A NEW SCENARIO OF TURBULENCE THEORY AND AN APPLICATION TO PIPE TURBULENCE

Tsutomu Kambe[1]
[1] *Former Professor, University of Tokyo, Tokyo, Japan*

Abstract A new general scenario of turbulence theory is proposed and applied to pipe-flow turbulence. The theory supports transverse traveling waves accompnied with considerabl dissipation of energy by internal friction (Joule-like effect). Its predictions are consistent with some characteristic features of pipe turbulence found in recent experiments and transition analyses: (i) *existence of traveling waves* and their important function and (ii) *existence of two large scales* (LSM and VLSM). In fact, predicted waves are characterized by two scales (wave-length and damping-length). Bulk energy dissipation is expressed unexpectedly in a form analogous to a model of eddy-viscosity.

INTRODUCTION

In the turn of the century, a number of publications on turbulence have appeared. To mention a few, in addition to self-contained textbooks on turbulence such as [1], a new-formulation or a formulation-by-analogy of the turbulence theory are proposed. Insufficiencies of the theory or ill-defined concepts are reviewed by some of papers or books such as [2, 3, 4, 5].

Large scale feature is one of the main subjects in the study of wall turbulence: boundary layer, pipe flow and channel flow [6, (a)$\sim$(d)]. There is substantial evidence of existence of large scales, LSM (large-scale motions) and VLSM (very-large-scale motions), containing a significant fraction of total kinetic energy. In pipe turbulence, VLSM-motions are not only energetic, but also contribute a large fraction of the Reynolds shear stress [7, (a)$\sim$(d)]. From observed power spectrum $\Phi_{uu}(k_x)$ of streamwise velocity u (x-direction) with respect to the streamwize wave-number k_x, a pre-multiplied spectrum is defined by $k_x \Phi_{uu}(k_x)$. It is now recognized that the pre-multiplied spectra have two *peaks*, at LSM of $\lambda_x/R \approx 1 \sim 3$ and at VLSM of $\lambda_x/R \approx 15 \sim 30$, and decays beyond VLSM, where R is the pipe radius and λ_x the streamwise scale. The power spectrum takes a scaling form $\Phi_{uu} \propto k_x^{-1}$ between the two, while $\Phi_{uu} \propto k_x^{-5/3}$ at higher k_x's. These features were known since Bullock *et al.* (1978) [6, b]. The spectrum range of $\Phi_{uu} \propto k_x^{-1}$ contains essentially all the streamwise kinetic energy.

It is clarified by recent studies of pipe flow [8, (a)$\sim$(e)] that the flow supports traveling waves with multi-fold rotational symmetries in turbulent state. From the experimental study of transitional flows in a pipe [8, (e)], formation of finite-length slugs is observed at transition from laminar to turbulent state. The structure of a slug has turbulence properties similar to those of developed pipe-turbulence, and its front and back faces propagate forward and backward respectively with respect to the flow-velocity averaged over the cross-section.

From the aboves, following two features can be summarized and in fact are regarded to be essential in the present formulation and analysis. (i) Turbulence fields support propagation of waves which are transverse. They exist even in incompressible fluids. Existence of transverse traveling waves in turbulent pipe flows is confirmed both experimentally and computationally. (ii) The scale VLSM is regarded as the largest coherent scale of pipe turbulence containing essentially all the streamwise kinetic energy. Mechanical origin of the scale VLSM is not clear at the moment. Present analysis of the next section gives a hint why it is much larger than the characteristic scale R of the experimental device.

New scenario of turbulence theory is currently developed. In one approach, the system of fluid equations is transformed to that of Maxwell-type equations and applied to turbulence [2, 4]. However in [5, (b)], it is proposed that a new field should be introduced in turbulence and governed by Maxwell-type equations. This approach is based on a *Theorem* stating that *current conservation* implies a field of Maxwell equations [5, (a)]. It is called *Transverse-Wave* (TW) field here (it was termed *Vortex Field* by Scofield & Huq [5, (b)]). In fluid turbulence, the current conservation is a basic property, so that one can introduce a TW-field in turbulence. The TW-field accompanies its own mechanism of energy dissipation by a *fluid*-Joule effect. Present study is carried out according to *this scenario*, and new findings are obtained regarding some features of pipe turbulence. It is in fact unexpected to find that *bulk* dissipation of the fluid-Joule effect takes a form analogous to a model of eddy-viscosity.

Alternative pipe-flow solutions consisting of streamwise roll, streaks and waves were studied by [8, (c)] and [9]. Present scenario gives a new insight into this problem. A small perturbation velocity $\boldsymbol{u}$ to a steady streaky flow $\boldsymbol{U}$ excites a *TW*-field described by a *fluid*-vector-potential $\boldsymbol{a}$, defining *fluid*-electric field $\boldsymbol{e}$ and *fluid*-magnetic field $\boldsymbol{b}$ respectively by

$$\boldsymbol{e} \equiv -\partial_t \boldsymbol{a} - \nabla\phi, \qquad \boldsymbol{b} \equiv \nabla \times \boldsymbol{a}.$$

Time-dependent current flux is given by $\boldsymbol{j} = \rho\boldsymbol{u} + \boldsymbol{j}_d$ where $\rho\boldsymbol{u}$ is a convection current of density ρ, while $\boldsymbol{j}_d = \sigma\boldsymbol{e}$ is a *drift* current with σ a positive constant (inverse internal friction), which is a *fluid*-Ohm's law. From the Maxwell-type equations of Eq.(1) of the next section, one can derive the following transverse-wave equation under forcing and damping:

$$\nabla^2 \boldsymbol{e} - c_t^{-2}\partial_t^2 \boldsymbol{e} = \rho\mu\,\partial_t \boldsymbol{u} + \mu\sigma\partial_t \boldsymbol{e}, \qquad [\textit{wave equation}]$$

where $c_t = 1/\sqrt{\mu\epsilon}$ is the wave velocity with μ and ϵ field parameters introduced there. The first term on the right is a forcing term driven by the fluctuating flow $\boldsymbol{u}$ and the second is a damping term due to the fluid-Joule current $\sigma\boldsymbol{e}$. Note that the *TW*-field exerts *fluid*-Lorentz-force $\boldsymbol{F}_L$ on the flow, while external forces were introduced in [8, (c)] and [9].

A NEW SCENARIO OF TURBULENCE THEORY

(i) *Governing Equations*: In order to represent the TW-field, we introduce a four-vector potential $(\phi, \boldsymbol{a})$ depending on time t and space coordinates $\boldsymbol{x} = (x_1, x_2, x_3)$. Then the dynamics of TW-field is defined by the following equations:

$$\partial_t \boldsymbol{b} + \operatorname{curl} \boldsymbol{e} = 0, \qquad \operatorname{div} \boldsymbol{b} = 0, \qquad -\partial_t \boldsymbol{d} + \operatorname{curl} \boldsymbol{h} = \boldsymbol{j}, \qquad \operatorname{div} \boldsymbol{d} = \rho, \qquad (1:\ a, b, c, d)$$

where $\boldsymbol{d} = \epsilon \boldsymbol{e}$ and $\boldsymbol{h} = \mu^{-1} \boldsymbol{b}$ with ϵ and μ being field parameters analogous to those of the Electromagnetic (EM) theory. Here, lower-case bold-letters are used for the TW-field vectors in order to show clear analogy to the Maxwell equations of EM theory with corresponding upper-case letters. Transverse waves are naturally supported in the system $(1a) \sim (1d)$. In turbulence field too, transverse waves are naturally accommodated by the equation system defined below.

From $(1a) \sim (1d)$, conservation equations of energy and momentum are immediately derived:

$$\partial_t w + \operatorname{div} \boldsymbol{q}_{TW} = -\boldsymbol{j} \cdot \boldsymbol{e}, \quad \partial_t \boldsymbol{g} + \partial_{x_j} T_{ij} = -\boldsymbol{F}_L, \quad \text{where } \boldsymbol{q}_{TW} = \boldsymbol{e} \times \boldsymbol{h}, \ \ \boldsymbol{F}_L \equiv \rho \boldsymbol{e} + \boldsymbol{j} \times \boldsymbol{b}, \qquad (2:\ a, b, c, d)$$

$w = \frac{1}{2}(\boldsymbol{e} \cdot \boldsymbol{d} + \boldsymbol{h} \cdot \boldsymbol{b})$ is a field energy density, $\boldsymbol{q}_{TW}$ is *fluid*-Poynting-vector (TW-energy flux), and $\boldsymbol{g} = \boldsymbol{d} \times \boldsymbol{b} = \boldsymbol{q}/c_t^2$ is a field momentum, and T_{ij} is *fluid*-Maxwell-stress, where $c_t = 1/\sqrt{\epsilon\mu}$ is the velocity of transverse waves. The right hand sides of $(2a)$ and $(2b)$ are *fluid*-Joule term and *fluid*-Lorentz-force reaction, respectively.

Whole field consists of *Fluid Flow* (FF) and *TW* field. The energy equation is given by $\partial_t \big[\rho(\frac{1}{2}v^2 + \varepsilon) + w\big] + \operatorname{div}(\boldsymbol{q}_{FF} + \boldsymbol{q}_{TW}) = 0$, where ε is the internal energy of fluid, and $\boldsymbol{q}_{FF}$ is the FF-energy flux, given by $\rho \boldsymbol{v}(\frac{1}{2}v^2 + \varepsilon + P) - \boldsymbol{v} \cdot \tau^{(vis)} - \kappa_T \nabla T$, with $\tau^{(vis)}$ the viscous stress tensor, T the temperature and κ_T the thermal diffusivity. The FF-momentum equation is given by

$$\partial_t \rho \boldsymbol{v} + \partial_{x_j} \Pi_{ij} = \boldsymbol{F}_L, \qquad \text{where} \quad \Pi_{ij} = \rho v_i v_j + p\delta_{ij} - \tau_{ij}^{(vis)} \qquad (4)$$

([5, (b)]). Adding $(2b)$ and (4) side by side leads to the momentum equation of whole field: $\partial_t(\rho \boldsymbol{v} + \boldsymbol{g}) + \partial_j(\Pi_{ij} + T_{ij}) = 0$.

(ii) *Dissipation*: Our system is dissipative, and the *fluid*-Joule heat is given by $Q_j = \boldsymbol{j}_d \cdot \boldsymbol{e} = \sigma|\boldsymbol{e}|^2 (> 0)$. Owing to this dissipative heat, the equation for the specific entropy s is described by $\rho T(D/Dt)s = Q_j + Q_{vis}$, where D/Dt is the convective derivative and Q_{vis} the viscous heat. The first term is new due to the Joule heat.

(iii) *Application to pipe turbulence*: Let us examine whether the present theory gives any insight into observed features of pipe turbulence, compactly summarized in the *Introduction*. In view of transverse waves existing in turbulence, the problem of wave propagation within the pipe turbulence reduces to that of wave guide filled with a medium characterized by the parameters ϵ and μ. This is studied by the equations $(1a \sim 1d)$ with assuming that the wave propagation is one-dimensional along the pipe axis (denoted by x-axis) with a frequency ω. Its cross-section is described by the coordinates (r, θ) for $0 \le r < R$ and $0 \le \theta < 2\pi$. Owing to the drift current $\boldsymbol{j}_d$, there is damping in the wave propagation.

Expressing the wave amplitude with a factor $e^{in\theta}$ to denote an n-fold symmetry (n: an integer), a component of $\boldsymbol{e}$ (or $\boldsymbol{h}$, or $\boldsymbol{a}$) is represented by a traveling wave: $\Psi = \psi(r) e^{-k_i x}\, e^{i(k_r x - \omega t)}\, e^{in\theta}$, where the wave-number is expressed by a complex form, $k = k_r + ik_i$, to account for the damping effect. From the system $(1a \sim 1d)$, it is found that $k_i = \mu\sigma(\omega/k_r) = \mu\sigma c_t$ with the wave length $\lambda = 2\pi/k_r$. The cross-stream mode is given by $J_n(\kappa r) e^{in\theta}$ where $\kappa = \sqrt{k_0^2 - k_r^2 + k_i^2}$ and $k_0 = \omega/c_t$. This implies that the damping distance of waves is given by $d \sim 1/k_i \sim (\mu\sigma c_t)^{-1}$. If we use a complex frequency $\omega = \omega_r + i\omega_i$ instead of wavenumber, we obtain a decay time, $\tau_d \sim 1/|\omega_i| \sim d/c_t$.

This solution has two characteristic scales λ and d. The two scales LSM and VLSM observed in the experiments are considered to be related to the traveling waves examined above. Thus, it is likely that the wave length λ corresponds to LSM, while VLSM could be related to the damping distance d because no larger scale is observed in experiments.

According to the present theory, the bulk rate of Joule dissipation takes a form analogous to that of eddy-viscosity with the coefficient derived by the present theory as $\nu_{joule} \sim c_t d$, while any eddy-viscosity ν_{eddy} is a model.

Summary: A new scenario of turbulence theory is proposed by introducing a new TW field to the turbulence field without self-contradiction. This formulation is equipped with a mechanism of energy dissipation by an internal friction resistance, which is comparable with the eddy-viscosity in order of magnitude. This theory predicts traveling waves in pipe turbulence, which have two characteristic scales of a wave-length λ and a length d of wave damping. Significance of λ and d is discussed in relation to LSM and VLSM of wall turbulence.

References

[1] Stephen B. Pope. *Turbulent Flows*. Cmbridge Univ. Press, 2000.

[2] H. Marmanis. Analogy between the Navier-Stokes equations and Maxwell's equations: Application to turbulence. *Phys. Fluids* **10**: 1428–1437, 1998.

[3] Arkady Tsinober. *An Informal Conceptual Introduction to Turbulence* (2nd ed., *Series*: Fluid Mechanics and Its Application 92). Springer, 2009.

[4] T. Kambe. A new formulation of equations of compressible fluids by analogy with Maxwell's equation, *Fluid Dyn. Res.* **42**, 2010.

[5] D F Scofield and Pablo Huq: (a) *Phys. Lett. A* **373**: 3476–3482, 2010; (b) *Fluid Dyn. Res.* **46**, 055514 (22p), 2014.

[6] (a) J. Jimenez. The largest scales of turbulent wall flows. *Center for Turbulence Research, Ann. Res. Briefs* (Stanford Univ): 137–154, 1998, reviews previous studies of long streamwise structures with $E \sim k^{-1}$ range; (b) Bullock, Cooper & Abernathy. *J. Fluid Mech.* **88**: 585–608, 1978; (c) Perry, Henbest & Chong. *J. Fluid Mech.* **165**: 585–608, 1986; (d) Priymak, & Miyazaki. *Phys. Fluids* **6**: 3454–3464, 1994.

[7] (a) K.C. Kim, and R.J. Adrian. *Phys. Fluids* **11**: 417–422, 1999; (b) M. Guala, S.E. Hommema, and R.J. Adrian. *J. Fluid Mech.* **554**: 521–542, 2006; (c) Monty, Stewart, Williams & Chong. *J. Fluid Mech.* **589**, 147–156, 2007; (d) Rosenberg, Hultmark, Vallikivi, Bailey & Smits. *J. Fluid Mech.* 731, 46-63, 2013.

[8] (a) L. Boberg, and U. Brosa. *Z. Naturforsch.* **43a**: 697–726, 1988; (b) U. Brosa. *Z. Naturforsch.* **46a**: 473–480 1991; (c) H. Faisst, and B. Eckhardt. *Phys. Rev. Lett.* **91**: 224502, 2003; (d) B. Hof, C.W.H. van Doorne, J. Westerweel, F.T.M. Nieustadt, H. Faisst, B. Eckhardt, H. Wedin, R.R. Kerswell and F. Waleffe. *Science* **305**: 1594–1598, 2004; (e) M. Nishi, B.Ünsal, F. Durst, and G. Biswas. *J. Fluid Mech.* **614**: 425–446, 2008.

[9] H. Wedin and R.R. Kerswell. Exact coherent structures in pipe flow: travelling wave solutions. *J. Fluid Mech.* **508**: 333–371, 2004.