\]

To give a symmetrical correction procedure, the flow coefficient and pressure rise at the reference conditions are used to determine the coefficients from the reference specific speed. This requires that a small iteration is first made to determine the specific speed at the reference conditions with \(f_{\text{ref}} = 0.012\), taking into account the shift in the flow and pressure characteristics.