

INTERMITTENCY IN WEAK MAGNETOHYDRODYNAMIC TURBULENCE

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Abstract Three-dimensional incompressible magnetohydrodynamic (MHD) turbulence with a strong uniform magnetic field \mathbf{b}_0 may be governed by the regime of weak turbulence. At leading order, it is known that the asymptotic regime of weak MHD turbulence is achieved *via* three-wave resonant interactions with the scattering of two of these waves on a third/2D mode for which $k_{\parallel} = 0$. For zero cross-helicity, the expected exact solution is an energy spectrum in k_{\perp}^{-2} . Higher-order statistics has, however, never been reported in the literature. Therefore, we have recently investigated this question with high resolution direct numerical simulations [10]. We found the presence of strong intermittency when the vector separation of structure functions is taken transverse to \mathbf{b}_0 . This result may be explained by the influence of the 2D modes whose regime belongs to strong turbulence. In addition to shed light on the origin of this intermittency, we derived a log–Poisson law, $\zeta_p = p/8 + 1 - (1/4)^{p/2}$, which fits perfectly the data and highlights the important role of parallel current sheets.

INTRODUCTION

Recently, growing interest has been given to the study of intermittency in the weak turbulence (WT) regime [4]. WT is the study of the long-time statistical behavior of a sea of weakly interacting nonlinear dispersive waves for which a natural asymptotic closure may be obtained. The energy transfer between waves occurs mostly among resonant sets of waves and the resulting energy distribution, far from a thermodynamic equilibrium, is characterized by a wide power law spectrum that can be derived exactly [11]. WT is a very common natural phenomenon studied in, *e.g.* nonlinear optics [3], superfluid helium and processes of Bose-Einstein condensation [9], rotating fluids [6] and space plasmas [5]. Intermittency has been observed in the situation where coherent structures like sea foam [12] or freak ocean waves [8] are present. In these particular examples, intermittency is linked to the breakdown of the weak nonlinearity assumption induced by the WT dynamics itself and therefore cannot be considered as an *intrinsic* property of this regime.

Weak MHD turbulence differs significantly from other cases because of the singular role played by the 2D modes for which $k_{\parallel} = 0$ (\mathbf{k} is the wavevector in Fourier space and the subscript \parallel indicates the component of \mathbf{k} parallel to the guide field \mathbf{b}_0). Since Alfvén waves have frequencies $\omega_k^{\pm} = \pm k_{\parallel} v_A$ (with v_A the Alfvén speed) and only counter-propagating waves can interact, the three-wave resonance condition $\omega_{k_1}^+ + \omega_{k_2}^- = \omega_{k_3}^{\pm}$ and $\mathbf{k}_1 + \mathbf{k}_2 = \mathbf{k}_3$, implies that at least one mode must have $k_{\parallel} = 0$. This mode which acts as a catalyst for the nonlinear interaction, is not a wave but rather a kind of two-dimensional condensate with a characteristic Alfvén time $\tau_A \sim 1/(k_{\parallel} v_A) = +\infty$ and cannot be treated by WT. The standard way to overcome this complication has been to assume that the k_{\parallel} spectrum of Alfvén waves is continuous across $k_{\parallel} = 0$. Under this assumption a k_{\perp}^{-2} energy spectrum was predicted analytically in the simplest case of zero cross-helicity with a direct cascade towards small-scales [7]. This prediction has been confirmed observationally [14] and numerically [1, 13]. We have re-investigated this regime with high resolution (with $1536^2 \times 128$ collocation points) direct numerical simulations [10]. We present below the main result, *i.e.* the intermittency law corresponding to weak MHD turbulence.

INTERMITTENCY LAW

To derive an intermittency law for weak MHD turbulence, we first need to introduce the symmetric structure functions,

$$S_p = \left\langle (\delta z^+)^{p/2} \right\rangle \left\langle (\delta z^-)^{p/2} \right\rangle = C_p \ell_{\perp}^{\zeta(p)}, \quad (1)$$

where $\mathbf{z}^{\pm} = \mathbf{v} \pm \mathbf{b}$ are the Elsässer fields (\mathbf{v} is the velocity and \mathbf{b} the magnetic field normalized to a velocity) and δz^{\pm} are the increments of these fields (between two points separated by a vector ℓ_{\perp} transverse to \mathbf{b}_0). We want to build a model that fits the exact solution of WT (energy spectrum in k_{\perp}^{-2}) which in physical space implies $\zeta(2) = 1$. Then, following the original development [15] we define: $S_p = C_p \langle \varepsilon^{p/2} \rangle \ell_{\perp}^{p/2}$, where $\langle \varepsilon \rangle$ is the mean dissipation rate of energy and $\langle \varepsilon^{p/2} \rangle \sim \ell_{\perp}^{\mu_{p/2}}$. The latter relation is the so-called refined similarity hypothesis. The log–Poisson distribution for the dissipation leads to the general relation [15]: $\mu_m = \mu(m) = -m\Delta + C_0(1 - \beta^m)$, where Δ and β are linked to the co-dimension C_0 of the dissipative structures such that $C_0 = \Delta/(1 - \beta)$. In our case, the direct numerical simulations [10] show clearly the domination of current sheets for which the co-dimension is $C_0 = 1$. The system is closed by defining the value of Δ which is related to the dissipation of the most singular structures, such that $\ell_{\perp}^{-\Delta} \sim E_{\infty}/\tau_{\infty}$,

where E_∞ is the energy dissipated in these most singular structures and $\tau_\infty \sim \ell/v_\ell$ is the associated time-scale. Δ may be obtained by considering the following remarks. Weak MHD turbulence behaves very differently from isotropic MHD because in the former case the regime is driven at leading order by three-wave resonant interactions, with the scattering of two of these waves on a 2D/third mode. These 2D modes are also important to characterize dissipative structures: these structures, which look like vorticity/current sheets, are strongly elongated along the parallel direction and are therefore mainly localized around the $k_{\parallel} = 0$ plane in Fourier space. If we assume that the dynamics of the 2D modes are similar to the dynamics of two-dimensional strong turbulence, then it seems appropriate to consider that the time-scale entering in the intermittency relation may be determined by [2] $v_\ell \sim \ell_\perp^{1/4}$, hence the value $\Delta = 3/4$. With this all considered, we finally obtain the intermittency model:

$$\zeta_p = \frac{p}{8} + 1 - \left(\frac{1}{4}\right)^{p/2}. \quad (2)$$

The model fits perfectly our direct numerical simulation [10]. Note that the parameter β in this model measures the degree of intermittency: non-intermittent turbulence corresponds to $\beta = 1$ whereas the limit $\beta = 0$ represents an extremely intermittent state in which the dissipation is concentrated in one singular structure. According to the value obtained here $\beta \simeq 1/4$ (with $C_0 \simeq 1$), we might conclude that weak MHD turbulence appear more intermittent than strong isotropic MHD turbulence for which $\beta = 1/3$.

CONCLUSION

The key novel result in this work is that, for the first time, strong intermittency is shown to exist in such weakly interacting systems. The intermittency, as manifested in the statistical scaling exponents $\zeta(p)$, is modelled with a log-Poisson law which provides excellent agreement with the results from the simulations. This model, which is based on the She-Leveque type intermittency model [15] with additional physical insight from WT, allows one to make statements regarding the topology of the structures responsible for the intermittency directly from the behaviour of the $\zeta(p)$ function. In the case of our work where the 2D modes play a central role *via* the dissipative structures, the model shows that the topology of these structures is highly parallel sheet-like.

Our results are important for the interpretation of plasma turbulence observations and provides objective insights to the normally heated discussions on what constitutes turbulence in systems such as plasmas which host a rich variety of waves and instabilities and at the same time are inherently nonlinear. The results of our work seem to suggest that the quintessential signature of turbulence in the form of intermittency is not simply a property of strong turbulence but it may also be found in a medium where WT is present. Incompressible MHD is a singular example in weak turbulence because its dynamics, and so its existence, depends on the 2D modes. Since the 2D modes are responsible for intermittency, we can conclude that intermittency will always be found in weak MHD turbulence. This fact has previously been ignored due to the over-emphasis in the literature on spectra and the random phase approximation. Our simulations show that phase synchronization plays an important role, even in WT, whilst retaining some of the exact analytical results pertaining to spectra.

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