NONHELICAL INVERSE TRANSFER OF A DECAYING TURBULENT MAGNETIC FIELD

Axel Brandenburg[1,2], Tina Kahniashvili[3,4] & Alexander G. Tevzadze[5]
[1]*Nordita, KTH Royal Institute of Technology and Stockholm University, 10691 Stockholm, Sweden*
[2]*Department of Astronomy, AlbaNova University Center, Stockholm University, 10691 Stockholm, Sweden*
[3]*The McWilliams Center for Cosmology, Carnegie Mellon University, Pittsburgh, PA 15213, USA*
[4]*Department of Physics, Laurentian University, Sudbury, ON P3E 2C, Canada*
[5]*Abastumani Astrophysical Observatory, Ilia State University, Tbilisi, GE-0194, Georgia*
[6]*Faculty of Exact and Natural Sciences, Tbilisi State University, 1 Chavchavadze Ave., Tbilisi, 0128, Georgia*

Abstract In the presence of magnetic helicity, inverse transfer from small to large scales is well known in magnetohydrodynamic (MHD) turbulence and has applications in astrophysics, cosmology, and fusion plasmas. Using high resolution direct numerical simulations of magnetically dominated self-similarly decaying MHD turbulence, we report a similar inverse transfer even in the absence of magnetic helicity. We compute for the first time spectral energy transfer rates to show that this inverse transfer is about half as strong as with helicity, but in both cases the magnetic gain at large scales results from velocity at similar scales interacting with smaller-scale magnetic fields. This suggests that both inverse transfers are a consequence of a universal mechanisms for magnetically dominated turbulence. Possible explanations include inverse cascading of the mean squared vector potential associated with local near two-dimensionality and the shallower k^2 subinertial range spectrum of kinetic energy forcing the magnetic field with a k^4 subinertial range to attain larger-scale coherence. The inertial range shows a clear k^{-2} spectrum and is the first example of fully isotropic magnetically dominated MHD turbulence exhibiting weak turbulence scaling.

DECAY SIMULATIONS

We solve the compressible MHD equations for $\boldsymbol{u}$, the gas density ρ at constant sound speed c_s, and the magnetic vector potential $\boldsymbol{A}$, so $\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A}$. Following our earlier work [1, 2, 3], we initialize our decaying DNS by restarting them from a snapshot of a driven DNS, where a random forcing was applied in the evolution equation for $\boldsymbol{A}$ rather than $\boldsymbol{u}$. To allow for sufficient scale separation, we take $k_0/k_1 = 60$. We use the PENCIL CODE (http://pencil-code.googlecode.com/) at a resolution of 2304^3 meshpoints on 9216 processors. The code uses sixth order finite differences and a third order accurate time stepping scheme. Our magnetic and kinetic energy spectra are normalized such that $\int E_\mathrm{M}(k,t)\,dk = \mathcal{E}_\mathrm{M}(t) = v_\mathrm{A}^2/2$ and $\int E_\mathrm{K}(k,t)\,dk = \mathcal{E}_\mathrm{K}(t) = u_\mathrm{rms}^2/2$ are magnetic and kinetic energies per unit mass. The magnetic integral scale is defined as $\xi_\mathrm{M} = k_\mathrm{M}^{-1}(t) = \int k^{-1} E_\mathrm{M}(k,t)\,dk/\mathcal{E}_\mathrm{M}(t)$. Time is given in initial Alfvén times $\tau_\mathrm{A} = (v_{\mathrm{A}0} k_0)^{-1}$.

In Fig. 1 we show $E_\mathrm{M}(k,t)$ and $E_\mathrm{K}(k,t)$. We find an inertial range with weak turbulence scaling,

$$E_\mathrm{WT}(k,t) = C_\mathrm{WT}(\epsilon v_\mathrm{A} k_\mathrm{M})^{1/2} k^{-2}, \qquad (1)$$

where $k_\mathrm{M}^{-1}(t) = \int k^{-1} E_\mathrm{M}(k,t)\,dk/\mathcal{E}_\mathrm{M}(t)$ is the integral scale and k_M has been used in place of $k_\parallel$. The prefactor is $C_\mathrm{WT} \approx 1.9$; see the inset. In agreement with earlier work [2, 4], $\mathcal{E}_\mathrm{M}$ decays like t^{-1}.

At small wavenumbers the k^4 and k^2 subinertial ranges respectively for $E_\mathrm{M}(k,t)$ and $E_\mathrm{K}(k,t)$ are carried over from the initial conditions. The k^4 Batchelor spectrum is in agreement with the causality requirement [5, 6] for the divergence-free vector field $\boldsymbol{B}$. The velocity is driven entirely by the magnetic field and follows a white noise spectrum, $E_\mathrm{K}(k) \propto k^2$ [6]. The resulting difference in the scaling implies that, although magnetic energy dominates over kinetic, the two spectra must cross at sufficiently small wavenumbers. This idea may also apply to incompressible [7] and relativistic [8] simulations, where inverse nonhelical transfer has recently been confirmed.

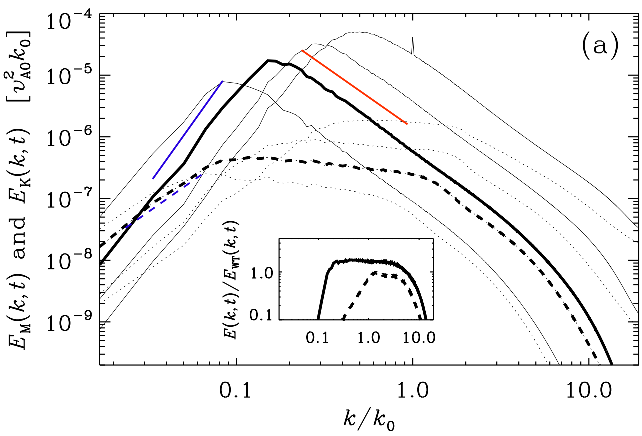

Figure 1. Magnetic (solid lines) and kinetic (dashed lines) energy spectra for Run A at times $t/\tau_\mathrm{A} = 18$, 130, 450, and 1800; the time $t/\tau_\mathrm{A} = 450$ is shown as bold lines. The straight lines indicate the slopes k^4 (solid, blue), k^2 (dashed, blue), and k^{-2} (red, solid). The inset show E_M and E_K compensated by E_WT.

NATURE OF INVERSE TRANSFER

To quantify the nature of inverse transfer we show in Fig. 2 representations of the spectral transfer function $T_{kpq} = \langle \boldsymbol{J}^k \cdot (\boldsymbol{u}^p \times \boldsymbol{B}^q)\rangle$ and compare with the corresponding helical case of Ref. [3], but with 1024^3 mesh points and at a comparable time. Here, the superscripts indicate the radius of a shell in wavenumber space of Fourier filtered vector

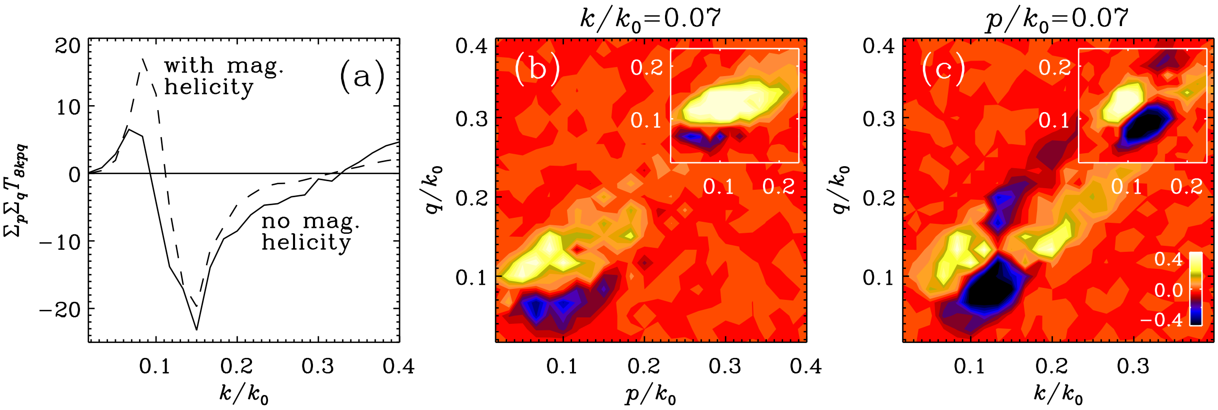

Figure 2. (Color online) Spectral transfer function T_{kpq}, (a) as a function of k and summed over all p and q, (b) as a function of p and q for $k/k_1 = 4$, and (c) as a function of k and q for $p/k_1 = 4$. The dashed line in (a) and the insets in (b) and (c) show the corresponding case for a DNS with helicity; both for $\mathrm{Pr_M} = 1$.

fields; see Ref. [9] for such an analysis in driven helical turbulence. The transfer function T_{kpq} quantifies the gain of magnetic energy at wavenumber k from interactions of velocities at wavenumber p and magnetic fields at wavenumber q. Fig. 2(a) shows a gain for $k/k_0 < 0.1$, which is about half of that for the helical case. The corresponding losses for $k/k_0 > 0.1$ are about equal in the two cases. In both cases, the magnetic gain at $k/k_0 = 0.07 = 4/60$ results from $\boldsymbol{u}^p$ with $0 < p/k_0 < 0.2$ interacting with $\boldsymbol{B}^q$ at $q/k_0 > 0.1$; see the light yellow shades in Fig. 2(b). Note that work done by the Lorentz force is $\langle \boldsymbol{u}^p \cdot (\boldsymbol{J}^k \times \boldsymbol{B}^q)\rangle = -T_{kpq}$. Thus, negative values of T_{kpq} quantify the gain of *kinetic* energy at wavenumber p from interactions of magnetic fields at wavenumbers k and q. Blue dark shades in Fig. 2(c) indicate therefore that the gain of kinetic energy at $p/k_0 = 0.07$ results from magnetic interactions at wavenumbers k and q of around $0.1\,k_0$. These results support the interpretation that the increase of spectral power at large scales is similar to the inverse transfer in the helical case; see [10] for information concerning the total energy transfer.

To exclude that the inverse energy transfer is a consequence of the invariance of magnetic helicity, $\mathcal{H}_\mathrm{M}(t) = \langle \boldsymbol{A}\cdot\boldsymbol{B}\rangle$, we compare ξ_M with its lower bound $\xi_\mathrm{M}^\mathrm{min} = |\mathcal{H}_\mathrm{M}|/2\mathcal{E}_\mathrm{M}$ [2]. In nonhelical MHD turbulence, ξ_M is known to grow like $t^{1/2}$ [2, 4]. Even though the initial condition was produced with nonhelical plane waves, we find $\mathcal{H}_\mathrm{M} \neq 0$ due to fluctuations. Since $\mathcal{H}_\mathrm{M}$ is conserved and $\mathcal{E}_\mathrm{M}$ decays like t^{-1}, $\xi_\mathrm{M}^\mathrm{min}$ grows linearly and faster than $\xi_\mathrm{M} \sim t^{1/2}$, so they will meet at $t/\tau_\mathrm{A} = 10^5$ and then continue to grow as $t^{-2/3}$, but at $t/\tau_\mathrm{A} = 10^3$ this cannot explain the inverse transfer. By contrast, we cannot exclude the possibility of the quasi two-dimensional mean squared vector potential, $\langle \boldsymbol{A}_\mathrm{2D}^2\rangle$, being approximately conserved [10]. This could explain the $\xi_\mathrm{M} \sim t^{1/2}$ scaling and the inverse transfer if the flow was locally two-dimensional [11].

Our results support the idea of the weak turbulence k^{-2} scaling for strong magnetic field that is here for the first time globally isotropic and not an imposed one [13]. At small scales, however, approximate equipartition is still possible. The decay is slower than for usual MHD turbulence which is arguably governed by the Loitsyansky invariant [14]. Future investigations of the differences between these types of turbulence are warranted [10]. Interestingly, the extended plateau in the velocity spectrum around the position of the magnetic peak may be important for producing observationally detectable broad gravitational wave spectra [15].

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