A NEW SPECTRAL MODEL FOR SHEAR-DRIVEN HOMOGENEOUS ANISOTROPIC TURBULENT FLOWS

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Abstract A new system of governing equations for spherically-averaged descriptors, which allows to calculate incompressible homogeneous turbulent flows, is derived in the present study. Remarkable features of this model are that it makes a distinction between directional and polarization anisotropies, which are treated separately, and that no heuristic tuning of arbitrary constants is required. Spherical averaging allows to obtain a model for anisotropic turbulence which is as versatile as the classical Eddy-Damped Quasi-Normal Markovian (EDQNM) model for isotropic turbulence, i.e. this model can calculate anisotropic turbulent flows at both very high and low Reynolds numbers, with good resolution of both large and small scales and over very long evolution times. The present model is particulary suited for the study of shear-driven turbulent flows and their return to isotropy.

CLOSED EQUATIONS FOR THE TWO-POINT SECOND-ORDER CORRELATION TENSOR

The present model is derived starting from the governing equation of the second-ordrer spectral tensor $\hat{R}_{ij}(\boldsymbol{k},t)$, which is the Fourier transform of the two-point second-order correlation tensor $R_{ij}(\boldsymbol{r},t) = \langle u_i(\boldsymbol{x},t)u_j(\boldsymbol{x}+\boldsymbol{r},t)\rangle$, where $u_i(\boldsymbol{x},t)$ is the fluctuating velocity field, $\boldsymbol{r}$ the vector separating the two points in physical space, and the operator $\langle\rangle$ denotes ensemble average. By virtue of incompressibility, the tensor $\hat{R}_{ij}(\boldsymbol{k},t)$ can be generated from scalar spectra according to:

$$\hat{R}_{ij}(\boldsymbol{k},t) = \mathcal{E}(\boldsymbol{k},t)P_{ij}(\boldsymbol{k}) + \Re\left(Z(\boldsymbol{k},t)N_i(\boldsymbol{k})N_j(\boldsymbol{k})\right) \tag{1}$$

where $\mathcal{E}(\boldsymbol{k},t)$ is the energy density in 3D Fourier space, from which is quantified directional anisotropy, and $Z(\boldsymbol{k},t)$ characterizes polarization anisotropy. $P_{ij}(\boldsymbol{k})$ denotes the projection operator onto the plan perpenticular to $\boldsymbol{k}$ and $N_i(\boldsymbol{k})$ refers to the helical modes. By virtue of this decomposition, the governing equation of $\hat{R}_{ij}$ is equivalent to a set of two equations in terms of $\mathcal{E}$ and Z. These generalized Lin equations include exact terms, linear in terms of $\mathcal{E}$ and Z, inherited from Rapid Distortion Theory, and call into play spectral transfer terms denoted $T^{(\mathcal{E})}(\boldsymbol{k},t)$ and $T^{(Z)}(\boldsymbol{k},t)$, which are mediated by third-order correlations and need to be closed. Nonlocal expression of the latter terms (in terms of $\mathcal{E}$ and Z) is achieved by mean of an EDQNM closure assumption: it is applied to the equation for three-point third-order correlations. Quasi-Normal (QN) closure for fourth-order moments is corrected by an eddy-damping (ED) term, assuming that the departure from Gaussianity is moderate so that fourth-order cumulants act as a relaxation of third-order ones. Finally, a Markovianization (M) procedure, which discards the explicit effets of production by mean-velocity gradients in third-order terms, is employed in order to obtain tractable expressions for $T^{(\mathcal{E})}(\boldsymbol{k},t)$ and $T^{(Z)}(\boldsymbol{k},t)$.

GOVERNING EQUATIONS FOR SPHERICALLY-AVERAGED DESCRIPTORS

The $\boldsymbol{k}$ dependence of $\mathcal{E}(\boldsymbol{k},t)$ and $Z(\boldsymbol{k},t)$ makes their governing equations, once closed, difficult to be solved from a practical point of view. In order to circumvent these difficulties, one solution is to integrate analytically the latter over a sphere of radius k. This analytical integration requires a representation of the tensor $\hat{R}_{ij}(\boldsymbol{k},t)$. Here, we choose the representation described in [1] which is written in terms of $\mathcal{E}(\boldsymbol{k},t)$ and $Z(\boldsymbol{k},t)$ as:

$$\mathcal{E}(\boldsymbol{k},t) = \frac{E(k,t)}{4\pi k^2}\left(1 - 15H_{ij}^{(dir)}(k,t)\frac{k_ik_j}{k^2}\right) \;\; ; \;\; Z(\boldsymbol{k},t) = \frac{5}{2}\frac{E(k,t)}{4\pi k^2}H_{ij}^{(pol)}(k,t)N_i^*(\boldsymbol{k})N_j^*(\boldsymbol{k}) \tag{2}$$

Equation (2) involves the tensors $H_{ij}^{(dir)}(k,t)$ and $H_{ij}^{(pol)}(k,t)$ which depend only on k and measure respectively directional and polarization anisotropies according to:

$$2E(k,t)H_{ij}^{(dir)}(k,t) = \iint_{S_k}\hat{R}_{ij}^{(dir)}(\boldsymbol{k},t)\mathrm{d}^2\boldsymbol{k} \;\; ; \;\; 2E(k,t)H_{ij}^{(pol)}(k,t) = \iint_{S_k}\hat{R}_{ij}^{(pol)}(\boldsymbol{k},t)\mathrm{d}^2\boldsymbol{k} \tag{3}$$

where $\hat{R}_{ij}^{(dir)}(\boldsymbol{k},t)$ and $\hat{R}_{ij}^{(pol)}(\boldsymbol{k},t)$ refer respectively to the directional and polarization parts of $\hat{R}_{ij}(\boldsymbol{k},t)$. $E(k,t)$ is the kinetic energy spectrum and $\iint_{S_k}\mathrm{d}^2\boldsymbol{k}$ denotes integration over a spherical shell of radius k. The degree of anisotropy permitted by the representation (2) is restricted by realizability requirements. Injecting this representation into the generalized Lin equations allows to integrate analytically the latter over a sphere of radius k and to derive a system of equations in terms of the spherically-averaged descriptors $E(k,t)$, $H_{ij}^{(dir)}(k,t)$ and $H_{ij}^{(pol)}(k,t)$. The latter completely determine the second-order spectral tensor $\hat{R}_{ij}(\boldsymbol{k},t)$, restricted to moderate anisotropy. The resulting system is of the form:

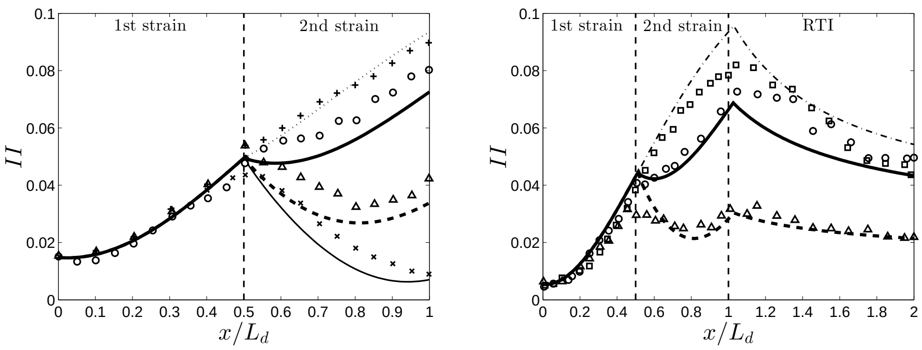
$$\left(\frac{\partial}{\partial t} + 2\nu k^2\right) E(k,t) = \mathcal{S}^L(k,t) + T(k,t) \ \ ; \ \ \left(\frac{\partial}{\partial t} + 2\nu k^2\right) E(k,t) H_{ij}^{(dir)}(k,t) = \mathcal{S}_{ij}^{L(dir)}(k,t) + \mathcal{S}_{ij}^{NL(dir)}(k,t) \quad (4)$$

$$\left(\frac{\partial}{\partial t} + 2\nu k^2\right) E(k,t) H_{ij}^{(pol)}(k,t) = \mathcal{S}_{ij}^{L(pol)}(k,t) + \mathcal{S}_{ij}^{NL(pol)}(k,t) \quad (5)$$

The tensors $\mathcal{S}^L(k,t)$, $\mathcal{S}_{ij}^{L(dir)}(k,t)$ and $\mathcal{S}_{ij}^{L(pol)}(k,t)$ account for the interactions with the mean flow and derive from the linear terms in the generalized Lin equations, whereas $T(k,t)$, $\mathcal{S}_{ij}^{NL(dir)}(k,t)$ and $\mathcal{S}_{ij}^{NL(pol)}(k,t)$ correspond to nonlinear transfer terms and derive from the expressions of $T^{(\mathcal{E})}(\boldsymbol{k},t)$ and $T^{(Z)}(\boldsymbol{k},t)$ closed by the EDQNM approximation.

VALIDATION AND RESULTS

The predictions of the present model are compared with the experiments of Gence and Mathieu ([2],[3]). In these studies, two successive plane strains with different orientations are applied to grid-generated turbulence, and the return to isotropy (RTI) of the turbulence thus obtained is investigated in the latter experiment. Experimental datas for the downstream evolution of the invariant $II = b_{ij}b_{ji}$, where b_{ij} is the dimensionless deviatoric part of the Reynolds stress tensor, are reported in figure 1 along with numerical results obtained with the system of governing equations (4)-(5). The latter allows to correctly capture the evolution of anisotropy and a good agreement between experimental and numerical results is observed, especially taking into account the uncertainty in the initial condition and a possible homogeneity fault in the experimental device. Some of these results can be predicted by the best 'full RSM' single-point models, but often they need the addition of coupled structure tensors (in line with Kassinos *et al.*, see [1, 5]). For instance, the directional anisotropy tensor $E(k,t)H_{ij}^{(dir)}(k,t)$ in eq. (3) gives the spectrum of their 'dimensionality tensor'. Accordingly, the prediction of such a sale-by-scale (k by k here) information for distribution of energy and anisotropy is the most interesting added value of our model, as a true two-point one. Many other results will be presented and discussed, especially for non-axisymmetric turbulence, possibly subjected to rotational mean flows (e.g pure plane shear) among arbitrary mean velocity gradients for which our complete model equations are valid.



(a) (b)

Figure 1. Evolution of the invariant II versus the position in the distorting duct of length L_d for the experiments [2] (a) and [3] (b). Symbols correspond to experimental datas and lines are obtained with the system of governing equations (4)-(5). Various values of the angle α between the principal axes of the two successive plane strains are investigated: $\alpha = 0$ (□,), $\alpha = \frac{\pi}{8}$ (+,), $\alpha = \frac{\pi}{4}$ (○,), $\alpha = \frac{3\pi}{8}$ (△,) and $\alpha = \frac{\pi}{2}$ (×,).

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