TURBULENCE IN MIXED-PHASE CLOUDS

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Abstract Mixed-phase clouds play an important role in the correct determination of the Earth’s radiative balance but are not as well understood as warm clouds. It has been known for some time that turbulence is important in generating and sustaining mixed-phase clouds and here we present a simple model of a turbulent mixed-phase cloud by coupling a simple adiabatic mixed-phase cloud model with a Lagrangian stochastic model for the vertical velocity. We demonstrate that this model can indeed generate and maintain both liquid water and ice and reproduces qualitatively many features of mixed-phase clouds.

THE MODEL

Clouds containing both liquid water and ice simultaneously, known as mixed-phase clouds, are not as well understood as warm clouds and are poorly modelled by general circulation models (GCMs). For example, GCMs exhibit a positive bias in sea-surface temperatures in the Southern Ocean and occurs because GCMs cannot produce liquid water and ice simultaneously: the diagnosis of liquid water depends on the value of the mean supersaturation over water in a grid box which is typically negative over the Southern Ocean. However, in a turbulent environment it is possible to have both a negative mean supersaturation over water and liquid water at the same time.

It is known that turbulence is an important mechanism both for generating and sustaining mixed-phase clouds \cite{1, 2, 3, 4, 5}. Turbulent velocity fluctuations can produce updrafts of sufficient strength to activate liquid water and limit the evaporation of droplets and the growth of ice particles that would result. To date, studies of turbulence in mixed-phase clouds have mostly been theoretical with various restrictive assumptions. Here we consider the diffusional growth of mixed-phase clouds by coupling a Lagrangian stochastic model with a simple adiabatic mixed-phase cloud model.

We assume that the only phase transitions are ice-vapour and liquid-vapour transitions; there is no ice-liquid transition. For simplicity we assume that the ice particles are spherical and that the ice particles and droplets are each monodisperse. We also assume that there is no sedimentation, inertia, mixing between different cloud parcels, solute or curvature effects, ventilation effects, collisions or coalescence. The adiabatic cloud model consists of equations for pressure, \( p \), and temperature, \( T \), at time \( t \):

\[
\begin{align*}
\frac{dp}{dt} &= -\frac{gp}{R_d} w, \\
\frac{dT}{dt} &= \frac{L_v}{c_{pa}} \frac{dq_l}{dt} + \frac{L_s}{c_{pa}} \frac{dq_i}{dt} - \frac{g}{c_{pa}} w
\end{align*}
\]

\((g\) is acceleration due to gravity, \( R_d \) is dry air gas constant, \( w \) is vertical velocity, \( L_v \) is latent heating of respectively vapourisation or sublimation, \( c_{pa} \) is the specific heat capacity of dry air) supersaturation over liquid water, \( s_w \),

\[
\frac{1}{1 + s_w} \frac{ds_w}{dt} = A_1(T) w - A_2(p, T) \frac{dq_l}{dt} - B_2(p, T) \frac{dq_i}{dt}
\]

and the ice particle radius, \( r_i \), and ice mixing ratio, \( q_i \),

\[
\frac{dr_i}{dt} = B_3(T) \frac{s_i}{r_i}; \quad \frac{dq_i}{dt} = 4\pi \frac{\rho_i}{\rho_a} N_i B_3(T) s_i r_i
\]

\((N_i \) is the ice particle number density, \( \rho_x \) is the density of ice or dry air; the exact form of the coefficients, \( A_x \) and \( B_x \), is unimportant). Equations for the droplet radius, \( r_m \), and liquid water mixing ratio, \( q_l \), are analogous to those for \( r_i \) and \( q_i \). The relationship between \( s_w \) and the supersaturation over ice, \( s_i \), is given by \( s_i = \xi(T)(s_w + 1) - 1 \) where \( \xi(T) = e_{sw}(T)/e_{s1}(T) \) and \( e_{sw} \) is the saturation vapour pressure with respect to liquid water or ice. For simplicity, liquid droplets and ice particles are activated whenever \( s_w > 0 \) or \( s_i > 0 \) respectively. If their size falls below a critical value, they are maintained at that size.

We assume that the turbulence is statistically homogeneous and that the velocity fluctuations are modelled according to

\[
dw = -\frac{w}{T_0} dt + \sqrt{C_0 \varepsilon} dW
\]

where \( \varepsilon \) is the mean kinetic energy dissipation rate, \( C_0 \) is the constant of proportionality in the Lagrangian second-order structure function, \( dW \) is the increment of a Wiener process and the decorrelation time scale is given by \( T_0 = 2\sigma_w^2/C_0 \varepsilon \) in which \( \sigma_w^2 \) is the vertical velocity variance. Here we take \( C_0 = 5 \).

RESULTS

We consider cases with and without liquid water present initially and also where \( N_i \) is relatively small, \( N_i = 10^9 \text{ m}^{-3} \), or large \( N_i = 10^9 \text{ m}^{-3} \). In the results below, \( s_w = 0 \) initially. Figure 1a shows that if there is no turbulence then any
liquid water that is present initially decays rapidly. In contrast, \( r_w \) grows (after an initial decrease) in the presence of turbulence (and no mean flow). While similar results can be obtained with a constant updraught (of suitable value), \( s_w \) is positive for this case. In contrast, when there is only turbulence, figure 1b shows that the mean \( s_w \) becomes negative while the root-mean-square (rms) \( s_w \) is positive and increasing. The results in figure 1 were obtained for small \( N_i \); when \( N_i \) is large, the evolution of the droplets is sensitive to the value of \( s_w \). If \( s_w \) is not sufficiently large, then the liquid water decays to zero. When it is larger, the droplets start to evaporate initially as a result of competition from the ice particles but then start to grow again.

**Figure 1.** Evolution of (a) the mean \( r_w \) (over many realisations of the flow) and (b) mean \( s_w \) and rms \( s_w \) for \( N_i = 10^3 \text{ m}^{-3} \).

Figure 2 shows how the probability density functions (pdfs) of \( s_w \) and \( s_i \) evolve with time in the case of no initial liquid water. A small positive tail in the pdf of \( s_w \) ensures that droplets form and grow with time. It can be seen that the peak of both \( p(s_w) \) and \( p(s_i) \) occurs at water saturation. The long negative (subsaturated) tail of the pdfs is a result of droplets and ice particles evaporating. In this case \( \frac{dg_w}{dt} \) is small for both phases and so the supersaturation is dominated by \( w \). Hence, we see an approximately uniform distribution. If the pdfs are conditioned on \( r_w \) larger than some threshold then the extent of this tail decreases. For large \( N_i \), the peaks of \( p(s_w) \) and \( p(s_i) \) occur below water supersaturation although there is a 'kink' at \( s_w = 0 \) indicating that liquid water does form (although it decays with time).

**Figure 2.** Pdfs of (a) \( s_w \) and (b) \( s_i \) for \( N_i = 10^3 \text{ m}^{-3} \) at different times with no liquid water present initially.

**References**


