PERTURBATIONS EVOLUTION AND STREAKS FORMATION IN TURBULENT CHANNEL FLOW

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Abstract Evolution of initially small perturbations superimposed on turbulent channel flow is investigated via DNS. On the linear stage perturbations grow exponentially in average as $\sim \exp(\lambda t)$ with $\lambda^+ \approx 0.021$ in accordance with previous findings [1, 2]. Maximum perturbation amplitude is located at distance $y^+ = 12$ from the wall, thus, the perturbation growth may be attributed to streak instability. However, in contrast to existing models of streak instability the growing patterns appear and disappear occasionally as localized spots in $x - z$ plane. Streaks in perturbation field appear on the nonlinear stage of evolution. Perturbation amplitude saturates finally on the level of $\sqrt{2}$ of that of turbulence fluctuations in underlying turbulent flow indicating that disturbed flow is uncorrelated with original undisturbed flow field.

FORMULATION

Consider statistically steady turbulent channel flow $\mathbf{u}(t, x, y, z)$ which at time moment $t = 0$ is disturbed by small-amplitude perturbation with velocity field $\mathbf{u}'(t = 0, x, y, z)$. Here, x, y and z are the streamwise, wall-normal and the spanwise directions respectively. Due to stochasticity of the process original flow $\mathbf{u}$ and disturbed flow $\mathbf{u}_2 = \mathbf{u}+\mathbf{u}'$ diverge exponentially in time at the linear stage of perturbation evolution, so that in average $\|\mathbf{u}'\| \sim \exp \lambda t$ with $\lambda^+ \approx 0.021$ being the highest Lyapounov exponent [1, 2]. At $t \gg 0$ the field $\mathbf{u}_2$ represents the same turbulent flow as does $\mathbf{u}$. Thus, $\mathbf{u}$ and $\mathbf{u}_2$ at this limiting stage possess identical statistics and are uncorrelated. The latter yields $\overline{u'_i} = 0$, $\overline{u'_i u'_j} = 2\overline{u_i u_j}$ (overline denotes statistical averaging). Besides, it may be shown that in the limiting stage $\mathbf{u}$ and $\mathbf{u}'$ have identical spatial power spectra. In particular this means, that perturbation field $\mathbf{u}'$ must possess streaky structures as does original field $\mathbf{u}$. One of the most interesting questions is when (on which stage of evolution) streaks first appear in perturbation velocity field $\mathbf{u}'$.

Both $\mathbf{u}$ and $\mathbf{u}_2$ satisfy Navier-Stokes equations, thus for perturbation velocity $\mathbf{u}'$ one has equations

$$\frac{\partial \mathbf{u}'}{\partial t} + (\mathbf{u}'\nabla)\mathbf{u} + (\mathbf{u}\nabla)\mathbf{u}' - \nu\nabla^2\mathbf{u}' + \nabla p' = -(\mathbf{u}'\nabla)\mathbf{u}' \quad (1)$$

$$\nabla \mathbf{u}' = 0 \quad (2)$$

Eq-s (1,2) for a plane channel geometry with no-slip conditions on the rigid walls and periodic conditions in homogeneous streamwise and spanwise directions are solved numerically. Turbulent flow field $\mathbf{u}(t, x, y, z)$ which enter eq-n (1) as a coefficient is obtained from the Navier-Stokes equations which are solved in parallel with (1,2). Finite-difference algorithm of [3] was used. Initial perturbations were set randomly with the amplitude of $10^{-6}U_0$. Simulations were performed for three Reynolds numbers: $Re = 3000$, $Re = 4200$ and $Re = 7500$, where $Re = U_0 h/\nu$, U_0 is Poiseuille flow centerline velocity ($3/2$ of the bulk velocity), h is the channel half-height, ν is kinematic viscosity. Corresponding friction-velocity Reynolds numbers are $Re_\tau = 133$, $Re_\tau = 182$ and $Re_\tau = 301$.

RESULTS

As it was predicted, after initial reorganization perturbations grow almost exponentially $\sim \exp(\lambda t)$ with λ close to value found in [1, 2], $\lambda^+ = 0.021$. The linear stage of evolution continues up to perturbation amplitude reachs the amplitude of about $10^{-2}U_0$. After that the nonlinear stage starts and perturbation amplitude finally saturates on the limiting value of about $10^{-1}U_0$. Maximum perturbation amplitude at the linear stage attains at distance $y^+ = 12$ from the wall. Distribution of the amplitude along the wall-normal coordinate is more concentrated in comparison with similar distribution in developed turbulent flow. In other respects, fluctuation intensities of different velocity components distributed qualitatively similar both in perturbation field and in turbulent field. This also applies to distributions of vorticity fluctuations except one important detail. In turbulent field ω_z fluctuations have strongly pronounced maximum at the wall, while in perturbation field maximum ω_z fluctuations are at the same distance $y^+ = 12$ and are not much higher than other components of vorticity fluctuations. Such a distinction can be attributed to absence in perturbation field of near-wall streaks, which cause the majority of near-wall ω_z fluctuations in turbulent flow. Streamwise-velocity distributions in turbulent flow and in perturbation field shown in figure 1 confirm this conjecture. Although some x-elongation is visible in perturbation field structures, they are much different from the near-wall streaky structures in turbulent velocity. Taking in account a priory information that streaks must present in perturbation field in limiting stage, one can conclude that streaks formation is a nonlinear effect. Indeed, perturbation field in limiting stage after nonlinear saturation do contain streaky structures with the same statistics as in turbulent flow.

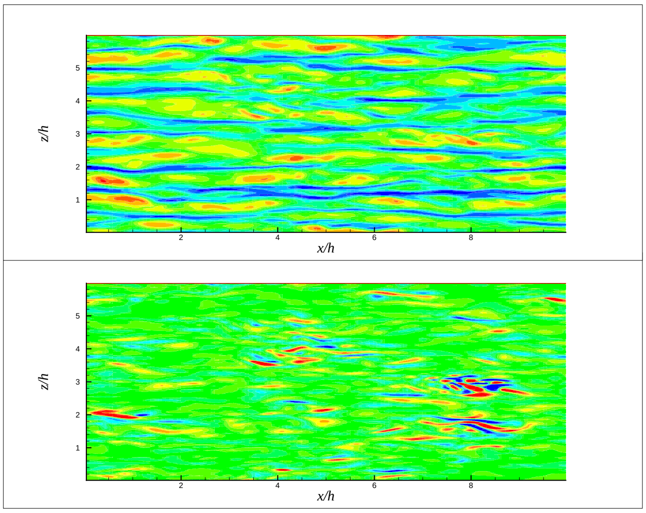

Figure 1. Instantaneous u_x-velocity distribution at $y^+ = 12$ in turbulent flow (up) and in perturbation field at linear stage of evolution (down).

Another distinctive feature of velocity distribution in perturbation field in figure 1 is extreme nonuniformity in perturbation patterns distribution over $x - z$ plane. This differ from results of the most of streak-instability models, considering streaks as a regular array of x-independent structures. Premultiplied 2-dimensional u_x-velocity power spectra $k_x k_z E_{u_x u_x}(k_x, k_z)$ in turbulent flow and in perturbation field in linear stage at $y^+ = 12$ are shown in figure 2. Since power spectra of perturbation field in limiting stage coincide with those in turbulent regime, the left spectrum in the figure equally belongs to perturbation field in limiting stage. One can see, that as a consequence of nonlinear transformation the scales in perturbation field undergoes significant enlargement in all spatial directions. While the most energetic u_x-structures in linear stage of perturbation evolution are of sizes $\lambda_x^+ \approx 200$, $\lambda_z^+ \approx 60$, in limiting stage as well as in turbulent flow the most energetic structures are more than three times longer, $\lambda_x^+ \approx 650$ and almost twice wider $\lambda_z^+ \approx 100$.

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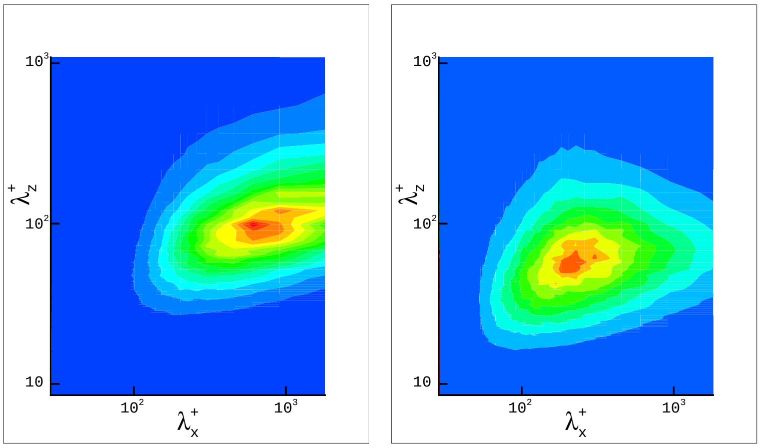

Figure 2. Premultiplied 2-dimensional power spectra of turbulent fluctuations (left) and perturbation field at linear stage of evolution (right) at $y^+ = 12$.

References

[1] N. Nikitin. On the rate of spatial predictability in near-wall turbulence. *Journal of Fluid Mechanics* **614**: 495–507, 2008.

[2] N.V. Nikitin. Disturbance growth rate in turbulent wall flows. *Fluid Dynamics* **44(5)**: 652–657, 2009.

[3] N. Nikitin. Finite-difference method for incompressible Navier-Stokes equations in arbitrary orthogonal curvilinear coordinates. *Journal of Computational Physics* **217(2)**: 759–781, 2006.