DISENTANGLING INERTIAL WAVES FROM EDDY TURBULENCE IN A FORCED ROTATING TURBULENCE EXPERIMENT

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Abstract
We present a spatio-temporal analysis of a statistically stationary rotating turbulence experiment, aiming to extract a signature of inertial waves and to determine at what scales and frequencies they can be detected. This analysis is performed from two-point correlations of temporal Fourier transform of the velocity fields obtained from time-resolved stereoscopic particle image velocimetry measurements in the rotating frame. We quantify the degree of anisotropy of turbulence as a function of frequency and spatial scale normal to the rotation axis. We show that this space-time-dependent anisotropy is well described by the dispersion relation of linear inertial waves at large scale, while smaller scales are dominated by the nonlinear sweeping of the waves by the random motions at larger scales. This sweeping effect is dominated here by the low-frequency quasi-two-dimensional component of the turbulence, a prominent feature of our experiment which is not accounted for by the weak wave turbulence theory.

INTRODUCTION

It is a matter of debate whether weak wave turbulence theory is a good candidate to describe rotating turbulence in the rapidly rotating limit [1, 2]. For rotating, stratified or magnetohydrodynamic turbulence [3], waves can propagate and coexist with "classical" eddies and coherent structures, which advocates for a spatio-temporal description of such flows. While temporal fluctuations are usually slaved to the spatial ones via sweeping effects in classical turbulence, they are expected to be governed by the dispersion relation of the waves for time scales much smaller than the eddy turnover time.

Inertial waves in rotating fluids obey the anisotropic dispersion relation $\sigma(\mathbf{k}) = 2\Omega|k_\parallel|/|\mathbf{k}|$, where Ω is the rotation rate and $k_\parallel$ the component of the wave vector $\mathbf{k}$ along the rotation axis (referred to as the vertical axis by convention) [4]. Fluid motions of weak amplitude and slowly varying in time ($\sigma \ll 2\Omega$) can be described in terms of waves with nearly horizontal wave vectors: they tend to be 2D3C (two-dimensional and three-component), invariant along the rotation axis.

A key question to assess the relevance of the wave turbulence theory for rotating turbulence is to determine the range of scales and frequencies for which 3D fluctuations follow the inertial-wave dispersion relation. This requires a full spatio-temporal analysis, which is very demanding in general for wave-turbulence systems: the accessible range of scales is usually limited in experiments, whereas long integration times are prohibitive in numerical simulations. The case of rotating turbulence is particularly delicate because of the specific form of the dispersion relation: the frequency is not related to the wave number but to the wave vector orientation only. Such signature of inertial waves in spatio-temporal energy spectra were recently obtained numerically by Clark di Leoni *et al.* [5], and experimentally by Yarom and Sharon [6]. The aim of the present paper is to further analyze experimentally the range of spatio-temporal scales at which inertial waves can be detected in rotating turbulence.

RESULTS

We present here new results aiming to analyze the spatio-temporal signature of inertial waves in a stationary rotating turbulence experiment. Turbulence is produced by a set of vortex dipole generators which continuously inject turbulent fluctuations towards the center of a rotating water tank where measurements are performed (Fig. 1a). We perform a spatio-temporal analysis of the two-dimensional three-component (2D3C) velocity fields measured by stereoscopic PIV in a vertical plane. We consider the two-point spatial correlation of the temporal Fourier transform $\tilde{u}_i(\mathbf{x}, \sigma)$ of the velocity fields,

$$R(\mathbf{r}, \sigma) = \frac{1}{2}\,\langle \tilde{u}_i(\mathbf{x}, \sigma)\tilde{u}_i^*(\mathbf{x}+\mathbf{r}, \sigma) + \text{c.c.}\rangle\,, \qquad (1)$$

with * the complex conjugate. This correlation probes the energy distribution among vector separations $\mathbf{r}$ for each angular frequency σ. Maps of the correlation $R(\mathbf{r}, \sigma)$ are illustrated in Fig. 1b: the iso-R lines evolve gradually from quasi-vertical at small frequency ("cigar" anisotropy) to more horizontal for $\sigma \sim 2\Omega$ ("pancake" anisotropy). From these correlations we define a scale and frequency-dependent anisotropy ratio $A(\sigma, r_\perp)$. If the anisotropy of the two-point correlation R at frequency $\sigma \leqslant 2\Omega$ is governed by linear inertial waves, we expect A to be independent of the scale, and to be set by the dispersion relation of inertial waves.

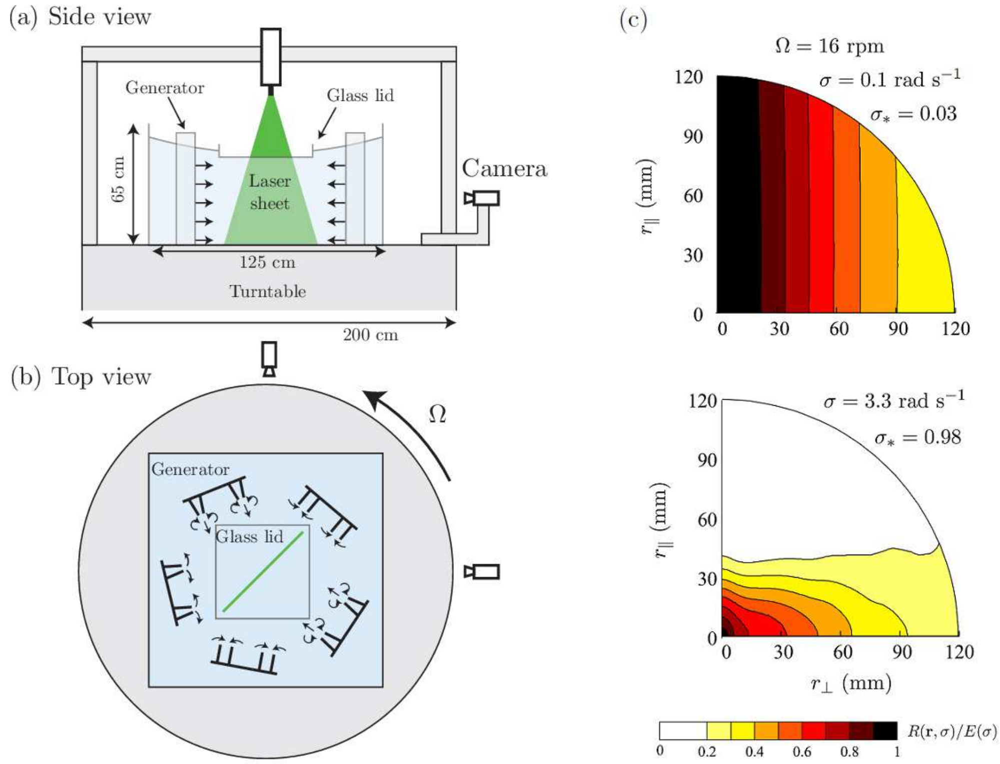

Figure 1. (a) Schematic of the experiment: an arena of 10 pairs of vertical flaps inject turbulent fluctuations in the center of a water tank mounted on a rotating platform [7, 8]. Three-component two-dimensional velocity measurements are performed in a vertical plane using stereoscopic PIV. (b) Maps of the normalized two-point correlation $R(\mathbf{r},\sigma)/R(0,\sigma)$ in the vertical plane $(r_\perp, r_\parallel)$ for $\Omega = 16$ rpm at small frequency (showing cigar anisotropy, reminiscent of the Talor-Proudman theorem) and large frequency (showing pancake anisotropy).

We find that the inertial-wave prediction provides a good description of the measured anisotropy ratio at large horizontal scales and large rotation rate. On the other hand, at smaller horizontal scale the inertial-wave prediction fails, with small frequencies significantly more isotropic than predicted by the inertial-wave argument. We identify the sweeping of the waves to be responsible for the scrambling of their spatio-temporal signature: An inertial wave propagating in a large-scale flow $\mathbf{U}$ evolving on a time scale slower than the wave period has a Doppler-shifted frequency, $\sigma = \sigma_i + \mathbf{k}\cdot\mathbf{U}$, where σ_i is the intrinsic frequency. In our experiment this sweeping is strongly dominated by the 2D mode, which is growing with the rotation rate. We believe that this sweeping effect by the 2D mode is generic in rotating turbulence, because of the accumulation of energy in the 2D mode allowed by the $3D \to 2D$ energy transfers at finite Rossby number. This 2D mode cannot be accounted for by the weak wave turbulence theory: our results question the relevance of this theory for rotating turbulence at the moderate Rossby numbers accessible in laboratory experiments, which are relevant to most geophysical and astrophysical flows.

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